CS4620/5620: Pipeline and Transformations

Professor: Kavita Bala

Course mechanics

Web  http://www.cs.cornell.edu/Courses/cs4620

Teaching Assistants (2 Ph.D., 2 ugrad, 1 nice guy)
  * John deCorato
  * Pramook Khungurn
  * Sean Ryan
  * Dan Schroeder
  * Nick Savva?

Piazza
Please sign up
Workload

- CS 4620/5620
  - 3 Homeworks
  - 4 programming assignments
  - No penalty for 1 late homework, then 10% per day

- CS 4621/5621
  - 3-4 programming assignments

- 2 prelims, no finals
  - Schedule will be updated

Course mechanics

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Mailing lists, ... etc. all on the web page

Practicum
- Not this Friday
- Will send out mail when a practicum is planned
Topics

- Graphics pipeline
- Geometric transformations
- Modeling in 2D and 3D
- Rendering 3D scenes
  - GPU and ray tracing
- Animation
- Images and image processing
  (sampling and reconstruction)
- Color science

Graphics pipeline

- rasterization
- interpolation
- z-buffer
- vertex and fragment ops
- texture mapping
**Geometric transformations**

- affine transforms
- perspective transforms
- viewing

rotate, then translate

translate, then rotate

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**Modeling**

- splines
- parametric surfaces
- triangle meshes
Rendering

- ray tracing
- shading & shadows
- transparency

Ray tracing

viewer (eye) -- viewing ray --> visible point

light source --> illumination

objects in scene
Object-order vs. Image-order

- Object-order
  
  for each triangle t {
    find pixels covered by t
    \( c(x,y) = \text{shade (visible point)} \)
  }

- Image-order
  
  for each pixel \( p(x,y) \) {
    intersect ray through \( p \) with scene
    \( c(x,y) = \text{shade (visible point)} \)
  }

- Hardware pipeline

- Ray tracers

Animation

- key frame animation
- subdivision surfaces
- physics-based animation
- particle systems
Images

- What is an image?
- Compositing
- Resampling

Graphics Pipeline

APPLICATION

COMMAND STREAM

GEOMETRY PROCESSING

TRANSFORMED GEOMETRY

RASTERIZATION

FRAGMENTS

FRAGMENT PROCESSING

FRAMEBUFFER IMAGE

DISPLAY
Math review

• Read:
  – Tiger, Chapter 2, 5: Misc Math, Linear Algebra
  – Gortler, Chapter 1, 2: Linear

• Vectors and points
• Vector operations
  – addition
  – scalar product
• More products
  – dot product
  – cross product
• Bases and orthogonality

• Vectors and points
  – P = (x, y, z)
  – V = (a, b, c)
• Vector operations
  – addition
  – scalar product
• Point operations
  – subtraction
Math review

• Vectors and points
  – P = (x, y, z)
  – V = (a, b, c)

• More products
  – dot product
    • geometric interpretation
  – cross product
    • geometric interpretation

Math review

• Bases and orthogonality

• Basis: 3 orthonormal axes
  – Unit length
  – Mutually perpendicular

• Represent a point/vector in the basis
Math review

- Linear transformations
- Matrices
  - Matrix-vector multiplication
  - Matrix-matrix multiplication
- Geometry of curves in 2D
  - Implicit representation
  - Explicit representation

Implicit representations

- Equation to tell whether v is on the curve
  \[ \{ v \mid f(v) = 0 \} \]
- Example: 2D line (orthogonal to u, distance k from 0)
  \[ \{ v \mid v \cdot u + k = 0 \} \]
- Example: circle (center p, radius r)
  \[ \{ v \mid (v - p) \cdot (v - p) - r^2 = 0 \} \]
Explicit representations

• Also called parametric
• Equation to map domain into plane
  \( \{ f(t) \mid t \in D \} \)
• Example: line (passes through \( p \), parallel to \( u \))
  \( \{ p + tu \mid t \in \mathbb{R} \} \)
• Example: circle (center \( p \), radius \( r \))
  \( \{ p + r[ \cos t \ \sin t ]^T \mid t \in [0, 2\pi) \} \)
• Like tracing out the path of a particle over time
• Variable \( t \) is the “parameter”

Transforming geometry

• Move a subset of the plane using a mapping from the plane to itself
  \( \cdot S \rightarrow \{ T(v) \mid v \in S \} \)
Transforming geometry

- Move a subset of the plane using a mapping from the plane to itself
  \[ S \rightarrow \{ T(v) \mid v \in S \} \]
- Parametric representation:
  \[ \{ f(t) \mid t \in D \} \rightarrow \{ T(f(t)) \mid t \in D \} \]
- Implicit representation:
  \[ \{ v \mid f(v) = 0 \} \rightarrow \{ T(v) \mid f(v) = 0 \} = \{ v \mid f(T^{-1}(v)) = 0 \} \]

Translation

- Simplest transformation: \[ T(v) = v + u \]
- Inverse: \[ T^{-1}(v) = v - u \]
- Example of transforming circle
Linear transformations using matrices

• One way to define a transformation is by matrix multiplication:
  \[ T(v) = Mv \]

• Such transformations are linear, which is to say:
  \[ T(au + v) = aT(u) + T(v) \]
  (and in fact all linear transformations can be written this way)
Geometry of 2D linear trans.

- 2x2 matrices have simple geometric interpretations
  - uniform scale
  - non-uniform scale
  - reflection
  - shear
  - rotation

Linear transformation gallery

- Uniform scale
  \[
  \begin{bmatrix}
  s & 0 \\
  0 & s
  \end{bmatrix}
  \begin{bmatrix}
  x \\
  y
  \end{bmatrix}
  =
  \begin{bmatrix}
  sx \\
  sy
  \end{bmatrix}
  \]

\[
\begin{bmatrix}
  1.5 & 0 \\
  0 & 1.5
\end{bmatrix}
\]

\[R\] to \[R\]
Linear transformation gallery

- Nonuniform scale

\[
\begin{bmatrix}
s_x & 0 \\
0 & s_y
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
s_xx \\
leon
\end{bmatrix}
= 
\begin{bmatrix}
1.5 & 0 \\
0 & 0.8
\end{bmatrix}
\]

Linear transformation gallery

- Reflection
  - can consider it a special case of nonuniform scale

\[
\begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix}
\]
Linear transformation gallery

• Shear
\[
\begin{bmatrix}
1 & a \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
x + ay \\
y \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 0.5 \\
0 & 1 \\
\end{bmatrix}
\]

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Linear transformation gallery

• Rotation
\[
\begin{bmatrix}
cos \theta & -sin \theta \\
sin \theta & cos \theta \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
x \cos \theta - y \sin \theta \\
x \sin \theta + y \cos \theta \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
0.866 & -.05 \\
0.5 & 0.866 \\
\end{bmatrix}
\]

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