Topics

- Graphics pipeline
- Rendering 3D scenes
  - ray tracing
  - GPU
- Images and image processing
  (featuring sampling and reconstruction)
- Geometric transformations
- Modeling in 2D and 3D
- Animation
- Color science

Graphics pipeline

- rasterization
- interpolation
- z-buffer
- vertex and fragment ops

Object-order vs. Image-order

- Object-order
  
  ```
  for each triangle t {
      find pixels covered by t
      c(x,y) = shaded result
  }
  ```

- Image-order
  
  ```
  for each pixel p=(x,y) {
      intersect ray through p with scene
      c(x,y) = shade (visible pixels)
  }
  ```

Workload

- CS 4620/5620
  - 4-5 Homeworks
  - 2-3 programming assignments
  - No penalty for 1 late homework, then 10% per day

- CS 4621/5621
  - 4-3 programming assignments

- 2 prelims, no finals
  - Schedule will be online shortly

Ray tracing

- Image-order
  
  ```
  for each pixel p=(x,y) {
      intersect ray through p with scene
      c(x,y) = shade (visible pixels)
  }
  ```

- Object-order
  
  ```
  for each triangle t {
      find pixels covered by t
      c(x,y) = shaded result
  }
  ```
Rasterizing lines

- Define line as a rectangle
- Specify by two endpoints
- Ideal image: black inside, white outside

Math review

- **Read:**
  - Chapter 2: Miscellaneous Math
  - Chapter 5: Linear Algebra
- Vectors and points
- Vector operations
  - addition
  - scalar product
- More products
  - dot product
  - cross product
- Bases and orthogonality

Math review

- Vectors and points
  - \( P = (x, y, z) \)
  - \( V = (a, b, c) \)
- Vector operations
  - addition
  - scalar product

Math review

- Notation for sets, functions, mappings
- Linear transformations
- Matrices
  - Matrix-vector multiplication
  - Matrix-matrix multiplication
- Geometry of curves in 2D
  - Implicit representation
  - Explicit representation
Implicit representations

- Equation to tell whether \( \mathbf{v} \) is on the curve
  \[ \{ \mathbf{v} \mid f(\mathbf{v}) = 0 \} \]
- Example: 2D line (orthogonal to \( \mathbf{u} \), distance \( k \) from \( \mathbf{0} \))
  \[ \{ \mathbf{v} \mid \mathbf{v} \cdot \mathbf{u} + k = 0 \} \]
- Example: circle (center \( \mathbf{p} \), radius \( r \))
  \[ \{ \mathbf{v} \mid (\mathbf{v} - \mathbf{p}) \cdot (\mathbf{v} - \mathbf{p}) - r^2 = 0 \} \]

Explicit representations

- Also called parametric
- Equation to map domain into plane
  \[ \{ f(t) \mid t \in D \} \]
- Example: line (containing \( \mathbf{p} \), parallel to \( \mathbf{u} \))
  \[ \{ \mathbf{p} + t\mathbf{u} \mid t \in \mathbb{R} \} \]
- Example: circle (center \( \mathbf{p} \), radius \( r \))
  \[ \{ \mathbf{p} + r[\cos t \sin t]^T \mid t \in [0, 2\pi] \} \]
- Like tracing out the path of a particle over time
- Variable \( t \) is the “parameter”

Transforming geometry

- Move a subset of the plane using a mapping from the plane to itself
  \[ S \rightarrow \{ T(\mathbf{v}) \mid \mathbf{v} \in S \} \]

Transforming geometry

- Move a subset of the plane using a mapping from the plane to itself
  \[ S \rightarrow \{ T(\mathbf{v}) \mid \mathbf{v} \in S \} \]
- Parametric representation:
  \[ \{ f(t) \mid t \in D \} \rightarrow \{ T(f(t)) \mid t \in D \} \]
- Implicit representation:
  \[ \{ \mathbf{v} \mid f(\mathbf{v}) = 0 \} \rightarrow \{ T(\mathbf{v}) \mid f(\mathbf{v}) = 0 \} \]
  \[ = \{ \mathbf{v} \mid f(T^{-1}(\mathbf{v})) = 0 \} \]

Translation

- Simplest transformation:  \( T(\mathbf{v}) = \mathbf{v} + \mathbf{u} \)
- Inverse:  \( T^{-1}(\mathbf{v}) = \mathbf{v} - \mathbf{u} \)
- Example of transforming circle

Linear transformations

- One way to define a transformation is by matrix multiplication:
  \[ T(\mathbf{v}) = M\mathbf{v} \]
- Such transformations are linear, which is to say:
  \[ T(a\mathbf{u} + \mathbf{v}) = aT(\mathbf{u}) + T(\mathbf{v}) \]
  (and in fact all linear transformations can be written this way)
Geometry of 2D linear trans.

- 2x2 matrices have simple geometric interpretations
  - uniform scale
  - non-uniform scale
  - reflection
  - shear
  - rotation

Linear transformation gallery

- Uniform scale
  \[
  \begin{pmatrix}
  s & 0 \\
  0 & s
  \end{pmatrix}
  \begin{pmatrix}
  x \\
  y
  \end{pmatrix}
  =
  \begin{pmatrix}
  sx \\
  sy
  \end{pmatrix}
  \]
  \[
  \begin{pmatrix}
  1.5 & 0 \\
  0 & 1.5
  \end{pmatrix}
  \]

- Nonuniform scale
  \[
  \begin{pmatrix}
  s_x & 0 \\
  0 & s_y
  \end{pmatrix}
  \begin{pmatrix}
  x \\
  y
  \end{pmatrix}
  =
  \begin{pmatrix}
  s_x x \\
  s_y y
  \end{pmatrix}
  \]
  \[
  \begin{pmatrix}
  1.5 & 0 \\
  0 & 0.8
  \end{pmatrix}
  \]

- Reflection
  - can consider it a special case of nonuniform scale
  \[
  \begin{pmatrix}
  -1 & 0 \\
  0 & 1
  \end{pmatrix}
  \]

- Shear
  \[
  \begin{pmatrix}
  1 & a \\
  0 & 1
  \end{pmatrix}
  \begin{pmatrix}
  x \\
  y
  \end{pmatrix}
  =
  \begin{pmatrix}
  x + ay \\
  y
  \end{pmatrix}
  \]
  \[
  \begin{pmatrix}
  1 & 0.5 \\
  0 & 1
  \end{pmatrix}
  \]

- Rotation
  \[
  \begin{pmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
  \end{pmatrix}
  \begin{pmatrix}
  x \\
  y
  \end{pmatrix}
  =
  \begin{pmatrix}
  x \cos \theta - y \sin \theta \\
  x \sin \theta + y \cos \theta
  \end{pmatrix}
  \]
  \[
  \begin{pmatrix}
  0.866 & -0.5 \\
  0.5 & 0.866
  \end{pmatrix}
  \]