Subdivision Overview

CS 4621 Lecture 1

Subdivision rules for curves

• New vertex positions are linear combinations of old positions

Subdivision curves

Figure 2.1: Example of subdivision for curves in the plane. On the left 4 points connected with straight line segments. To the right of it a refined version: 3 new points have been inserted “inbetween” the old points and again a piecewise linear curve connecting them is drawn. After two more steps of subdivision the curve starts to become rather smooth.

Subdivision surfaces

Figure 2.2: Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.
Generalizing from curves to surfaces

- Two parts to subdivision process
- Subdividing the mesh (computing new topology)
  - For curves: replace every segment with two segments
  - For surfaces: replace every face with some new faces
- Positioning the vertices (computing new geometry)
  - For curves: two rules (one for odd vertices, one for even)
    - New vertex’s position is a weighted average of positions of old vertices that are nearby along the sequence
  - For surfaces: two kinds of rules (still called odd and even)
    - New vertex’s position is a weighted average of positions of old vertices that are nearby in the mesh

Subdivision of meshes

- Quadrilaterals
  - Catmull-Clark 1978
- Triangles
  - Loop 1987

Loop regular rules

Catmull-Clark regular rules
Creases

- With splines, make creases by turning off continuity constraints
- With subdivision surfaces, make creases by marking edges “sharp”
  - use different rules for vertices with sharp edges
  - these rules produce B-splines that depend only on vertices along crease

**Boundaries**

- At boundaries the masks do not work
  - mesh is not manifold; edges do not have two triangles
- Solution: same as crease
  - shape of boundary is controlled only by vertices along boundary

Extraordinary vertices

- Vertices that don’t have the “standard” valence
- Unavoidable for most topologies
- Difference from splines
  - treatment of extraordinary vertices is really the only way subdivision surfaces are different from spline patches

Full Loop rules (triangle mesh)
Full Catmull-Clark rules (quad mesh)

- Mask for a face vertex
  - $\frac{1}{4}$
  - $\frac{1}{4}$
- Mask for an edge vertex
  - $\frac{1}{16}$
  - $\frac{1}{16}$
  - $\frac{1}{8}$
- Mask for a boundary odd vertex
  - $\frac{1}{2}$

Loop Subdivision Example

- Control polyhedron

Odd subdivision mask

Loop Subdivision Example

- Refined control polyhedron
Loop Subdivision Example

subdivision level 1

Loop Subdivision Example

even subdivision mask (ordinary vertex)

Loop Subdivision Example

subdivision level 1

Loop Subdivision Example

even subdivision mask (extraordinary vertex)
Loop Subdivision Example

subdivision level 1

subdivision level 2

subdivision level 3
Loop Subdivision Example

subdivision level 4

limit surface

Relationship to splines

- In regular regions, behavior is identical
- At extraordinary vertices, achieve $C^1$
  - near extraordinary, different from splines
- Linear everywhere
  - mapping from parameter space to 3D is a linear combination of the control points
  - “emergent” basis functions per control point
    - match the splines in regular regions
    - “custom” basis functions around extraordinary vertices

Loop vs. Catmull-Clark

Loop

Catmull-Clark
**Loop vs. Catmull-Clark**

*Loop* (after splitting faces)

*Catmull-Clark*

Loop with creases

Catmull-Clark with creases

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**Loop vs. Catmull-Clark**

*Loop* (after splitting faces)

*Catmull-Clark*
Geri’s Game

- Pixar short film to test subdivision in production
  - Catmull-Clark (quad mesh) surfaces
  - complex geometry
  - extensive use of creases
  - subdivision surfaces to support cloth dynamics

[DeRose et al. SIGGRAPH 1998]