1 IR lowering

After doing the translations described thus far, we arrive at an IR version of the program code. However, this code is still not very assembly-like in various respects: it contains complex expressions and complex statements (because of SEQ), and statements inside expressions (because of ESEQ). Statements inside expressions mean that an expression can cause side effects, and statements can cause multiple side effects. Another difference is the CJUMP statement can jump to two different places, whereas in assembly, a conditional branch instruction falls through to the next instruction if the condition is false.

To bring the IR closer to assembly we can flatten statements and expressions, resulting in a canonical, lower-level IR in which:

- There are no nested SEQs.
- There are no ESEQs.
- Each statement contains at most one side effect (or call).
- The “false” target of a CJUMP always goes to the very next statement.
- All CALL nodes appear at the top of the tree, essentially as a kind of IR statement.

2 Canonical IR

We’ll express this IR lowering as yet another syntax-directed translation. Unlike the previous translations, the source and target of the translation are both varieties of IR. We can describe the target language with a grammar. First, since there are no nested SEQ nodes, the code becomes a linear sequence of other kinds nodes of nodes. For brevity, we will write this sequence as \( s_1; s_2; \ldots; s_n \), equivalent to source-level SEQ\((s_1, \ldots, s_n)\). The grammar for top-level statements is then:

\[
s ::= \text{MOVE}(\text{dest}, e) \\
| \text{MOVE}(\text{TEMP}(t), \text{CALL}(f, e_1, \ldots, e_n)) \\
| \text{EXP}((\text{CALL}(f, e_1, \ldots, e_n)) \\
| \text{JUMP}(e) \\
| \text{CJUMP}(e, l_1, l_2) \\
| \text{LABEL}(l) | \text{RETURN}
\]

Expressions \( e \) are the same as before but may not include ESEQ or CALL nodes.

3 Translation functions

We express the lowering transformation using two syntax-directed translation functions:

\[ \mathcal{L}[s] \text{ translates an IR statement } s \text{ to a sequence } s_1; \ldots; s_n \text{ of canonical IR statements that have the same effect. We write } \mathcal{L}[s] = s_1; \ldots; s_n, \text{ or as a shorthand, } \mathcal{L}[s] = \vec{s}. \]

\[ \mathcal{L}[e] \text{ translates an IR expression } e \text{ to a sequence of canonical IR statements } \vec{s} \text{ that have the same effect, and an expression } e' \text{ that has the same value if evaluated after the whole sequence of statements } \vec{s}. \text{ We write } \mathcal{L}[e] = \vec{s}; e' \text{ to denote this.} \]
Given these translation functions, we can apply $\mathcal{L}[s]$ to the IR for each function body to obtain a linear sequence of IR statements representing the function code. This will get us much closer to assembly code for each function.

### 4 Lowering expressions

Our goal is to convert an expression into one that has no side effects, the side effects being factored out into a sequence of statements that are hoisted to the top level of the generated code. If we use $\bullet$ to represent an empty sequence of statements, expressions that already have no side effects are trivial to lower. We can write these translations as an inference rule:

\[
\begin{align*}
  e &= \text{CONST}(i) \lor e = \text{NAME}(l) \lor e = \text{TEMP}(t) \\
  \mathcal{L}[e] &= \bullet; e
\end{align*}
\]

For other simple expressions, we just hoist the statements out of subexpressions:

\[

\begin{align*}
  \mathcal{L}[\text{MEM}(e)] &= \vec{s}; \text{MEM}(e') \\
  \mathcal{L}[\text{JUMP}(e)] &= \vec{s}; \text{JUMP}(e') \\
  \mathcal{L}[\text{CJUMP}(e, l_1, l_2)] &= \vec{s}; \text{CJUMP}(e', l_1, l_2)
\end{align*}
\]

Since we can hoist statements, we can eliminate \texttt{ESEQ} nodes completely:

\[

\begin{align*}
  \mathcal{L}[s] &= \vec{s} \\
  \mathcal{L}[\text{ESEQ}(s, e)] &= \vec{s}; \vec{s'}; e'
\end{align*}
\]

Call nodes must be hoisted too because they can cause side effects. And the side effects of computing arguments must be prevented from changing the computation of other arguments:

\[

\begin{align*}
  \mathcal{L}[e_i] &= \vec{s_i}; e'_i \quad \forall i \in 0..n \\
  \mathcal{L}[\text{CALL}(e_0, e_1, \ldots, e_n)] &= \vec{s}_0: \\
  &\phantom{=} \quad \text{MOVE(TMP}(t_0), e'_0); \\
  &\phantom{=} \quad \vec{s}_1: \\
  &\phantom{=} \quad \text{MOVE(TMP}(t_1), e'_1); \\
  &\phantom{=} \quad \ldots \\
  &\phantom{=} \quad \vec{s}_n: \\
  &\phantom{=} \quad \text{MOVE(TMP}(t_n), e'_n); \\
  &\phantom{=} \quad \text{MOVE(TMP}(t), \text{CALL}(t_0, t_1, \ldots, t_n)); \\
  &\phantom{=} \quad \text{TEMP}(t)
\end{align*}
\]

Binary operations are surprisingly tricky. A naive first cut at a lowering translation would be to simply hoist the side effects of both expressions to the top level:

\[

\begin{align*}
  \mathcal{L}[e_1] &= \vec{s_1}; e'_1 \\
  \mathcal{L}[e_2] &= \vec{s_2}; e'_2 \\
  \mathcal{L}[\text{OP}(e_1, e_2)] &= \vec{s_1}; \vec{s_2}; \text{OP}(e'_1, e'_2)
\end{align*}
\]

This naive translation has some nice features: beyond its simplicity, it also keeps the expressions $e'_1$ and $e'_2$ together as part of a larger expression, which helps with the quality of later code generation.

However, there’s a problem with this naive translation: it reorders the evaluation of $s_2$ and $e'_1$. Let us assume that the language, like Java, dictates that expressions are evaluated in left-to-right order. If executing statements $s_2$ changes the value that $e'_1$ computes, the lowering translation will change the behavior of the code. For example, $e_2$ might use a function call that changes the contents of an array that $e_1$ reads from.
Therefore, the rule above can only be used if the result of evaluating the expression \( OP(e_1, e_2) \) does not depend on the order in which \( e_1 \) and \( e_2 \) are evaluated. In this case, we say that \( e_1 \) and \( e_2 \) commute:

\[
\frac{\mathcal{L}[e_1] = s_1; e_1', \quad \mathcal{L}[e_2] = s_2; e_2'}{\mathcal{L}[OP(e_1, e_2)] = s_1; s_2; OP(e_1', e_2')}
\]

If they don’t commute, the solution is to evaluate \( e_1 \) first, store its result into a fresh temporary, and only then evaluate \( e_2 \):

\[
\frac{\mathcal{L}[e_1] = s_1; e_1', \quad \mathcal{L}[e_2] = s_2; e_2'}{\mathcal{L}[OP(e_1, e_2)] = s_1; MOVE(TMP(t_1), e_1'); s_2; OP(TMP(t_1), e_2')}
\]

We probably prefer the other rule when we can use it, because it keeps the subexpressions \( e_1' \) and \( e_2' \) together, possibly to be computed using the same assembly-language instruction. Breaking the code into more and smaller statements may eliminate some opportunities to generate efficient code. As we’ll see later when we do instruction selection, big expression trees are helpful for generating efficient assembly code.

5 Lowering statements

Recall that the lowering translation lifts all statements up into a single top-level sequence of statements.

Hence, we translate a sequence of statements by lowering each statement in the sequence to its own sequence of statements, and concatenating the resulting sequences into one big sequence:

\[
\mathcal{L}[SEQ(s_1, \ldots, s_n)] = \mathcal{L}[s_1]; \ldots; \mathcal{L}[s_n]
\]

To lower an \textbf{EXP} node, we just throw away the expression, because that is what \textbf{EXP} does. We write the translation rule as an inference rule:

\[
\frac{\mathcal{L}[e] = s; e'}{\mathcal{L}[\text{EXP}(e)] = s'}
\]

For statements such as \textbf{JUMP} and \textbf{CJUMP}, we flatten the expression to obtain a sequence of statements that are done before the jump:

\[
\frac{\mathcal{L}[e] = s; e'}{\mathcal{L}[\text{JUMP}(e)] = s; \text{JUMP}(e')} \quad \frac{\mathcal{L}[e] = s; e'}{\mathcal{L}[\text{CJUMP}(e, l_1, l_2)] = s; \text{CJUMP}(e', l_1, l_2)}
\]

Some statements we can leave alone:

\[
\mathcal{L}[\text{LABEL}(l)] = \text{LABEL}(l)
\]

A tricky case is \textbf{MOVE}. Assuming that the side effects of the two subexpressions are to be performed in left-to-right order, we’d like the following translation:

\[
\frac{\mathcal{L}[\text{DEST}] = s; dest', \quad \mathcal{L}[e] = s'; e'}{\mathcal{L}[\text{MOVE}(dest, e)] = s; s'; \text{MOVE}(dest', e')}
\]

But this translation is not safe in general. The problem is the same as with the translation of binary expressions: if the statements \( s' \) change the meaning of \( dest \), the translated program does something different from the original. The simpler \textbf{MOVE} rule above can only be used if \( e \) and \( dest \) commute.

Note that we haven’t defined what it means to lower a destination. This is trivial for temporaries, but \textbf{MEM} expressions can have a side effect to be hoisted out:

\[
\frac{\mathcal{L}[\text{TEMP}(t)] = \bullet; \text{TEMP}(t)}{\mathcal{L}[\text{MEM}(e)] = s; \text{MEM}(e')}
\]
Now, back to the MOVE rule. What if $e$ and dest don’t commute? They always commute if dest is a temporary, so we only need to worry about the case where dest is of the form $\text{MEM}(e_d)$. In that case we can capture the result of $e'$ in a fresh temporary:

$$
\mathcal{L}[e_d] = \vec{s}_d; e'_d \quad \mathcal{L}[e] = \vec{s}_e; e' \quad t \text{ is fresh}
$$

$$
\mathcal{L}[\text{MOVE}(\text{MEM}(e_d), e)] = \vec{s}_d; \text{MOVE}(\text{TEMP}(t), e'_d); \vec{s}_e; \text{MOVE}(\text{MEM}(\text{TEMP}(t)), e');
$$

## 6 Commuting statements and expressions

The rules for $OP$ and MOVE both rely on interchanging the order of a statement $s$ and an expression $e$. This can be done safely when the statement cannot alter the value of the expression. There are two ways in which this could happen: the statement could change the value of a temporary variable used by the expression, and the statement could change the value of a memory location used by the expression. It is easy to determine whether the statement updates a temporary used by the expression, because temporaries have unique names. Memory is harder because two memory location can be aliases. If the statement $s$ uses a memory location $\text{MEM}(e_1)$ as a destination, and the expression reads from the memory location $\text{MEM}(e_2)$, and $e_1$ might have the same value as $e_2$ at run time, we cannot safely interchanged the operations.

A simple, conservative approach is to assume that all MEM nodes are potentially aliases, so a statement and an expression that both use memory are never reordered. We can do better by exploiting the observation that two nodes of the form $\text{MEM}(\text{TEMP}(t) + \text{CONST}(k_1))$ and $\text{MEM}(\text{TEMP}(t) + \text{CONST}(k_2))$ cannot be aliases if $k_1 \neq k_2$.

To do a good job of reordering memory accesses, we want more help from analysis of the program at source level. Otherwise there is not enough information available at the IR level for alias analysis to be tractable.

A simple observation we can exploit is that if the source language is strongly typed, two MEM nodes with different types cannot be aliases. If we keep around source-level type information for each MEM node, which we might denote as $\text{MEM}_t(e)$, then these type annotations $t$ can help identify opportunities to reorder operations. Accesses to $\text{MEM}_t(e_1)$ and to $\text{MEM}_{t'}(e_2)$ cannot conflict if $t$ is not compatible with $t'$.

Sophisticated compilers incorporate some form of pointer analysis to determine which memory locations might be aliases. The typical output of pointer analysis is a label for each distinct MEM node, such that accesses to differently labeled MEM nodes cannot interfere with each other. The label could be as simple as an integer index, where $\text{MEM}_i$ and $\text{MEM}_j$ cannot be aliases unless $i = j$. We’ll see how to do such a pointer analysis in a later lecture.

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1We also need to know that the language run-time system will never map two virtual addresses to the same physical address.