1. Which of the following statements holds for every three languages $L_1$, $L_2$, $L_3$?
   (i) $(L_1 \cap L_2) L_3 = L_1 L_3 \cap L_2 L_3$
   (ii) $L_1^* = L_1^* \cdot L_1^*$
   (iii) $(L_1 \cup L_2) \cap L_3 = L_1 \cup (L_2 \cap L_3)$

   Please prove your claims.

2. Prove that for every non-empty language $L$, $\varepsilon \in L$ iff $L \subseteq LL$

3. (i) Prove that if $x$ and $y$ are both strings over the same 1-letter alphabet, then $xy = yx$

   (ii) Find strings $x, y$ over the alphabet $\{0,1\}$ such that $x \neq y$, both 0 and 1 appear in $x$ (and $y$), and yet $xy = yx$.

   (iii) **BONUS:** Find a general (as general as you can) condition on strings such that if $x, y$ satisfy this condition then $xy = xy$.

4. (i) Find an infinite set (W of Languages over $\{0,1\}$ so that the following two conditions hold (simultaneously):
a. Every intersection of finitely many members of W is non-empty

b. There is a subset of W whose intersection is empty

(ii) **BONUS:** Does there exist a set W that in addition to satisfying a & b above also satisfies:

   c. Every infinite subset of W has empty intersection

5. Find what are the languages computed by each of the following automata:

   ![Automaton Image](image)

   Explain your claims (there's no need to prove them).

6. Describe automata that compute each of the following languages:

   (i) \( L_{5,3} = \{ w \in \{0,1\}^* : |w| \text{ is divisible by either 3 or 5} \} \)
(ii) For a given string \( w \in \{0,1\}^* \), the language \( \{w\}^* \).