1. Suppose $L \subseteq \Sigma^*$, $L' \subseteq \Delta^*$, We need to find a function $\sigma : \Sigma^* \rightarrow \Delta^*$, such that for all $x \in \Sigma^*$:

   $x \in L \iff \sigma(x) \in L'$

Because $L'$ is non-trivial, $L'$ and $L'$ complement are not empty. We can pick some $y_1 \in L'$ and $y_2 \notin L'$. And define $\sigma$ in this way: for all $x \in \Sigma^*$

   $x \in L, \sigma(x) = y_1 \in L'$

   $x \notin L, \sigma(x) = y_2 \notin L'$

Now we only need to show that $\sigma$ is total and computable. Since $L$ is recursive, there exists some total Turing machine $T$ computing $L$. Now we can construct a total Turing machine $T'$ computing $\sigma$ in this way: on input $x$, $T'$ simulate $T$ on input $x$, if $T$ accepts $x$, $T'$ accepts and writes $y_1$ on its tape. Or if $T$ rejects $x$, $T'$ rejects and writes $y_2$ on its tape. Thus $\sigma$ is total and computable, which completes the proof.

2. Find a pair of languages $L, L'$ for which $L \leq_m L'$ but $L' \not\leq_m L$.

   Solution 2: Take a language $L = \phi \in R$ and $L' \in R$ but $L'$ non-trivial. By problem 1 we know that $L \leq_m L'$, however $L' \not\leq_m L$ because $L$ is trivial.

3. The claim is False. Prove by counterexample: Let $A = \text{HP}$, $B = \text{FIN}$, as defined in Kozen p.241. We have $A \leq_m B$ since $\text{HP} \leq_m \text{FIN}$. Now if the statement is true, then $\sim B \leq_m \sim A$ or equivalently $\sim \text{FIN} \leq_m \sim \text{HP}$. But $\sim \text{FIN}$ is harder than $\sim \text{HP}$ which is in turn harder than $\text{HP}$. Thus we reach a contradiction.