1. Prove that if \( E \) and \( F \) are events, which happen with probability \( P(E) \) and \( P(F) \) respectively, then:
\[
P(E \cap F) \geq P(E) + P(F) - 1.
\]

2. Let \( E_1, E_2, \ldots, E_n \) be events. Prove by induction that
\[
P(E_1 \cap E_2 \cap \cdots \cap E_n) \geq P(E_1) + P(E_2) + \cdots + P(E_n) - n + 1.
\]

3. Let \( E \) and \( F \) be independent events. Prove that, event not \( E \), denoted as \( \overline{E} \), and event not \( F \), denoted as \( \overline{F} \), are independent events.

4. (a) What is the probability that two people chosen at random were born on the same day of the week?
   (b) What is the probability in a group of \( n \) people chosen at random, there are at least two born on the same day of the week?
   (c) How many people chosen at random are needed to make the probability greater than \( \frac{1}{2} \) that there are at least two people born in the same month of the year?

5. Suppose we know 8\% of players use steroids and 92\% do not. We have a test that returns positive if the test thinks the player used steroids. When a player on steroids takes the test, they have a 96\% chance of testing positive. When a player not on steroids takes the test, they have a 9\% chance of testing positive.

Now, we test a player and the test returns positive. What is the probability the player was using steroids?