1. Consider the integers, \( I = \{ \ldots, -2, -1, 0, 1, 2, \ldots \} \), and a prime \( p \).
   (a) if, \( i \in I \), and \( i \geq 0 \) what is \( i \) (mod \( p \))?
   (b) if \( i \in I \), and \( i < 0 \) what is \( i \) (mod \( p \))? 

2. (a) Prove that modular arithmetic under \( p \), ie mod \( p \), is an equivalence relation.
   (b) Note that in modular arithmetic one represents each equivalence class by a representative of the class and defines arithmetic for the representative elements. Eg, in mod 4, the representative elements are 0, 1, 2, 3.
   If \( p = 3 \) what are the addition and multiplication tables for the representative elements?

3. In modular arithmetic some people get sloppy and use a sequence of operations that produce numbers greater than the mod, we are working in. For example in computing \( 2 \times 4 + 3 \times 3 \) mod 4, one may do:

\[
2 \times 4 + 3 \times 3 \mod 4 = (8) + (9) \mod 4 \\
= 17 \mod 4 \\
= 1 \mod 4,
\]

rather than:

\[
2 \times 4 + 3 \times 3 \mod 4 = (0) + (1) \mod 4 \\
= 1 \mod 4.
\]

Is it okay to produce numbers larger than the mod we are working in and if so why?

4. How many ways can one write seven as the sum of four nonnegative integers? (Note: Order does not matter, 6+1+0+0 is the same as 0+0+1+6, don’t count it twice.)

5. (a) Roll a \( k \)-sided dice, three times:
   - How many possible outcomes are there?
   - What is the probability of a face appearing exactly two times?

(b) Roll a 6-sided dice three times:
   - What is the probability of each face being different in three rolls?
   - What is the probability of a face appearing exactly two times?
   - What is the probability of a face appearing three times?
   - What is the sum of the above three probabilities?