1 Regular Expressions

Write a regular expression for the set of all strings of 0’s and 1’s that have two blocks of 1’s. A sample string your regular expression should represent:

000001111111000111000

1.1 Answer

+4 for correct notation and regular expression, +2 for each correct element of the regular expression, −1 for extra 0’s and 1’s in an otherwise correct solution.

\{ 0*11*00*11*0* \}

2 Recurrence Relation

Find a close form solution for the following recurrence equations: (Also, show that you checked the first four values of your solution)

Saved for next week.

3 Induction

+5 for correct base case, +5 for assuming \( g(n) \) is true in a correct way, +5 for manipulating \( g(n+1) \), and +5 for overall correctness

Prove by induction that \( g(n) = 2n^4 + 2n^2 + 4 \) is divisible by 4 for all \( n \geq 1 \).

Proof. Base Case: let \( n = 1 \), then \( g(n) = 8 \), which is divisible by 4.

Inductive Step: Assume \( g(n) \) is divisible by 4, now:

\[
\begin{align*}
g(n + 1) &= 2(n + 1)^4 + 2(n + 1)^2 + 4 \\
g(n + 1) &= 2n^4 + 8n^3 + 12n^2 + 8n + 2 + 2n^2 + 4n + 2 + 4 \\
g(n + 1) &= 2n^4 + 2n^2 + 4 + 8n^3 + 12n^2 + 12n + 4 \\
g(n + 1) &= g(n) + 4(2n^3 + 3n^2 + 3n + 1)
\end{align*}
\]

Now, by assumption \( g(n) \) is divisible by 4 and the second set of terms is a multiple of, each of the components of \( g(n + 1) \) is divisible by 4. Hence, \( g(n + 1) \) is divisible by 4. From the induction principle this implies \( g(n) \) is divisible by 4 for all \( n \geq 1 \).
4  Countably Infinite OR NOT!?!?

−3 for every incorrect answer, minimum 0, and Yes and No were accepted for all computer programs
For each set say whether or not the set is countably infinite:

• the set of all integers (Yes)
• all finite subsets of integers (Yes)
• all subsets of integers (No)
• the set of all reals (No)
• all finite subsets of reals (No)
• all subsets of reals (No)
• the set of all finite length strings (Yes)
• all finite subsets of the set of all finite length strings (Yes)
• all subsets of the set of all finite length strings (No)
• all computer programs (No)
• functions mapping integers to the set \{0, 1\} (No)
• functions mapping integers to integers (No)

5  Countable Concept

+5 for an example of a countable set, +5 for a good explanation of countability, +5 for an example of an uncountable set, +5 for a good explanation of why it is uncountable

In a concise way, write an explanation of countably infinite a high school student could understand. And, give an example of a set that is not countably infinite or finite and explain why.

5.1 Sample Answer

Let us say we have a monkey that never got bored. In fact we taught that monkey to count and he liked it so much that he just wanted to sit and count. At first we gave him the presidents of the US to count, and he counted them and got angry when he ran out of presidents to count. So we gave him the countries in the world to count, and he got angry when he ran out of countries! Fearing the wrath of this monkey, we gave him the negative integers to count. So he sat down and started counting, neatly looking at integer \(-10\) and then \(-11\), etc, he’s still counting. In fact, he’ll never run out of negative integers to count. But he can methodically progress through them and if he was given enough time he could lay out all the negative integers in the order he counted them. That is countably infinite.

Now, if we took this same monkey and asked him to count the real numbers, we’d have a very angry monkey indeed. He would start and count 0, then he would try to find the next number to count, let’s say 0.1, but that isn’t really the next number, because 0.00000001 should come before 0.1. And this process would continue, infact because there are an infinite number of reals inbetween any two real numbers, the monkey would never find what number should be counted second. That is uncountably infinite.
6 Countable Triplets

There were two interpretations of the problem, one printed all tuples of \((a,b,c)\) the other printed all sets of \(\{a,b,c\}\), answers to either were accepted. +5 for three nested loops, +5 for correct middle loop limits, +5 for correct inner loop limits, +5 for correctness

Sketch a algorithm to list all triples of a countably infinite set.

6.1 Answer

Since the set is countably infinite, we can list the set as \(e_1, e_2, \ldots e_i, \ldots\)

\begin{algorithm}
\begin{algorithmic}
\FOR{$n \leftarrow 1, \infty$}
\FOR{$m \leftarrow 1, n$}
\FOR{$p \leftarrow 1, n$}
\STATE print \((e_n, e_m, e_p)\)
\ENDFOR
\ENDFOR
\ENDFOR
\end{algorithmic}
\end{algorithm}