CS 2800 - Homework 9 - Due Wednesday April 21
at the beginning of lecture

INCLUDE THIS COVER PAGE WITH YOUR HOMEWORK

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You should justify/prove all your answers.

**Problem 1**

Show that the following are equivalent:

(a) A graph $G$ with $n$ nodes is connected and has no cycles.
(b) A graph $G$ with $n$ nodes is connected and has $n - 1$ edges.

**Problem 2**

Suppose $G$ is a graph with $n$ nodes and $m$ edges.

(a) What is the minimum number of connected components $G$ could have?
(b) What is the maximum number of connected components $G$ could have?

**Problem 3**

In this problem we will consider the $n$ dimensional cube. For all $n \geq 1$, the graph $\text{Cube}_n$ is defined as follows. The vertex set is all possible 0/1 strings of length $n$ and $\{u, v\}$ is an edge of $\text{Cube}_n$ iff $u$ and $v$ differ in exactly one position. For $n = 1$ we have a single edge, for $n = 2$ we have a square, for $n = 3$ we have the edges of a cube.

Find the chromatic number $\chi(\text{Cube}_n)$.

**Problem 4**

Consider the $n$ dimensional cubes defined in the last problem. For which $n$ is $\text{Cube}_n$ Eulerian?

**Problem 5**

We can define the notion of a random graph on $n$ nodes $V = \{1, \ldots, n\}$ as follows. For each possible edge $\{i, j\}$ we include the edge in the graph with probability $p$ and don’t include it with probability $1 - p$.

(a) What is the expected number of edges in a random graph?
(b) What is the expected number of triangles? (a triangle is a cycle of length 3)
(c) What is the probability that a random graph is complete?