19.A  From Textbook 4.4-6

\[ P(\text{ace or } \heartsuit) = P(\text{ace}) + P(\heartsuit) - P(\text{ace of } \heartsuit) = \frac{1}{13} + \frac{1}{4} - \frac{1}{13} \cdot \frac{1}{4} = \frac{16}{52} = \frac{4}{13} \approx 0.3077 \]

19.B  From Textbook 4.4-16

The first card of the hand can be any card.

\[ P(\text{flush}) = P(\text{second card is of the same suit}) \cdot P(\text{third card is OK}) \cdot P(\text{fourth}) \cdot P(\text{fifth}) \]
\[ = \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} \]
\[ = \frac{11 \cdot 3}{17 \cdot 5 \cdot 49 \cdot 4} = \frac{33}{16660} \]
\[ \approx 1.98 \times 10^{-3} \]

Alternate method: select a suit \( C_4^1 \) and all the possible combination of 5 cards of that suit \( C_{13}^5 \), the number of hands being \( C_{52}^5 \) and \( P(\text{flush}) = \frac{C_4^1 \cdot C_{13}^5}{C_{52}^5} \approx 1.98 \times 10^{-3} \)

19.C  From Textbook 4.4-28

Note: all the numbers are different, so once the first number has correctly been chosen, there are only 79 numbers left, among which only 10 have been selected by the lottery.

\[ P(\text{winning}) = P(\text{first number chosen is among the 11}) \]
\[ \cdot P(\text{second number chosen is among the 11}) \]
\[ \cdot P(\text{third number chosen is among the 11}) \]
\[ \cdot \ldots \]
\[ \cdot P(\text{seventh number chosen is among the 11}) \]
\[ = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{80 \cdot 79 \cdot 78 \cdot 77 \cdot 76 \cdot 75 \cdot 74} \]
\[ = \frac{3}{28,879,240} \]
\[ \approx 1.0388 \times 10^{-7} \]

Alternate method: there are \( C_{11}^7 \) winning tickets out there, the total number of tickets being \( C_{80}^7 \) and \( P(\text{flush}) = \frac{C_{11}^7}{C_{80}^7} \approx 1.0388 \times 10^{-7} \)

19.D  From Textbook 4.4-32

The total number of possible outcomes is \( 6 \cdot 6 = 36 \) when there are two dice, \( 6 \cdot 6 \cdot 6 = 216 \) when there are three dice.

The relevant pairs are \{ (2,6); (3,5); (4,4); (5,3); (6,2) \}, that is 5.

1/3
The relevant triplet are \{ (1,1,6); (1,2,5); (2,1,5); (1,3,4); (2,2,4); (3,1,4); (1,4,3); (2,3,3); (3,2,3); (4,1,3); (1,5,2); (2,4,2); (3,3,2); (4,2,2); (5,1,2); (1,6,1); (2,5,1); (3,4,1); (4,3,1); (5,2,1); (6,1,1) \}. There are 21 of them. Be careful to choose a way of ordering them. In my case the last number is decreasing, then the first one is increasing.

If you are smarter, only count those which are different modular a circular permutation: \{ (1,1,6); (1,2,5); (1,5,2); (1,3,4); (1,4,3); (2,2,4); (2,3,3) \}. There are 7 of them, and each one counts for 3.

\[
\begin{align*}
P(\text{making 8 with 2 dice}) &= \frac{5}{36} \approx 0.13889 \\
\end{align*}
\]

\[
\begin{align*}
P(\text{making 8 with 3 dice}) &= \frac{21}{216} \approx 0.09722 \\
\end{align*}
\]

20A From Textbook 4.5-6

\[
P(E) = 0.8 \quad \text{and} \quad P(F) = 0.6.
\]

\[
\begin{align*}
P(E \cap F) &= 1 - P(E \cap \overline{F}) \\
&= 1 - P(E \cup \overline{F}) \quad \text{by complementarity} \\
&= 1 - P(E) - P(\overline{F}) + P(E \cap \overline{F}) \quad \text{by De Morgan’s law} \\
&= 1 - (1 - P(E)) - (1 - P(F)) + P(E \cap \overline{F}) \quad \text{by Theorem 20.3} \\
&= P(E) + P(F) - 1 + P(E \cap \overline{F}) \quad \text{by complementarity} \\
&\geq P(E) + P(F) - 1 = 0.4 \quad \text{since } P(E \cap \overline{F}) \geq 0
\end{align*}
\]

20B From Textbook 4.5-10

We have \( P(E) \cdot P(F) = P(E \cap F) \) and we want to show \( P(E) \cdot P(F) = P(E \cap \overline{F}) \).

\[
\begin{align*}
P(E \cap \overline{F}) &= 1 - P(E \cap \overline{F}) \quad \text{by complementarity} \\
&= 1 - P(E \cup F) \quad \text{by De Morgan’s law} \\
&= 1 - P(E) - P(F) + P(E \cap F) \quad \text{by Theorem 20.3} \\
&= 1 - P(E) - P(F) + P(E) \cdot P(F) \quad \text{by hypothesis} \\
&= (1 - P(E)) \cdot (1 - P(F)) \quad \text{factorisation} \\
&= P(E) \cdot P(F) \quad \text{by complementarity}
\end{align*}
\]

20C From Textbook 4.5-16

Let \( E \) by the event exactly four heads appear when a fair coin is flipped five times. Let \( F \) be the event the first flip was a tails. Then \( P(F) = \frac{1}{2} \) and \( P(E \cap F) = \frac{1}{2^5} \).

\[
P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{16}
\]

2/3
20.D From Textbook 4.5-26ac

\[ P(\text{no success}) = (1-p)^n \]
\[ P(\text{at most one success}) = P(\text{no success}) + P(\text{exactly one success}) = (1-p)^n + np(1-p)^{n-1} \]

21.A From Textbook 4.5-30

For those who are courageous, they can compute

\[ E(N) = \sum_{n=0}^{10} n \cdot P(N=n) = \sum_{n=0}^{10} \frac{n \cdot C_{10}^n p^n (1-p)^{10-n}}{2^{10}} \]

For the others, it comes from \( E(X+Y) = E(X) + E(Y) \) that \( E(10X) = 10E(X) \). Let’s call \( X \) the number of heads out of one flipping of our biased coin. \( E(X) = 0.6 \) and

\[ E(N) = 10E(X) = 6 \]

21.B From Textbook 4.5-32

\[ P(\text{winning}) = \frac{1}{C_{50}^6} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45} \approx 6.29 \times 10^{-8} \]

\[ E = \$ \cdot P(\text{winning}) \cdot 10 \cdot 10^6 + P(\text{loosing}) \cdot 0 - 1 \]

\[ E \approx \$ - 1 + 6.29 \times 10^{-1} \]

\[ E \approx \$ - 0.37 \]

21.C From Textbook 4.5-34

Same explanation as for 4.5-30 Recall that \( E(\text{value of a die}) = \sum_{n=1}^{6} \frac{1}{6} n = \frac{21}{6} = \frac{441}{36} = 3.5 \)

\[ E = 3E(\text{value of a die}) = 3 \cdot 3.5 = 10.5 \]

21.D From Textbook 4.5-44

Same explanation as for 4.5-30, we have \( V(10X) = 10V(X) \) where \( X \) is the random variable number of times a 6 is obtained when rolling a fair die. df

\[ V = 10V(X) = 10[E(X^2) - E(X)^2] = 10[E(X) - E(X)^2] = 10(\frac{1}{6} - \left(\frac{1}{6}\right)^2) = \frac{25}{18} \approx 1.3889 \]