**Goal:** flexible sentence planning for argumentative text

**Techniques:**

- Combining *Text Structure* and *Upper Model*
- Aggregation
Tree Proof
add lines
Data base
lookup
Methods
transform
call
Verifier
Proverb
abstract/ verbalize
justify
start
start
Proof Tree
modify
apply
check
add lines
lookup
add lines
transform
modify
direct
User (Planner)
call
Proof Transform
Otter
MKRP
Setheo
LEO
INKA
Motivation

**Theorem** (Subgroup Criterion)
Let $G$ be a group, $S \subseteq G$, if for all $x, y$ in $S$, $y \ast x^{-1}$ is also in $S$, then the inverse of every element of $S$ is also in $S$.

<table>
<thead>
<tr>
<th>Initial Clause Set:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1 = {+(u \ast u^{-1} = e)}$</td>
</tr>
<tr>
<td>$C_2 = {+(e \ast w = w)}$</td>
</tr>
<tr>
<td>$C_3 = {- (x \in S), -(y \in S), -(x \ast y^{-1} = z), +(z \in S)}$</td>
</tr>
<tr>
<td>$C_4 = {+(v \in S)}$</td>
</tr>
<tr>
<td>$C_5 = {-(q^{-1} \in S)}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Resolution Steps:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_3, 4 &amp; C_3, 1 \rightarrow R_1$: ${- (x \in S), -(y \in S), -(x \ast y^{-1} = z), -(y' \in S), -(z \ast y'^{-1} = z'), +(z' \in S)}$</td>
</tr>
<tr>
<td>$R_1, 1 &amp; C_4, 1 \rightarrow R_2$: ${- (y \in S), -(v \ast y^{-1} = z), -(y' \in S), -(z \ast y'^{-1} = z'), +(z' \in S)}$</td>
</tr>
<tr>
<td>$R_2, 1 &amp; C_4, 1 \rightarrow R_3$: ${- (v \ast v^{-1} = z), -(y' \in S), -(z \ast y'^{-1} = z'), +(z' \in S)}$</td>
</tr>
<tr>
<td>$R_3, 2 &amp; C_4, 1 \rightarrow R_4$: ${- (v \ast v^{-1} = z), -(z \ast v^{-1} = z'), +(z' \in S)}$</td>
</tr>
<tr>
<td>$R_4, 1 &amp; C_1, 1 \rightarrow R_5$: ${- (e \ast v^{-1} = z'), +(z' \in S)} $</td>
</tr>
<tr>
<td>$R_5, 1 &amp; C_2, 1 \rightarrow R_6$: ${+(v^{-1} \in S)}$</td>
</tr>
<tr>
<td>$R_6, 1 &amp; C_5, 1 \rightarrow R_7$: $\square$</td>
</tr>
</tbody>
</table>

**Proof:**
Let $a$ be in $S$. According to the definition of inverse element, $a \ast a^{-1} = e$. According to our hypothesis, $e$ in $S$. $e \ast a^{-1} = a^{-1}$ according to the definition of unit. Again according to our hypothesis, $a^{-1}$ is in $S$. 
Previous Work

Machine Oriented Proofs
[Andrews 80, Miller 83, Pfenning 87, Lingenfelder 90]

Natural Deduction Proofs (ND)
[Chester 76, McDonald 83, Edgar & Pelletier 93]

Natural Language Proofs (NL)
Reconstructive Explanation in PROVERB

Problem
- Auto. Prover
- Refutation Graph
- Transformation

Proof in English
- TAG-GEN
- Input for TAG-GEN
- TAG Interface

Natural Deduction Proof
- Abstraction
- Text Planner
- Assertion Level Proof

Computational Model for Informal Mathematics
Computational Model for Proof Presentation

linguistic Specification:
The System *PROVERB*

**Macrolananner:** choice of content and order of the information to be conveyed

**Microplanner:** sentence scoping and planning of the internal structure of the sentences

**Realizer:** realization of the surface text by TAG-GEN
**PROVERBS** Macroplanner

**Task**: content determination

**Input**: an assertion level proof

**Output**: an ordered sequence of *proof communicative acts* (PCAs)

**Methods**:

- goal directed hierarchical planning (top-down)
- focus guided local organisation (bottom-up)
Case-Implicit

\[ F \quad G \]

- Proof: \( \frac{F \lor G, Q, Q}{Q} \) CASE

- Acts:
  
  1. Subproof
  2. (CASE-FIRST, Assumptions: \( F \))
  3. Subproof
  4. (CASE-SECOND, Assumptions: \( G \))
  5. Subproof

- Features: (top-down compulsory implicit)
Local Organization

\[
\begin{align*}
[1] &: \ P(a, b) \\
[3] &: \ Q(a, b) \\
[5] &: \ Q(a, b) \land R(b, c)
\end{align*}
\]

- **local focus** = [1]
  focal centers = \{a, b\}

- **next node** = [3],
  since [3] does not introduce any new objects and \{b\} \subset \{a, b\}
The Need for a Microplanner

• first version of \textit{PROVERB} without a microplanner:

  – no paraphrasing:
    \begin{itemize}
    \item Since $A, B$. \\
    [A leads to $B$.]
    \end{itemize}

  – rigid recursive verbalization:
    \begin{itemize}
    \item \textit{Set}(F) \land \textit{Subset}(F, G) \\
    \item $F$ is a set and $F$ is a subset of $G$. \\
    [F is a set and a subset of $G$.]
    [The set $F$ is a subset of $G$.]
    \end{itemize}

• only microplanning technique: derivation reference choice
  \Rightarrow \textit{preverbal message} (PM)
Example

(1) Let $F$ be a group and $U$ be a subgroup of $F$ and $1$ be a unit element of $F$ and $1_U$ be a unit element of $U$.
(2) According to the definition of unit element $1_U \in U$.
(3) Therefore there is an $X, X \in U$.
(4) Now suppose that $u_1$ is such an $X$.
(5) According to the definition of unit element $u_1 * 1_U = u_1$.
(6) Since $U$ is a subgroup of $F$, $U \subset F$.
(7) Therefore $u_1 \in F$.
(8) Similarly $1_U \in F$, since $1_U \in U$.
(9) Since $F$ is a group, $F$ is a semigroup.
(10) Since $u_1 * 1_U = u_1$, $1_U$ is a solution of the equation $u_1 * X = u_1$.
(11) Since $1$ is a unit element of $F$, $u_1 * 1 = u_1$.
(12) Since $1$ is a unit element of $F$, $1 \in F$.
(13) Since $u_1 \in F$, $1$ is a solution of the equation $u_1 * X = u_1$.
(14) Since $F$ is a group, $1_U = 1$ by the uniqueness of solution.
(15) This conclusion is independent of the choice of the element $u_1$. 
**PROVERBS**s Microplanner

**Task:** sentence scoping and sentence organisation

**Input:** a sequence of PCAs

**Output:** a *Text Structure*

**Methods:**

- progressive refinement of the Text Structure
- operations on the Text Structure
A Text Structure [Meteer, 91] contains information about:

- constituency
- structural relations between constituents
- semantic categories of the constituents
PROVERBS Upper Model

- adopted from [Bateman et al., 90]
PROVERBS Textual Semantic Categories

- adopted from [Panaget, 94]
Resource Trees

- Text Structure built up by *resource trees*
- *resource trees* consist of basic tree types:

  ![Resource Tree Diagram]

  **Kernel tree**

  Peter likes Mary

  **Composite trees**

  pretty Mary

  Peter and Mary
Paraphrasing

$Orth(C_1, C_2)$

$quality\text{-}relation(Orth, C_1, C_2)$

$process\text{-}relation(Orth, C_1, C_2)$

$property\text{-}ascription(Orth, conjunction(C_1, C_2))$

$<\text{lex be}>$

$\text{vp}$

$\text{head}$

$\text{argument}$

$conj(C_1, C_2)$

$\text{np}$

$C_1$ and $C_2$ are orthogonal

$\text{matrix}$

$Orth$

$\text{np}$

the orthogonality of $C_1$ and $C_2$

$\text{adjunct}$

$conj(C_1, C_2)$

$\text{modifier}$
The Architecture of \textit{PROVERB}

- Natural Deduction Proof
- Macroplanner
  - PCAs
- Microplanner
  - DRCC
    - Text Structure Expansion
    - Sentence Scoping
    - Lexical Choice
    - Ordering
    - Aggregation
    - Cue Word Insertion
    - Layout
  - PMs
  - TSG
    - Upper Model
    - Realization Classes
    - Textual Semantic Categories
    - Lexicon
  - Text Structure
- Transformer
- Realizer
Aggregation

Grouping (5)
- Logical Predicates (1)
- PMs (2)
- Logical Connectives (2)

Embedding (2)

Pattern (4)
- Chaining (3)
- Others (1)

Aggregation (11)
Predicate Grouping

\[ \text{Set}(F) \land \text{Set}(G) \]

"\( F \) is a set. \( G \) is a set."

\[ \downarrow \]

\[ \text{Set}(F \land G) \]

"\( F \) and \( G \) are sets."
Grouping of Implications

\[
\text{conjunction}(\text{implication}(a < b, a \neq b), \\
\text{implication}(a > b, a \neq b))
\]

“If \(a < b\) then \(a \neq b\). If \(a > b\) then \(a \neq b\).”

\[\downarrow\]

\[\text{implication}(\text{disjunction}(a < b, a > b), a \neq b)\]

“If \(a < b\) or \(a > b\) then \(a \neq b\).”
Embedding

Set(F) \land Subset(F, G)

“F is a set. F is a subset of G.”

\downarrow

Subset(Set(F), G)

“The set F is a subset of G.”
Pattern Based Aggregation

\[ (T'_1 \cdots C \cdots T'_k) \]

\[ M' \quad C' \]

\[ \text{derive-cont} \]

\[ \text{assume} \]

\[ (R \quad M \quad C) \]

\[ \text{derive-cont} \]

\[ \text{assume} \]

\[ (T'_1 \cdots T'_k) \]

\[ M' \quad C' \]

\[ \text{derive-cont} \]

\[ \text{derive-cont} \]

\[ \text{derive} \]

\[ \text{DefTrans} \]

\[ \sigma \subseteq \sigma^* \]

\[ \text{derive}(\epsilon, \text{DefTrans}, \sigma \subseteq \sigma^*) \]

```
“\( \sigma \subseteq \sigma^* \) by the definition of transitive closure.”
```

\[ \text{derive}((x, y) \in \sigma, \sigma \subseteq \sigma^*), \text{DefSubset}, (x, y) \in \sigma^* \]

```
“Since \((x, y) \in \sigma\) and \(\sigma \subseteq \sigma^*\), \((x, y) \in \sigma^*\) by the definition of subset.”
```

\[ \Rightarrow \]

\[ \text{derive}(\epsilon, \text{DefTrans}, \sigma \subseteq \sigma^*, \text{derive-cont}((x,y)\in \sigma), \text{DefSubset}, (x, y) \in \sigma^*) \]

```
“\( \sigma \subseteq \sigma^* \) by the definition of transitive closure, thus establishing \((x, y) \in \sigma^*\) by the definition of subset, since \((x,y) \in \sigma\).”
```
Theorem:

Let $F$ be a group, $U$ be a subgroup of $F$, and $1$ an element of $F$. Then $1 \in U$. 

Proof:

Let $F$ be a group, $U$ be a subgroup of $F$, and $1$ an element of $F$. Then $1 \in U$. 

Example (cont'd)
Conclusion

- Microplanning techniques necessary for mathematical proofs
- Text Structure combined with Upper Model and textual semantic categories
- Aggregation rules defined in terms of Upper Model concepts