1) The Problem

Constraint Satisfaction Problem (CSP)
- Constraint satisfaction problems (CSPs) are problems where you have a set of variables, each of which can take on a finite number of values, and a set of constraints that dictate which values the variables can take.
- The goal is to find an assignment of values to the variables that satisfies all the constraints.

2) Estimating Marginal Probabilities

Factor Graphs:
- Cast the problem as an inference problem in a graphical model, specifically a factor graph.
- Each factor in the factor graph corresponds to a constraint in the CSP.
- Each variable is represented by a variable node, which has factors connected to it.
- Factors are connected by edges that represent the constraint.

Representing clusters in a factor graph:
- A cluster is a set of variables that are connected to the same set of factors.
- The idea is to find clusters of variables that are likely to have similar values.

3) Clusters

Factor Graph for Clusters:
- To reason about clusters, we seek a factor graph representation that captures the dependencies between variables.
- The goal is to find a factor graph that approximates the original CSP.

4) Belief Propagation

Factor Graph and Cluster Beliefs:
- Inference in factor graphs is typically done using Belief Propagation (BP).
- BP is an iterative algorithm that passes messages between nodes to compute marginal probabilities.
- The messages encode the belief that a variable takes a certain value, given the values of its neighbors.

5) Results

Empirical Results: Z_\text{COL} for COL
- For factor graphs, we compute the partition function Z of the graph.
- Z_\text{COL} is a lower bound on the number of solutions.

Theoretical Results: Exactness of Z_\text{COL}
- For factor graphs, we can compute the exact number of solutions using Z_\text{COL}.
- Z_\text{COL} is exact for a 3-COL problem.

Z_\text{COL} is exact for all factor graphs.

Conclusion:
- The combination of clustering and Belief Propagation provides a powerful framework for solving CSPs.
- The approach is scalable and can be applied to large and complex problems.

Cornell University

Lukas Kroc, Ashish Sabharwal, Bart Selman