



Reusable Software Infrastructure for Stream Processing

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Thesis Defense

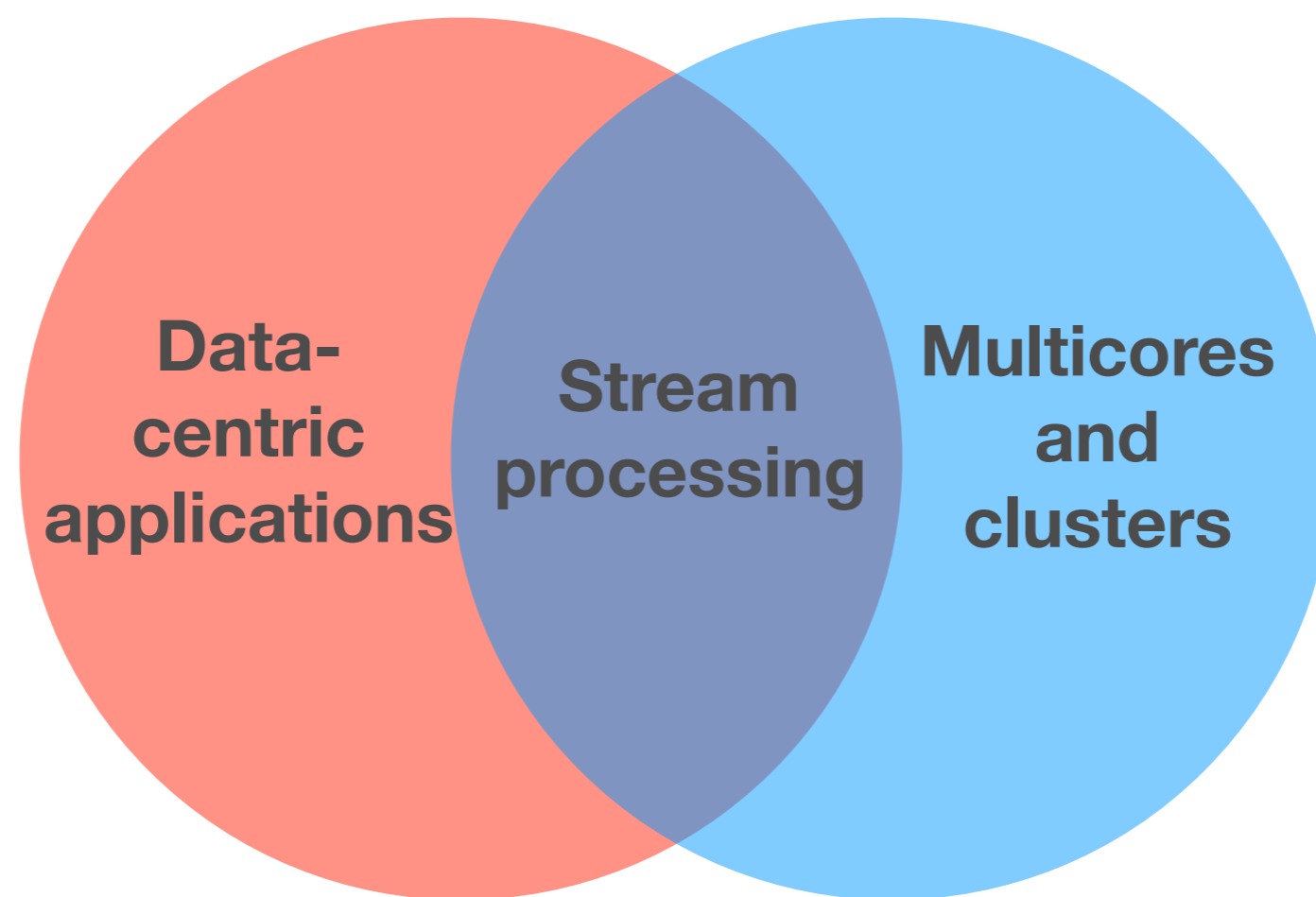


Stream Processing Is Everywhere

- ❏ **Netflix accounts for ~30% of downstream internet traffic.**
- ❏ **Algorithmic trading accounts for 50-60% of all trades in the U.S.**
- ❏ **A streaming application can predict the onset of sepsis in premature babies 24 hours sooner than experienced ICU nurses.**



At the Intersection of Two Trends



**Languages and optimizations
need to adapt**



Streaming Languages and Optimizations

Streaming Languages	Streaming Optimizations
<p>CQL, StreamIt, Sawzall, Hancock, Gigascope, Lime, etc.</p> <p>Represent an application as a graph of streams and operators</p> <p>Tailored to the needs of a particular application domain</p>	<p>Fusion, fission, placement, reordering, etc.</p> <p>Maximize utilization of available resources</p> <p>Often re-write the data-flow graph</p>



Stream Processing Needs Infrastructure

- ❖ Benefits of a *intermediate language (IL)* are well known
 - ❖ Increase portability
 - ❖ Share optimizations
- ❖ Streaming needs its own intermediate language
 - ❖ Need to reason across machines
 - ❖ Support different optimizations



Hypothesis

An intermediate language designed to meet the requirements of stream processing can serve as a common substrate for optimizations; assure implementation correctness; and reduce overall implementation effort.



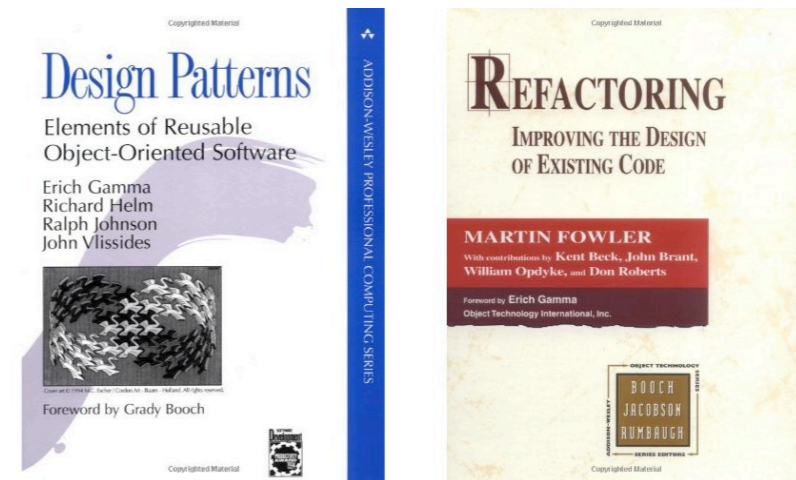
Thesis Components

- ❏ **A catalog of streaming optimizations identifies the requirements for a streaming IL**
- ❏ **A minimal calculus provides a general, formal semantics and enables reasoning about correctness**
- ❏ **An intermediate language provides a practical realization of the calculus**



Optimizations Catalog

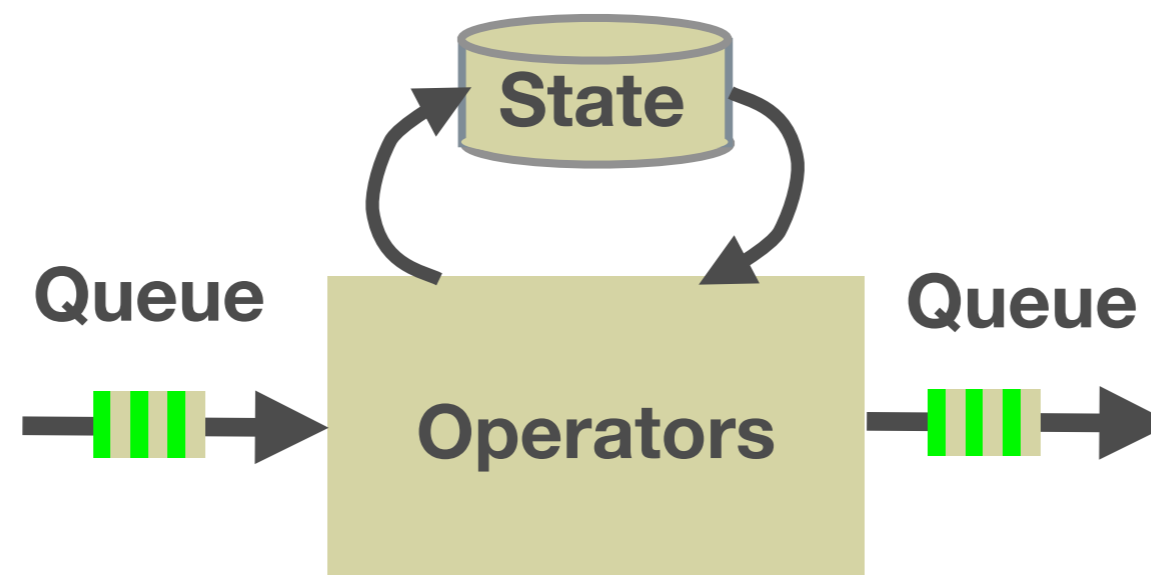
*A catalog, but organized
as a reference.*



- ❖ Resolves conflicting terminology (e.g. kernel = operator = box)
- ❖ Makes assumptions explicit (e.g. stream graph is a forrest)
- ❖ Identifies the requirements for implementing optimizations



Brooklet Calculus

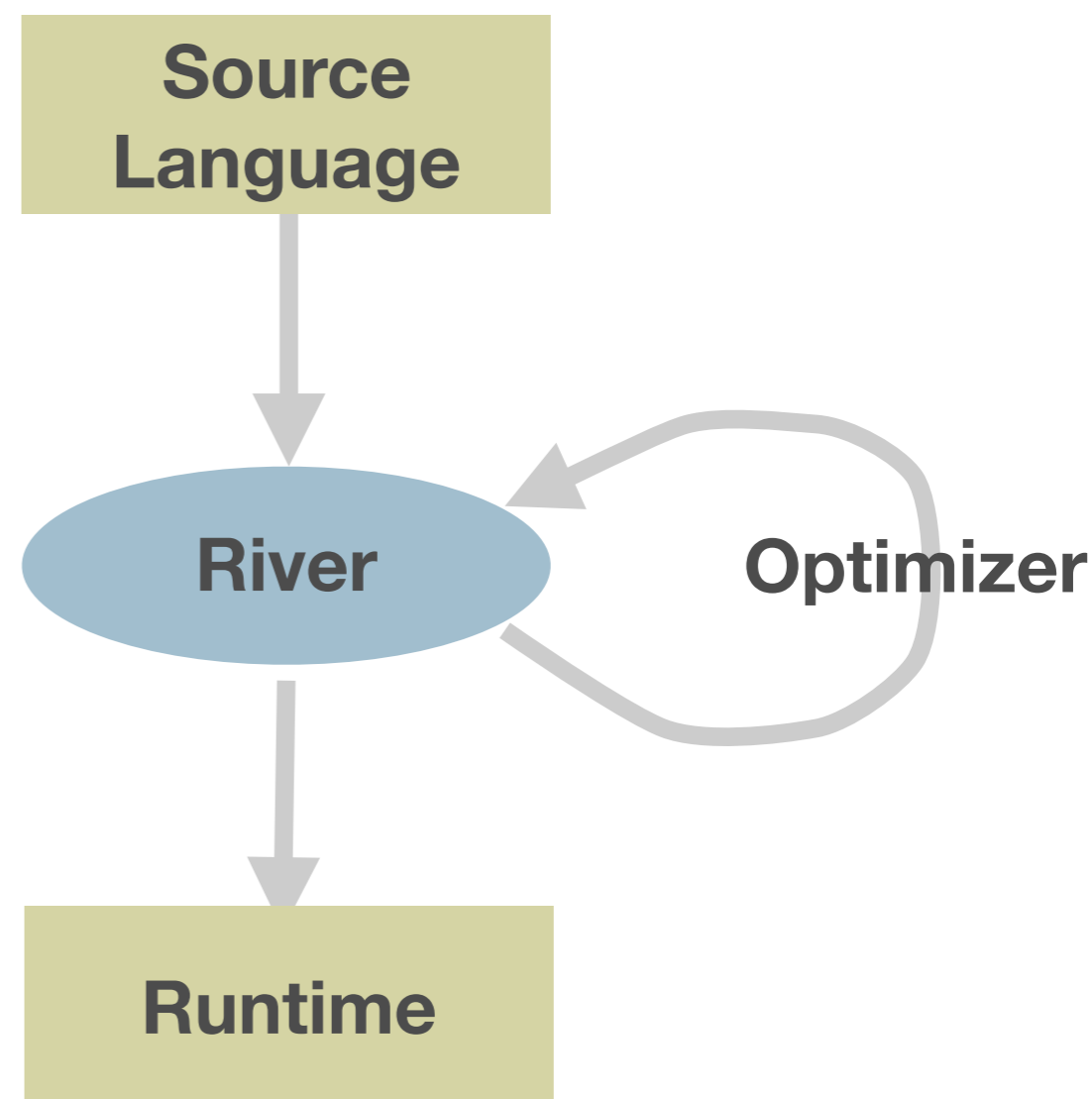


- Names operators and queues: fundamental components
- Explicit state and communication: need machinery
- Non-deterministic execution: reality of distributed systems
- Establishes a formal foundation for an IL



River IL

- ⦿ **Decouples front-ends from optimizations: portability and reuse**
- ⦿ **Concretizes Brooklet: operator implementations, concurrent execution, back-pressure**
- ⦿ **Modular parsers, type-checkers, code generators**
- ⦿ **Practical IL for streaming with a formal semantics**



Evaluation

Condition	Experiment
Meets the requirements of stream processing	Front-ends for CQL, StreamIt, Sawzall and benchmark applications
Serves as a common substrate for optimization	Operator fusion, fission, and placement optimizations
Assures implementation correctness	Formal translations of three languages, Safety proofs for three optimizations
Reduces overall implementation effort	Language agnostic optimizations applied to benchmarks illustrates reuse



Contributions

- A systematic exploration of the requirements for a streaming IL
- A formal foundation for the design of an IL
- An IL with a rigorously defined semantics that decouples front-ends from optimizations
- The first formal semantics for Sawzall
- The first distributed implementation of CQL



Outline of This Talk

- ❖ **A Catalog of Streaming Optimizations**
- ❖ **The Brooklet Core Calculus**
- ❖ **River: From a Calculus to an Execution Environment**
- ❖ **Related Work**
- ❖ **Outlook and Conclusions**



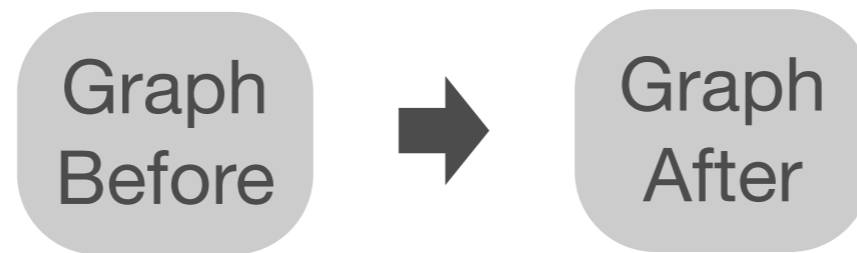
Optimizations Catalog

Identifying the Requirements for a Streaming IL



Optimization Name

Key Idea



Safety

- Preconditions for correctness

Profitability

throughput (higher is better)

- Micro-benchmark
- Runs on System S
- Relative numbers

Central trade-off factor

Variations

- Most influential published papers

Dynamism

- How to optimize at runtime

Items highlighted in red will be addressed in this talk



List of Optimizations

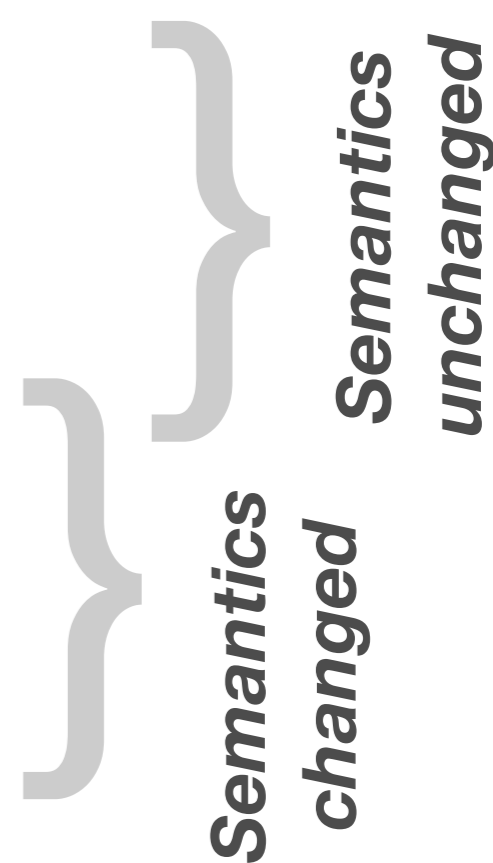
Graph changed

- Operator reordering
- Redundancy elimination
- Operator separation
- Fusion
- Fission

Graph unchanged

- Load balancing
- Placement
- State sharing
- Batching
- Algorithm Selection

- Load shedding



Operator Reordering

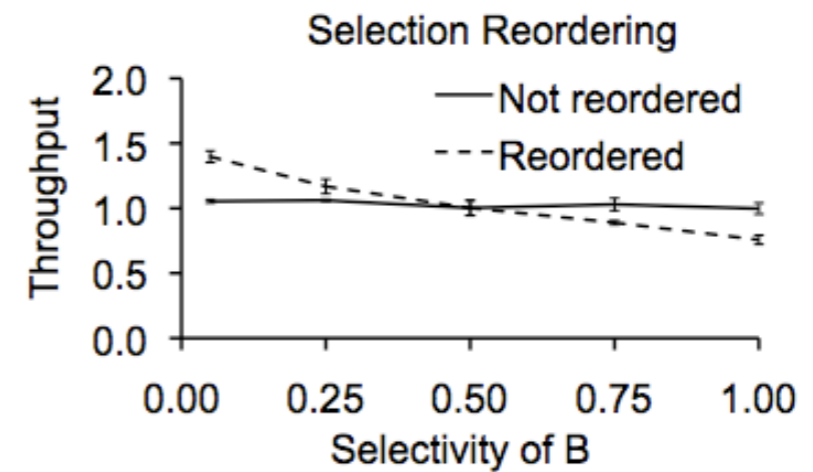
Move more selective operators upstream to filter data early.



Safety

- ⊞ Commutative
- ⊞ Attributes available

Profitability



Variations

- ⊞ **Algebraic**
- ⊞ Commutativity analysis
- ⊞ Synergies, e.g. fusion, fission

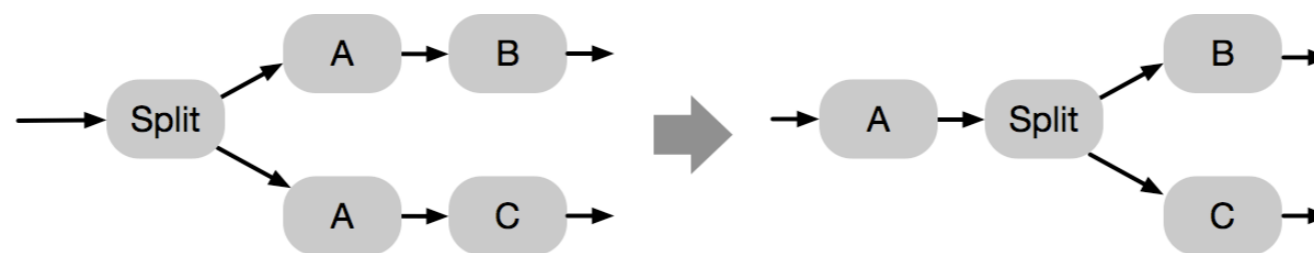
Dynamism

⊞ **Eddy**



Redundancy Elimination

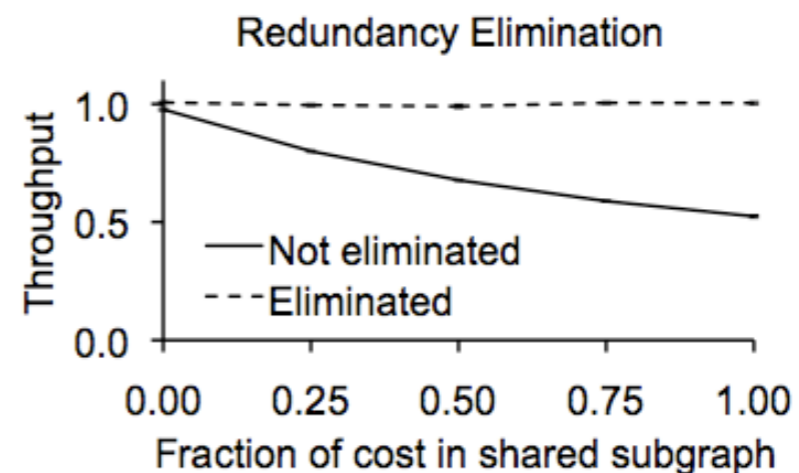
Combine or remove redundant operators.



Safety

- Same algorithm
- Data available

Profitability



Variations

- Many-query optimization**
- Eliminate no-op
- Eliminate idempotent op
- Eliminate dead subgraph**

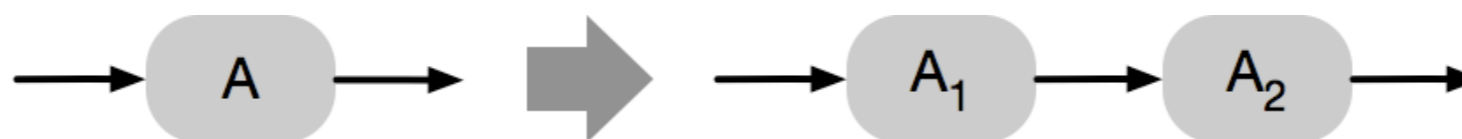
Dynamism

- In many-query case: share at submission time



Operator Separation

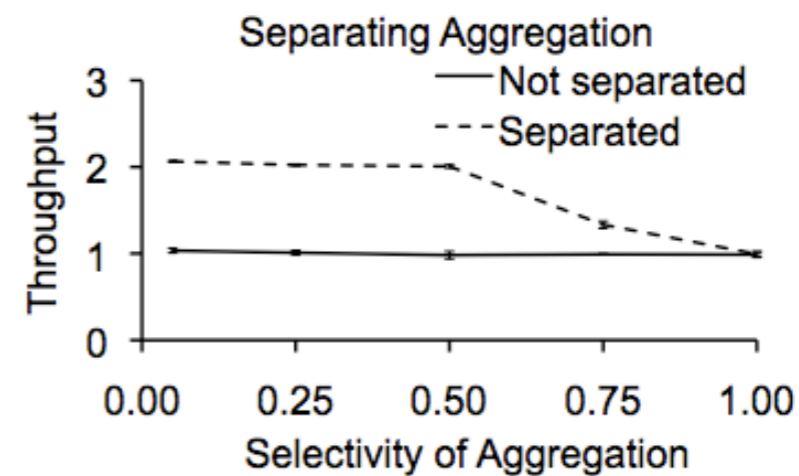
Break coarse-grained operators into finer steps.



Safety

⊞ Ensure $A_1(A_2(s)) = A(s)$

Profitability



Variations

- ⊞ Algebraic
- ⊞ Using special API
- ⊞ Dependency analysis
- ⊞ **Enable Reordering**

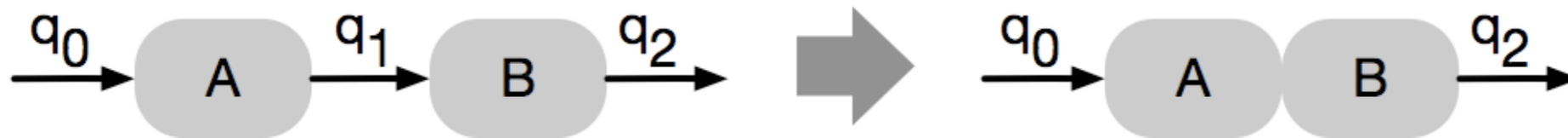
Dynamism

⊞ N/A



Fusion

Avoid the overhead of data serialization and transport.



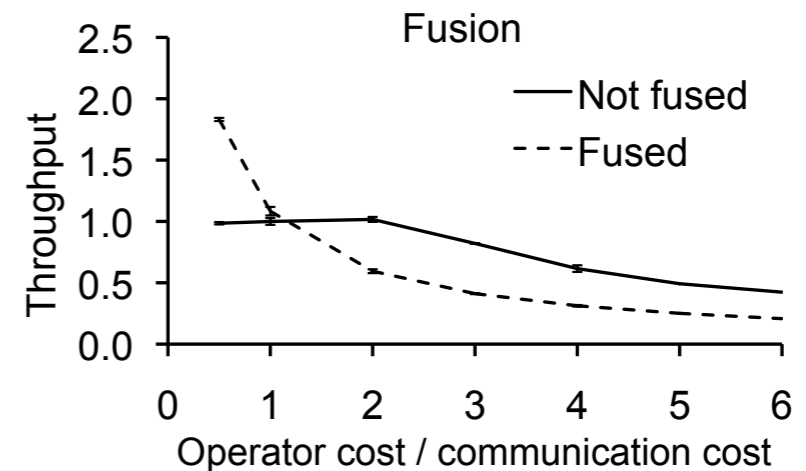
Safety

- ⊞ Have right resources
- ⊞ Have enough resources
- ⊞ No infinite recursion

Variations

- ⊞ **Single vs. multiple threads**
- ⊞ **Fusion enables traditional compiler optimizations**

Profitability



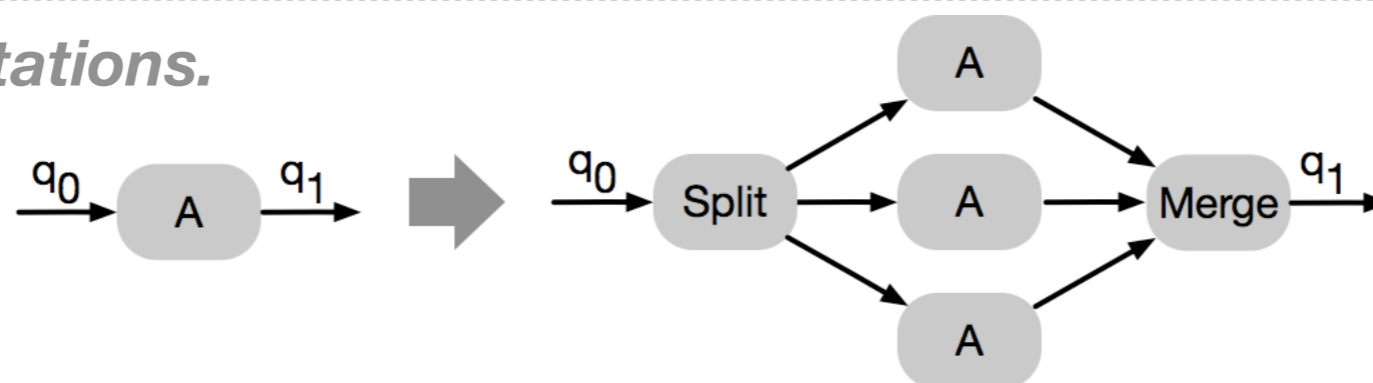
Dynamism

- ⊞ Online recompilation
- ⊞ Transport operators



Fission

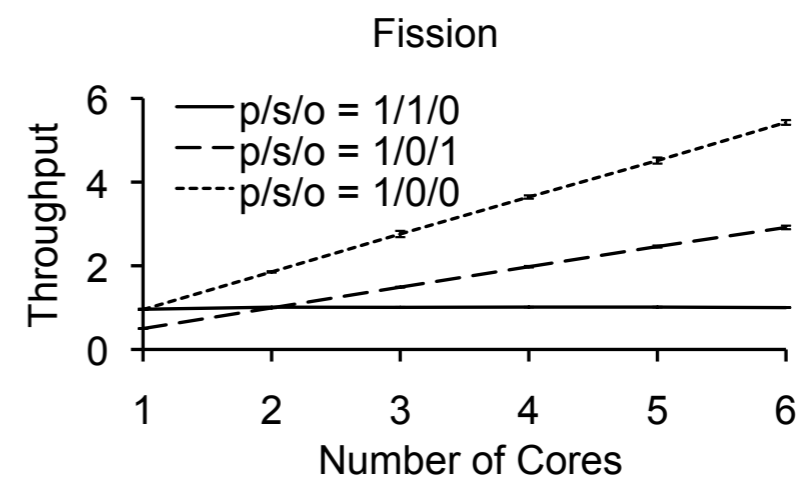
Parallelize computations.



Safety

- ⊞ No state or disjoint state
- ⊞ **Merge in order, if needed**

Profitability



Variations

- ⊞ Round-robin (no state)
- ⊞ **Hash by key (disjoint state)**
- ⊞ Duplicate

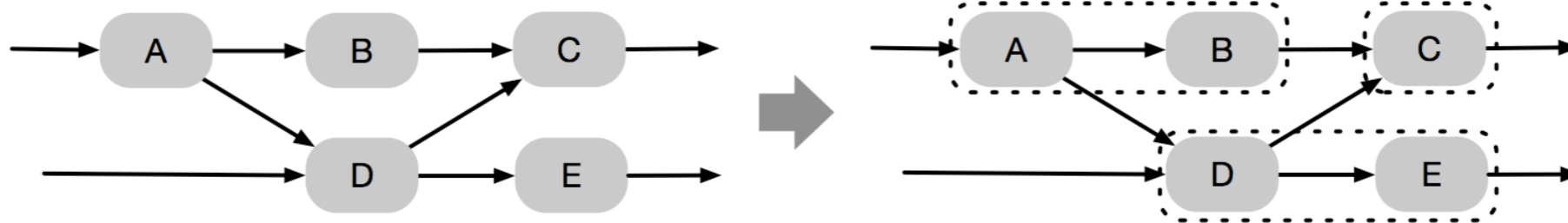
Dynamism

- ⊞ Elastic operators (learn width)
- ⊞ STM (resolve conflicts)



Placement

Assign operators to hosts and cores.



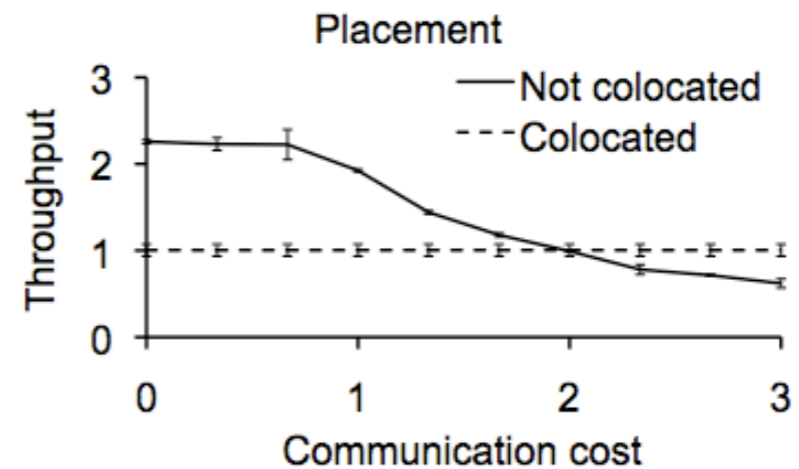
Safety

- ⊞ Have right resources
- ⊞ Have enough resources
- ⊞ Obey license/security
- ⊞ If dynamic, need migratability

Variations

- ⊞ Based on host resources vs. network resources, or both
- ⊞ Automatic vs. user-specified

Profitability



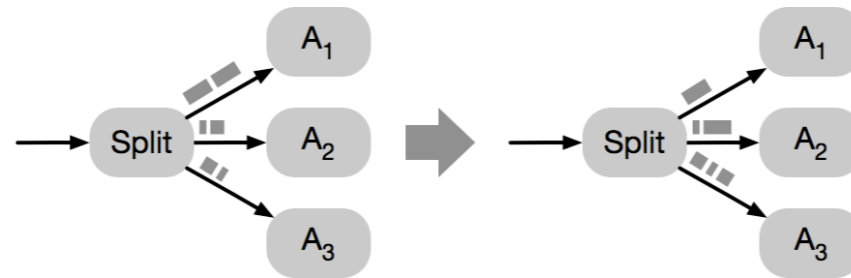
Dynamism

- ⊞ Submission-time
- ⊞ Online, via operator migration



Load Balancing

Distribute workload evenly across resources



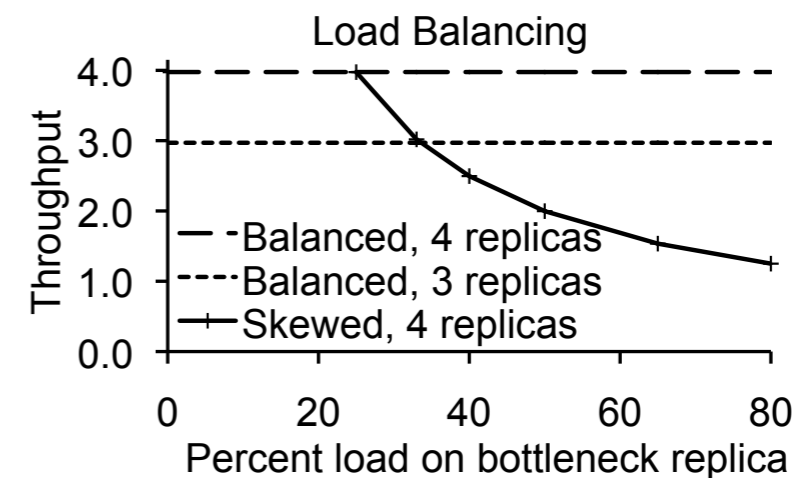
Safety

- ❏ Avoid starvation
- ❏ Ensure each worker is equally qualified
- ❏ Establish placement safety

Variations

- ❏ Balancing work while **placing operators**
- ❏ Balancing work by **re-routing data**

Profitability



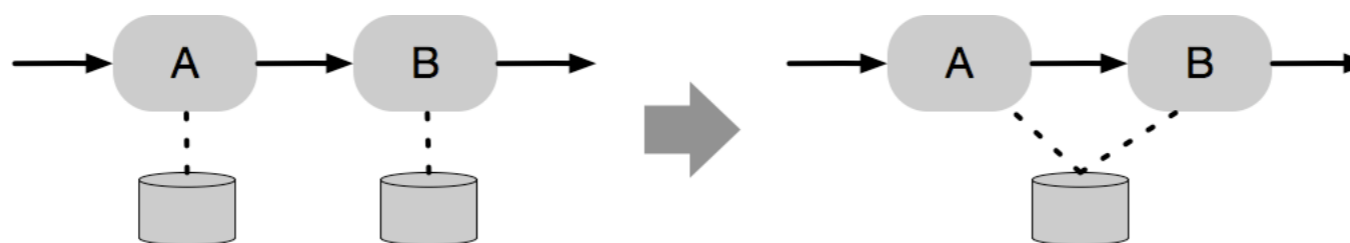
Dynamism

- ❏ Easier for routing than placement



State Sharing

Optimize for space by avoiding unnecessary copies of data.



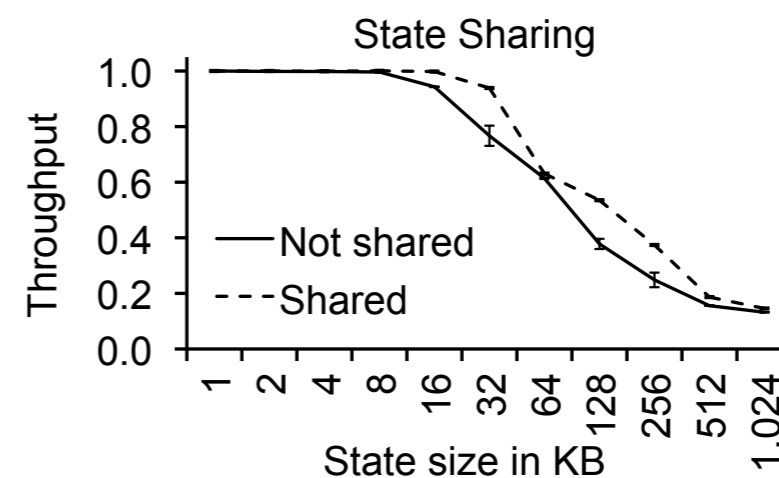
Safety

- Common access (usually fusion)
- No race conditions
- No memory leaks

Variations

- Sharing **queues**
- Sharing **windows**
- Sharing **operator state**

Profitability



Dynamism

N/A



Batching

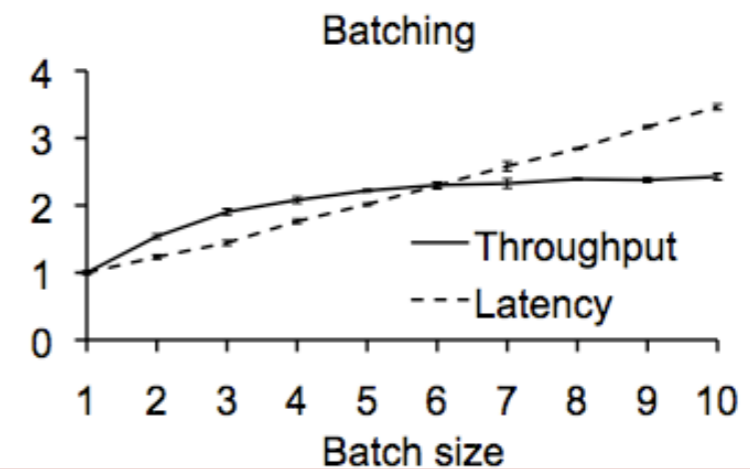
Process multiple data items in a single batch.



Safety

-  No deadlocks
-  Satisfy deadlines



Profitability



Variations

-  **Batching enables traditional compiler optimizations**

Dynamism

-  Batch controller
-  Train scheduling



Algorithm Selection

Use a faster algorithm for implementing an operator.

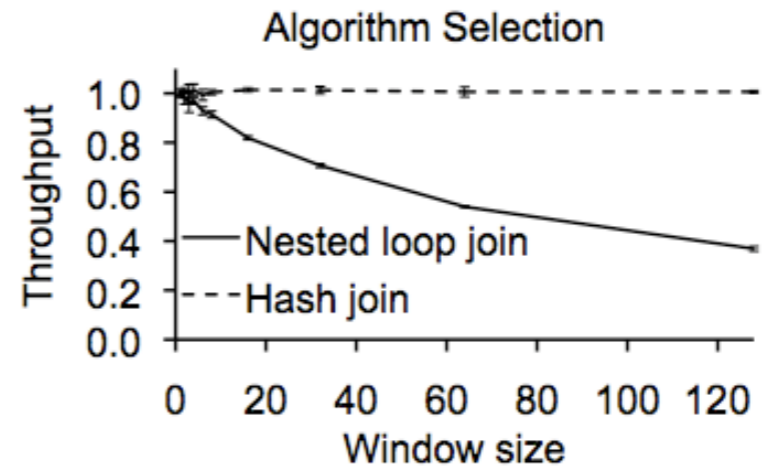


Safety

⊞ $A_\alpha(s) \cong A_\beta(s)$

⊞ **May not need to be safe**

Profitability



Variations

⊞ Algebraic

⊞ Auto-tuners

⊞ **General vs. specialized**

Dynamism

⊞ **Compile both versions, then select via control port**



Load Shedding

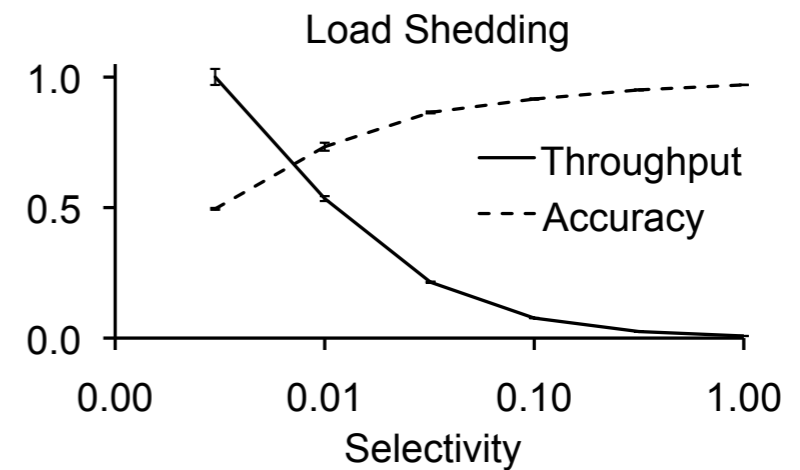
Degrade gracefully when overloaded.





Safety

-  **By definition, not safe!**
-  QoS trade-off

Profitability



Variations

-  Filtering data items
(variations: where in graph)
-  Algorithm selection

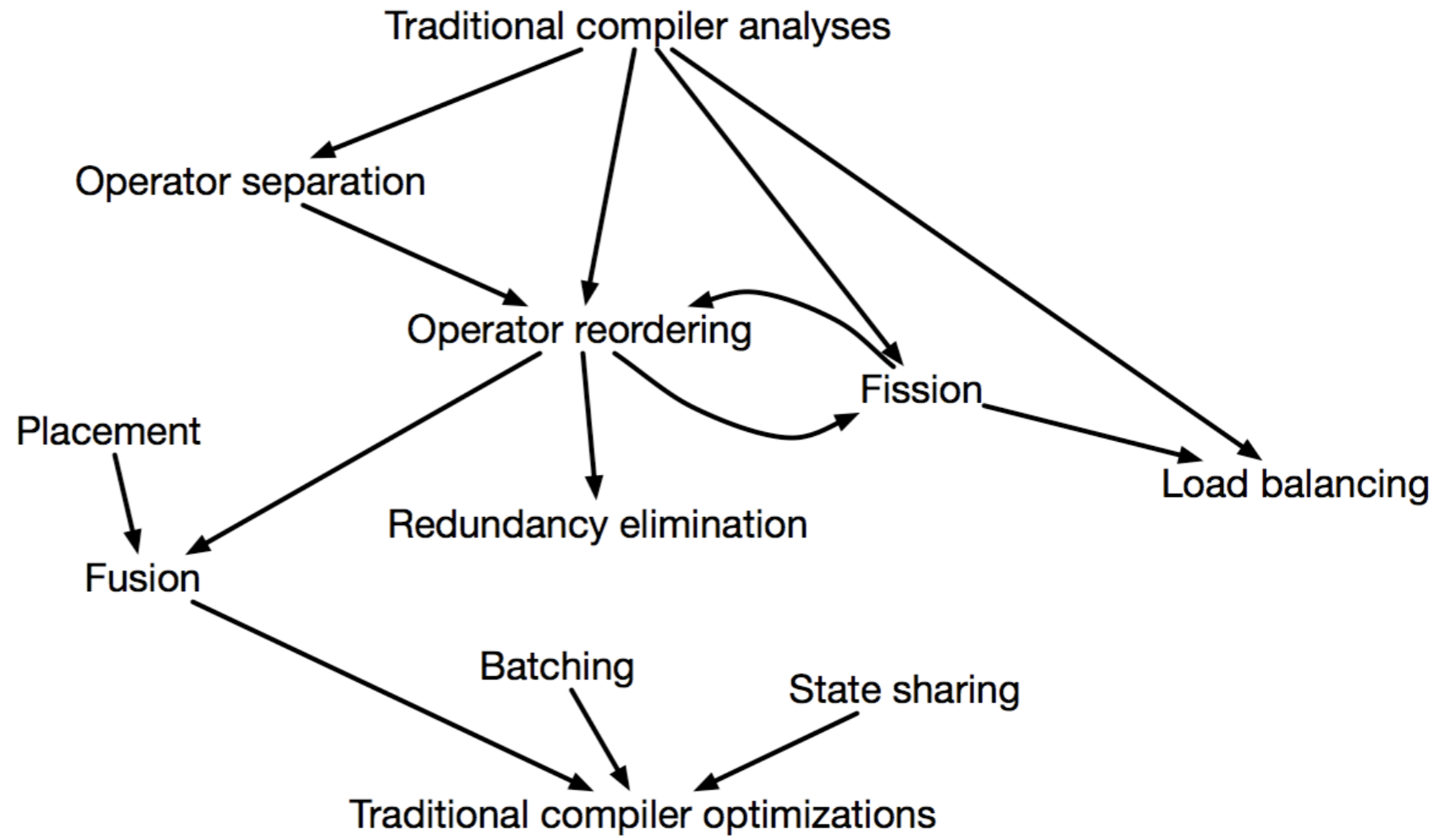
Dynamism

-  **Always dynamic**



Optimizations Enable Optimizations

Traditional → {
Stream
Stream → {
Stream
Stream → {
Traditional

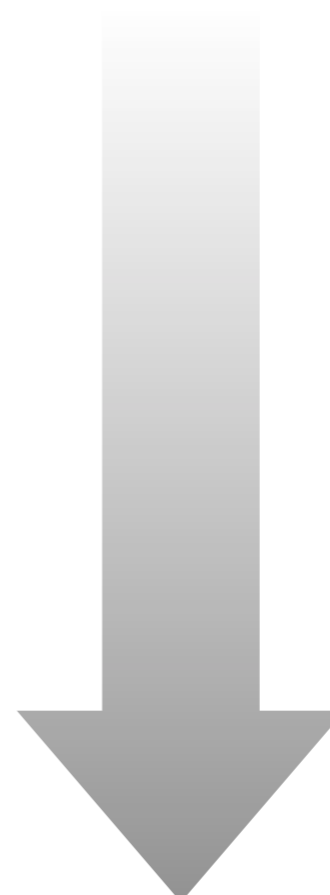


Languages Enable Optimizations

High-level
Easy to use
Optimizable



Mario
CEP patterns
StreamDatalog
StreamSQL
StreamIt
Graph GUI
SPL
Java API
Annotated C
C/Fortran



Low-level
General
Predictable



Hand-Optimized vs. Auto-Optimization

Hand-Optimized

- 🔹 Experts can get better performance
- 🔹 Better Control
- 🔹 Generality
- 🔹 Easier to build systems

Auto-Optimized

- 🔹 Better out-of-the-box experience
- 🔹 Portability
- 🔹 Application code is less cluttered



Requirements for an IL

Observation	Conclusion
4/11 depend on the order that operators execute	IL should be explicit how determinism is enforced
5/11 modify the topology	IL needs to model communication
8/11 depend on state	IL needs to model state
9/11 have dynamic variations	IL needs to support dynamism
11/11 have a unique requirement	IL must be extensible



A Universal Calculus For Stream Processing

A formal foundation for a streaming IL

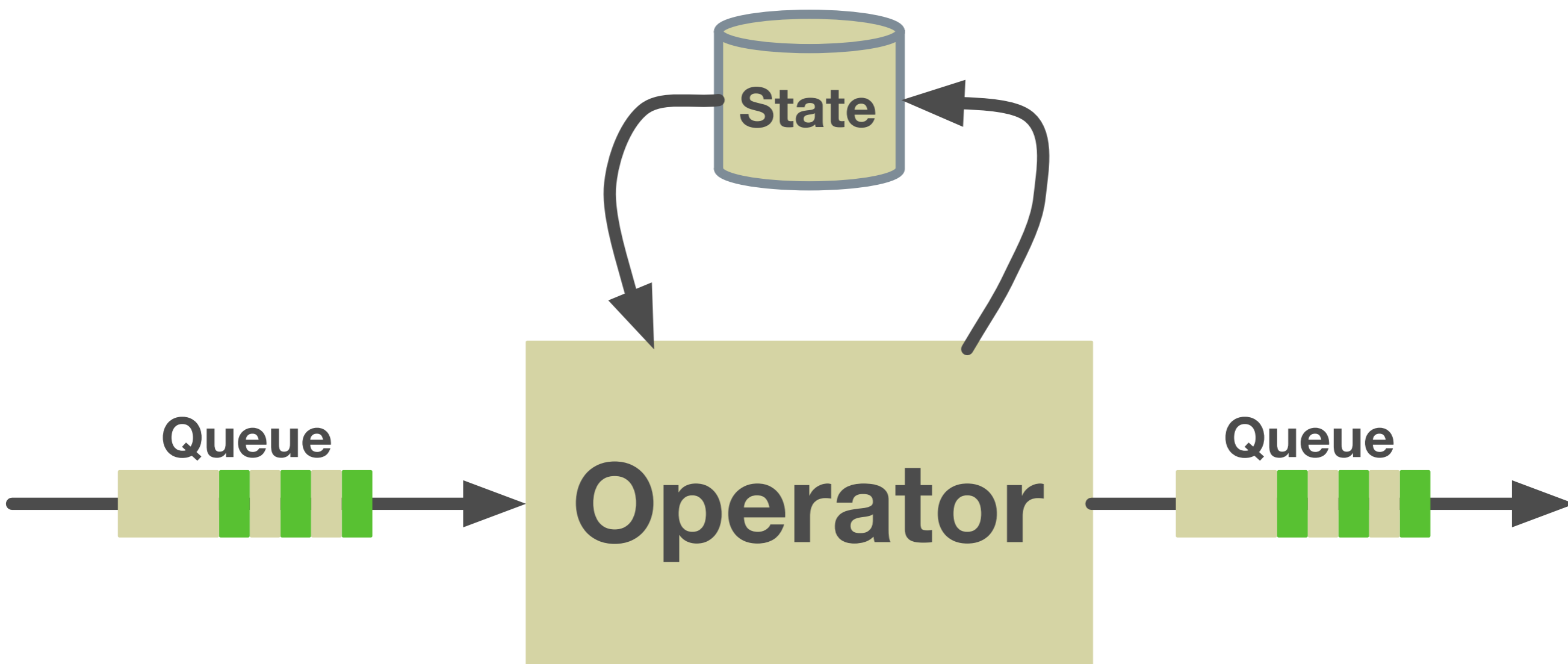


Design Goals

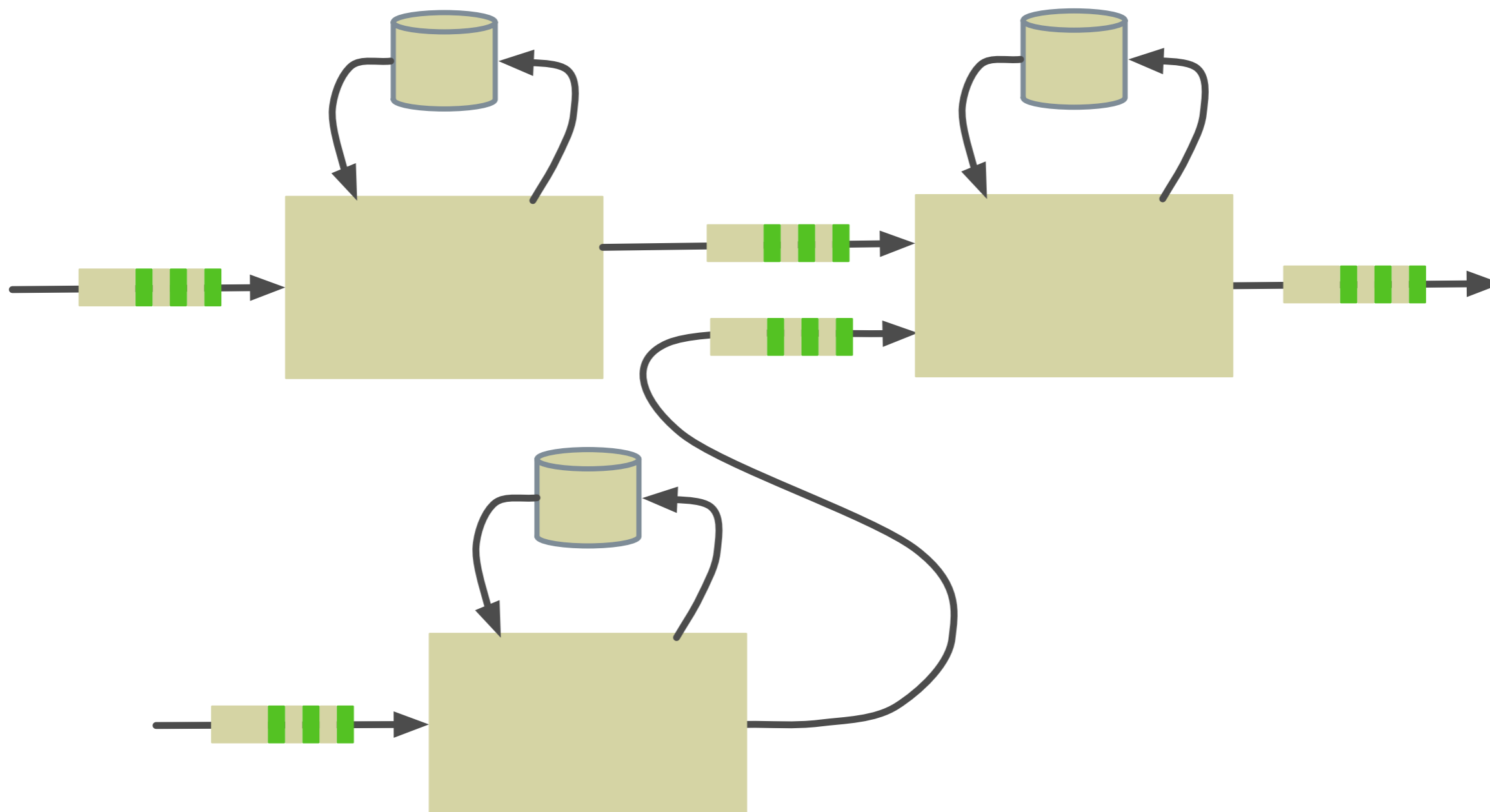
- Enable reasoning about correctness of optimizations
- Flexibility to represent diverse languages
- Formalize *three* of the requirements:
 - State, communication, and non-determinism
- Save dynamism for future work
- Extensibility is addressed in the IL



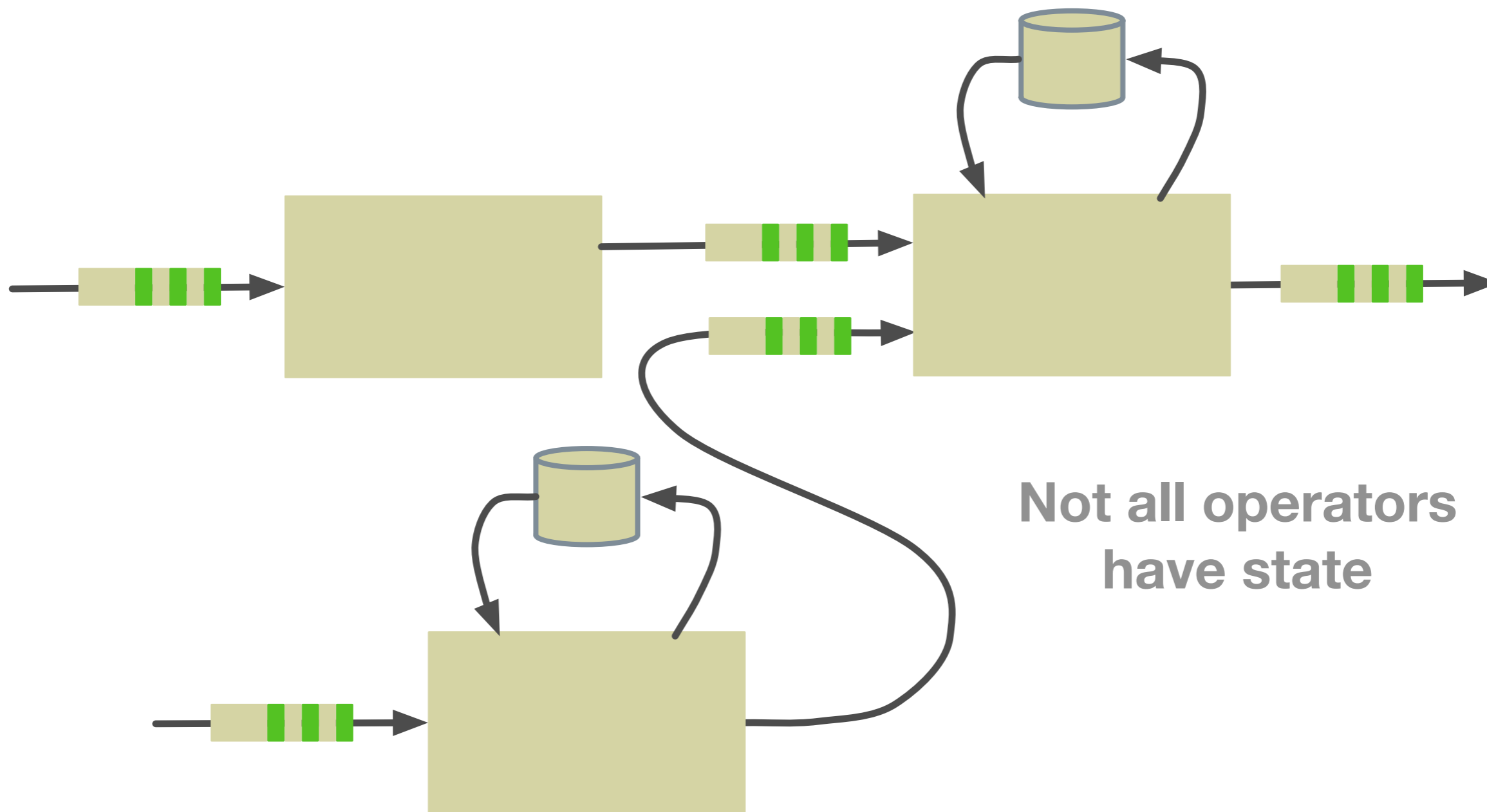
Elements of a Streaming App



Elements of a Streaming App



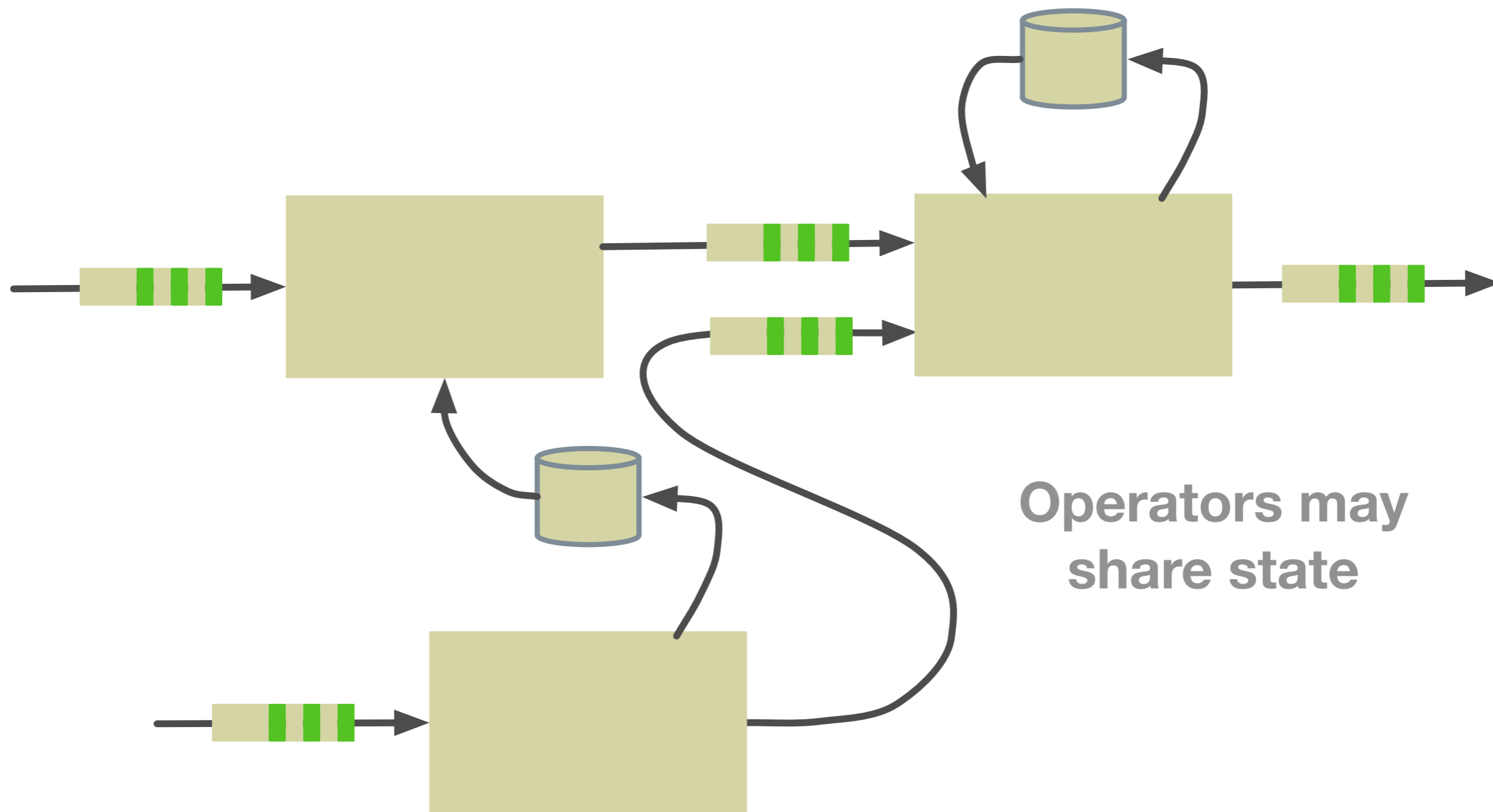
Elements of a Streaming App



Not all operators
have state



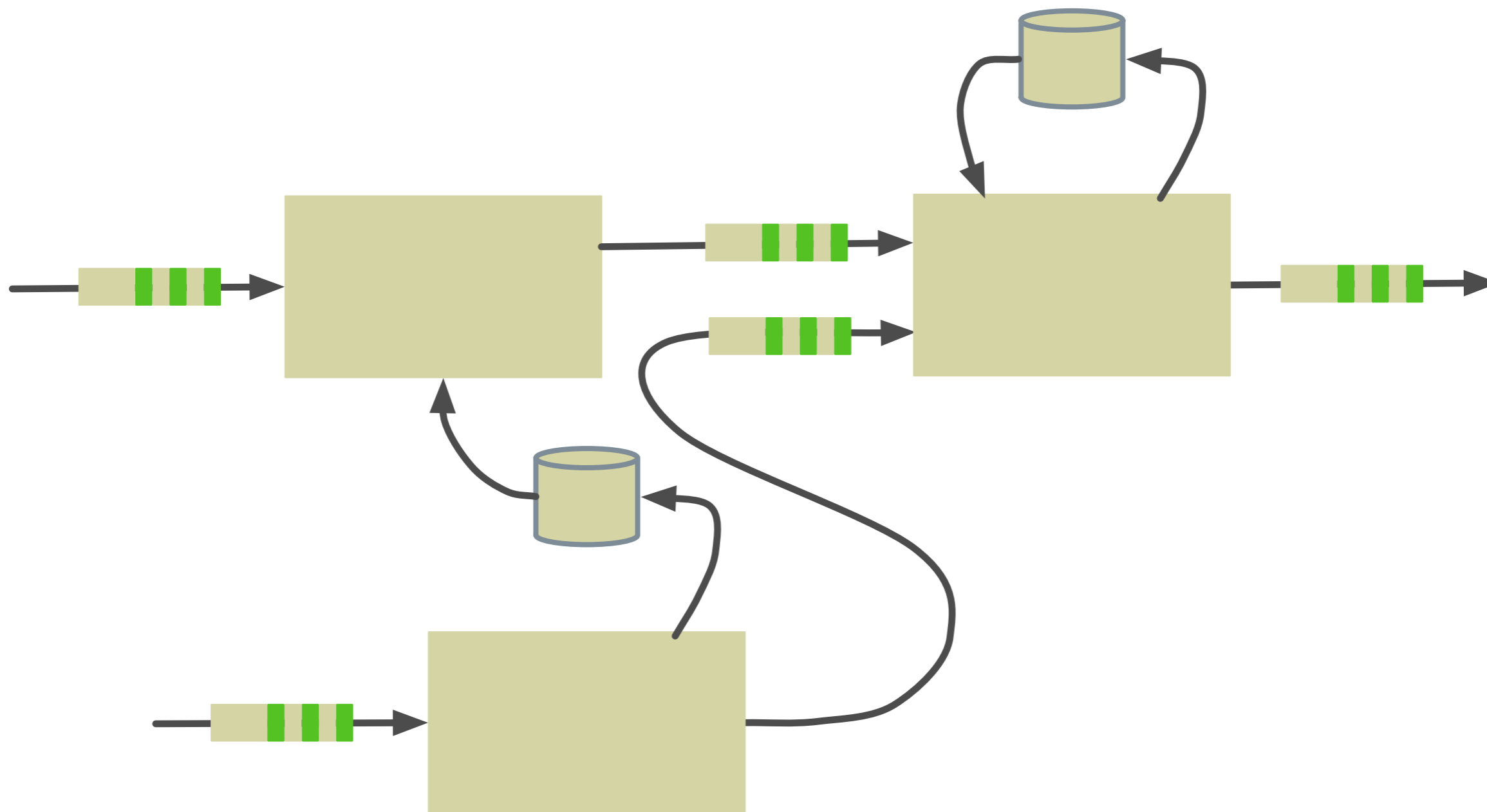
Elements of a Streaming App



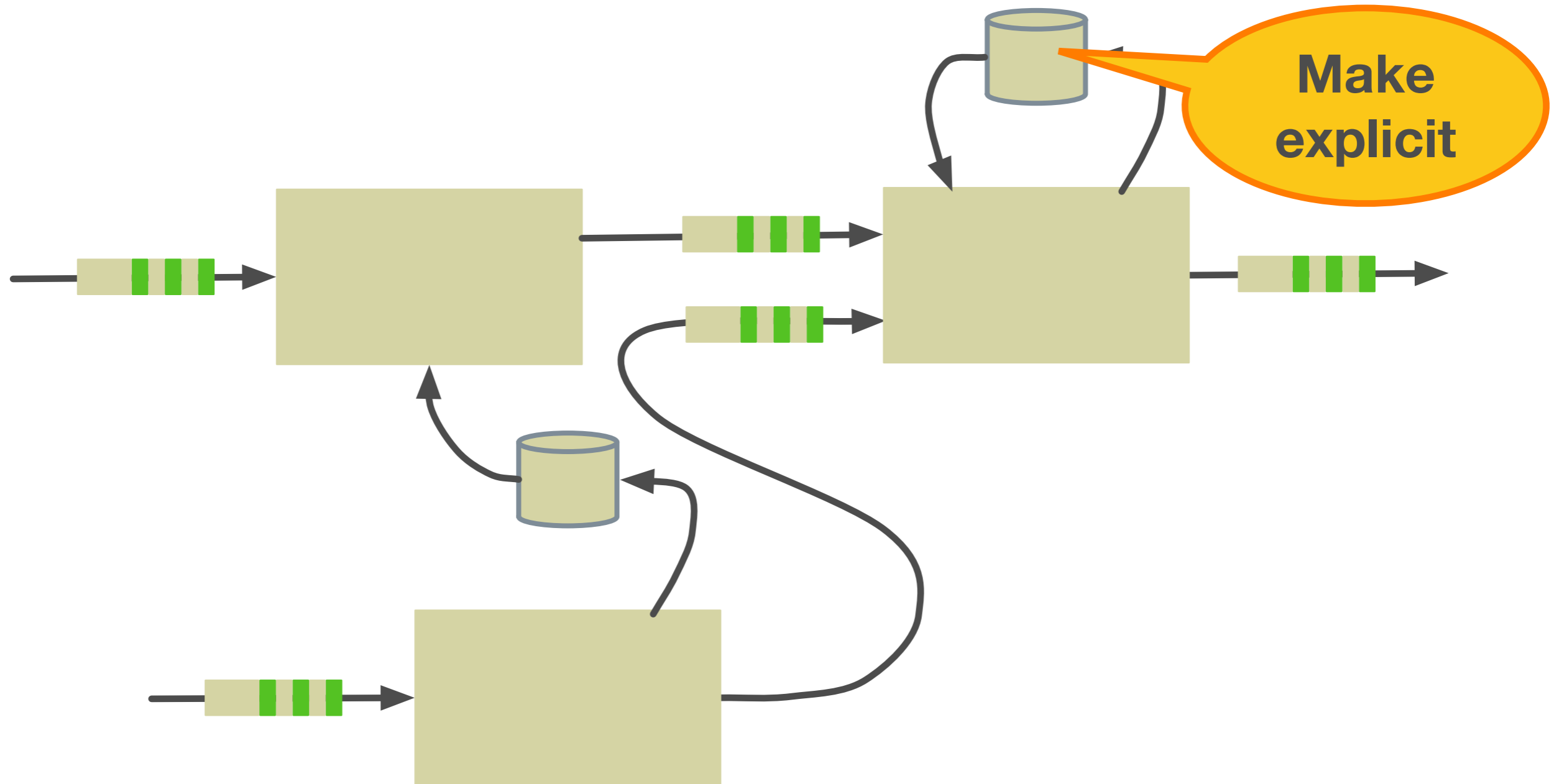
Operators may share state



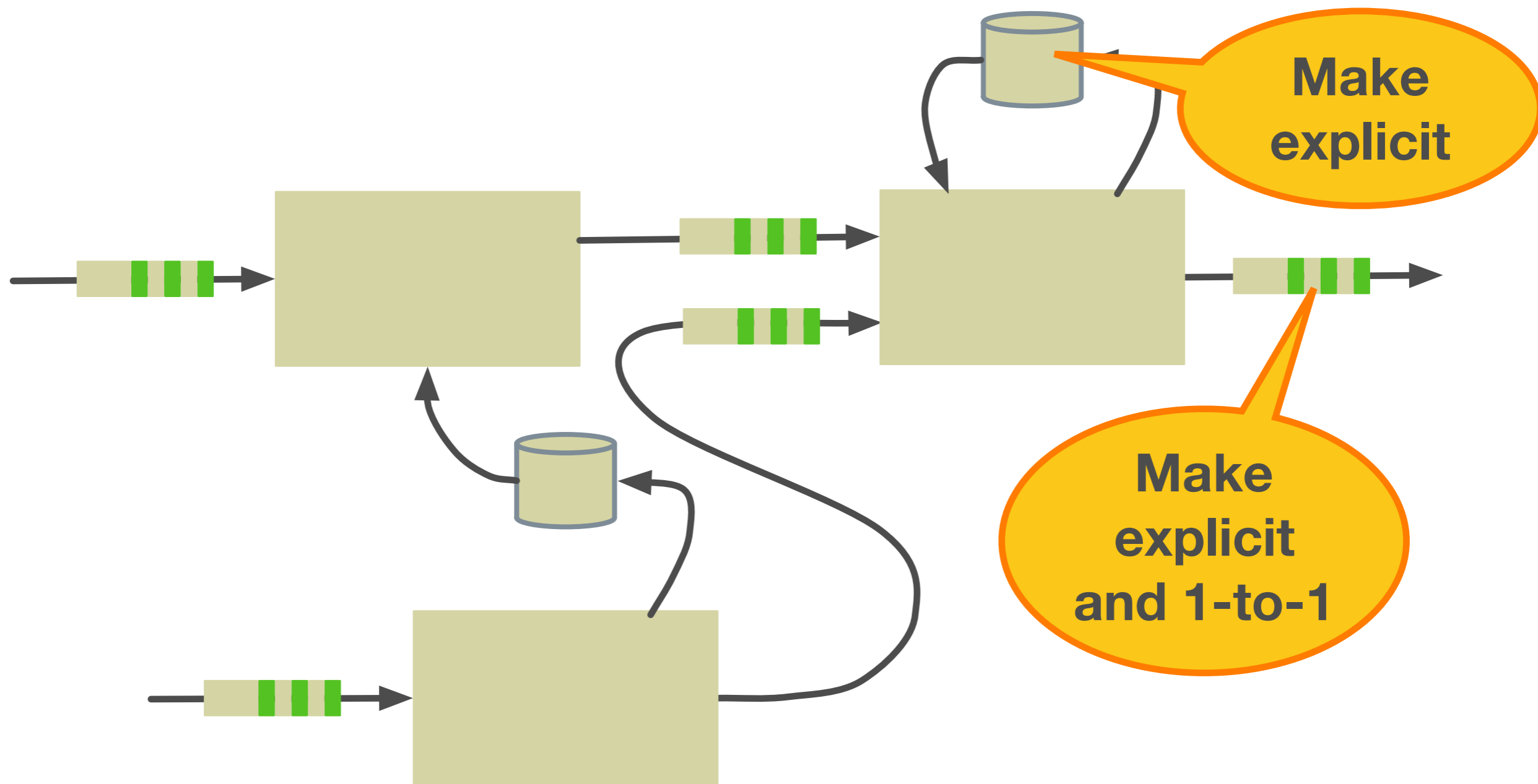
Requirements for Calculus



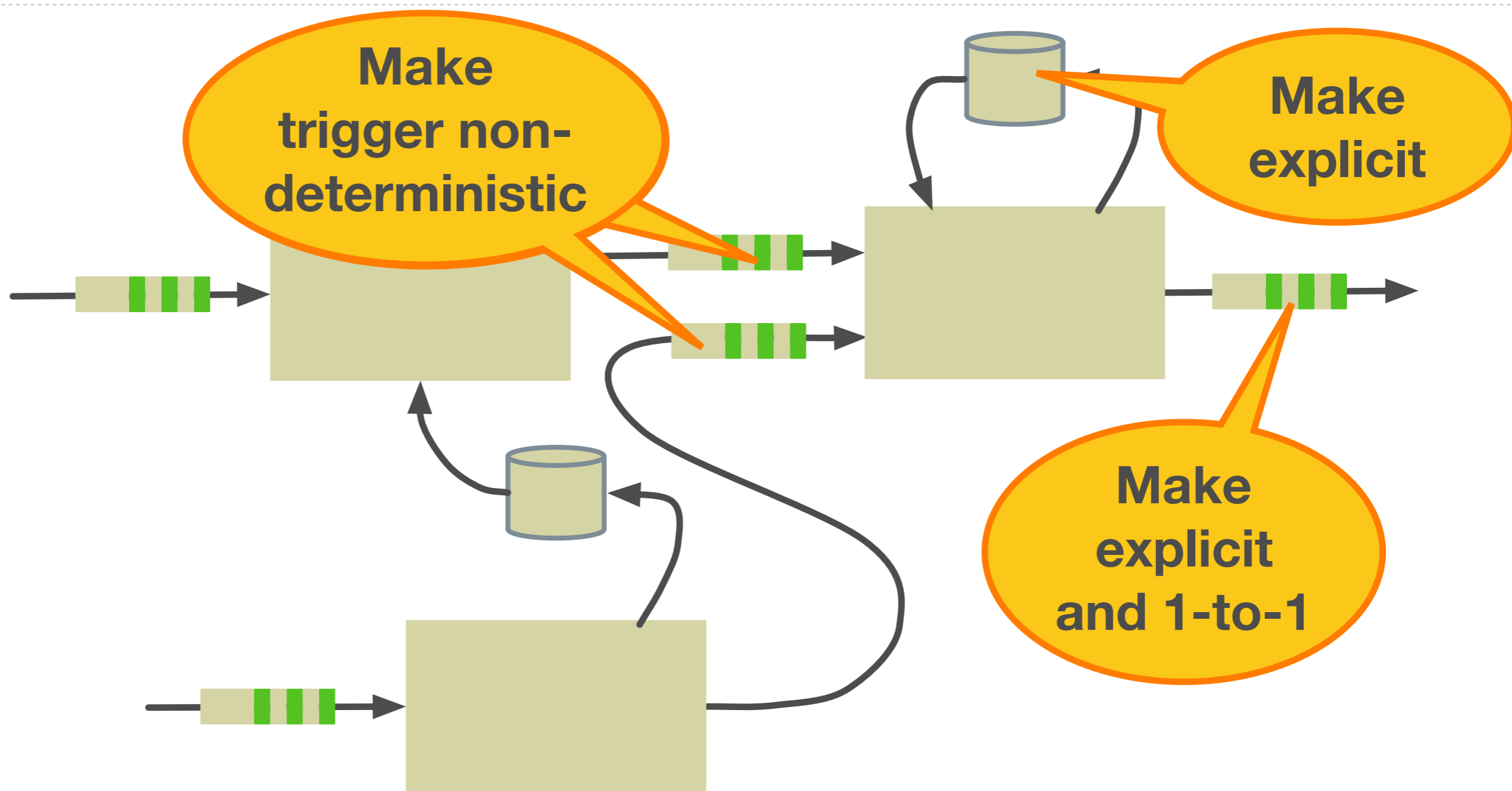
Requirements for Calculus



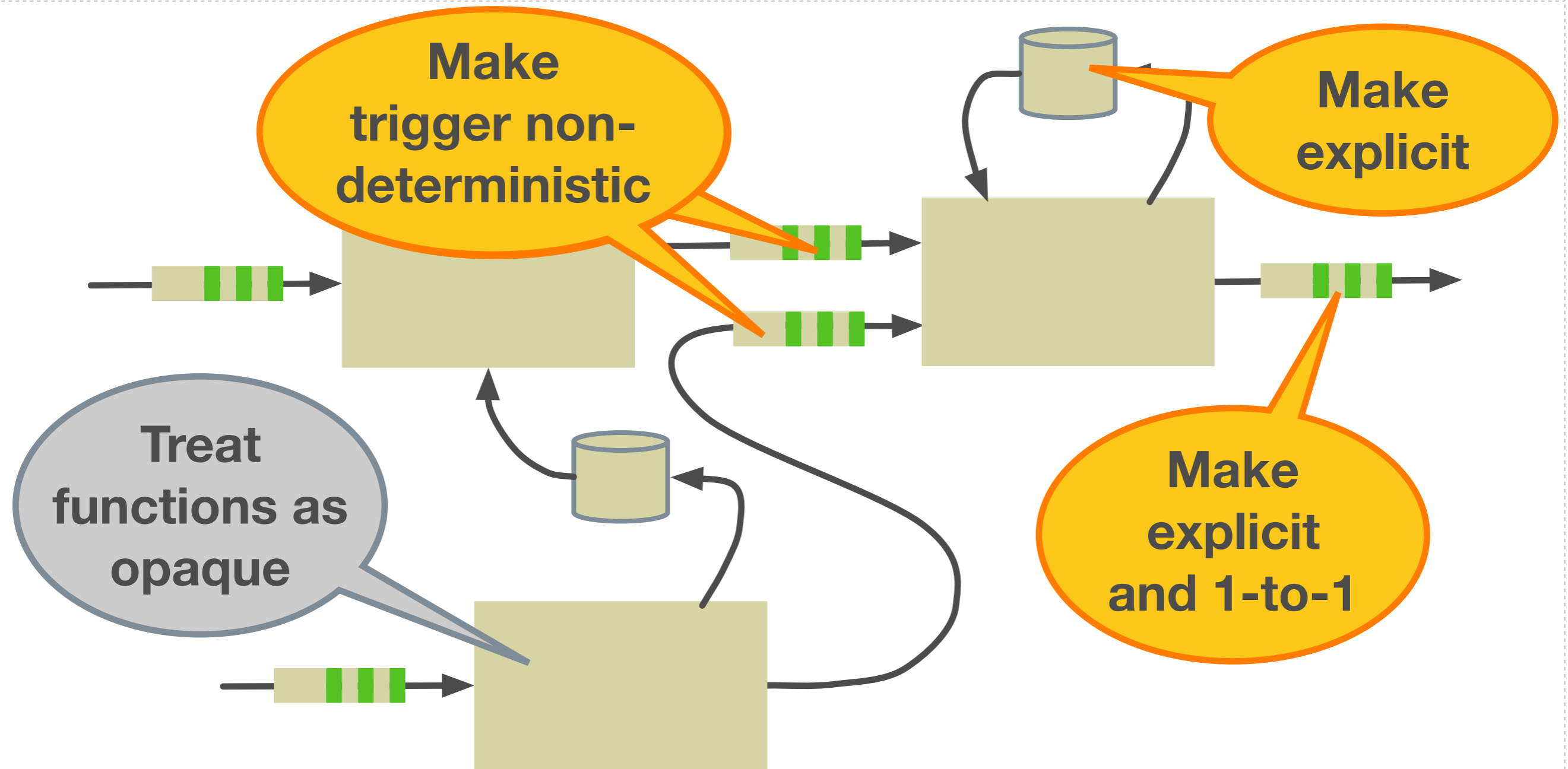
Requirements for Calculus



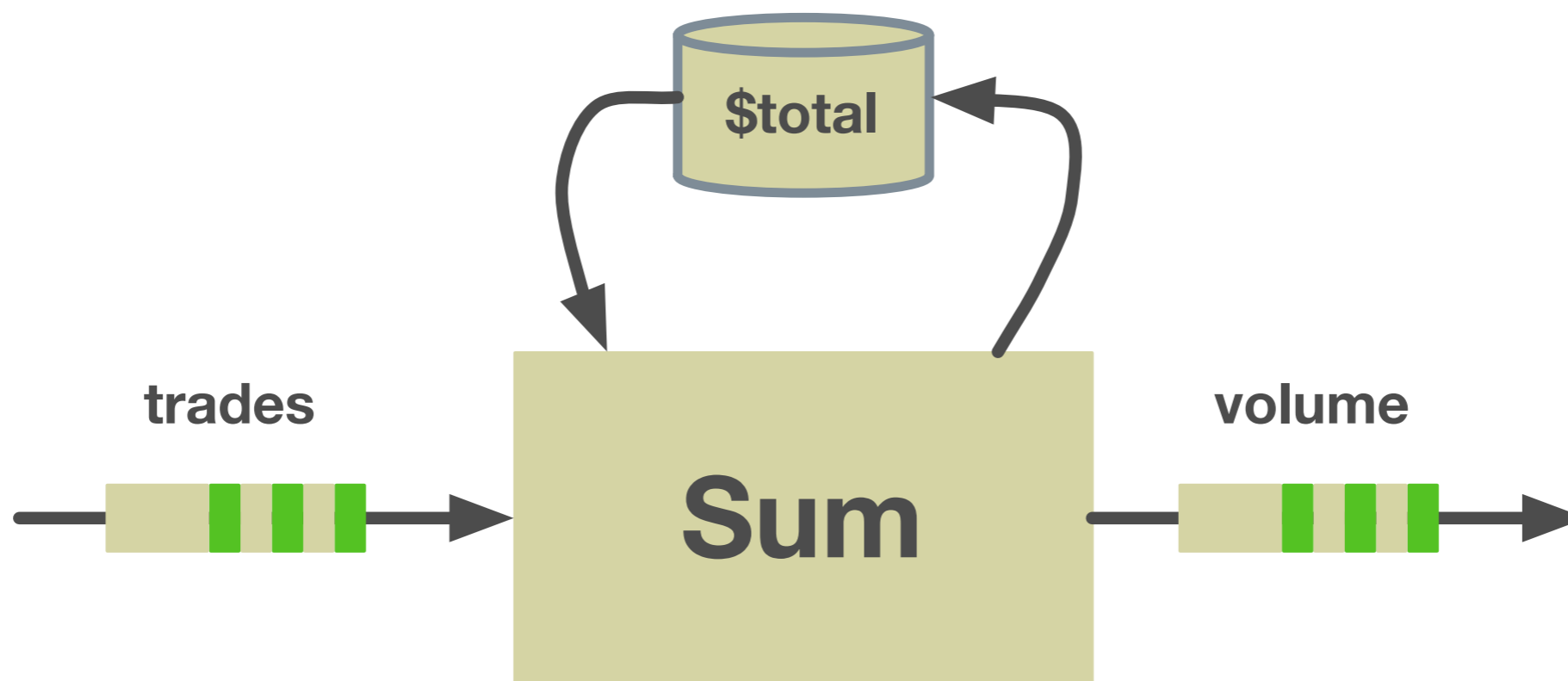
Requirements for Calculus



Requirements for Calculus



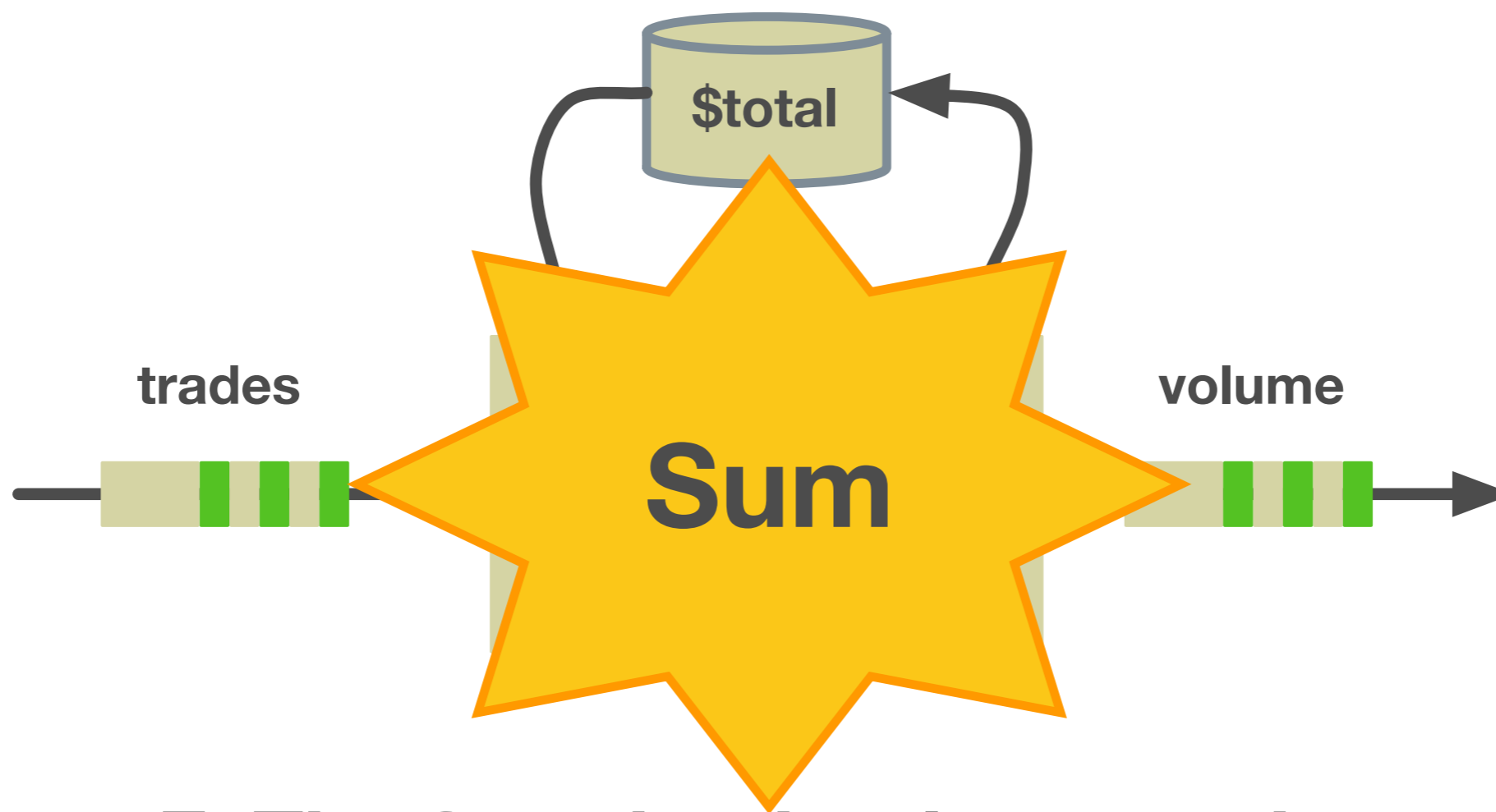
Brooklet Syntax



(volume, \$total) ← Sum(trades, \$total)



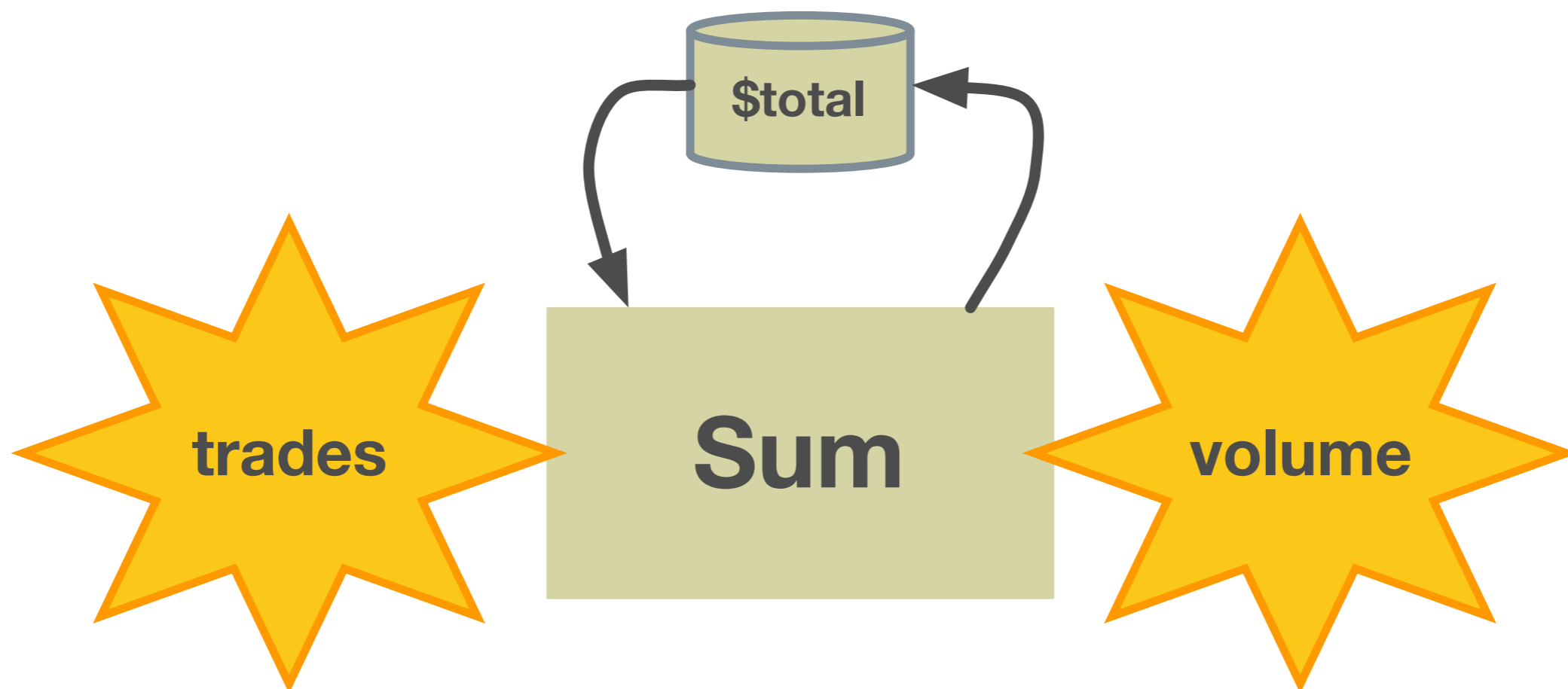
Function Environment



F: The function implementations



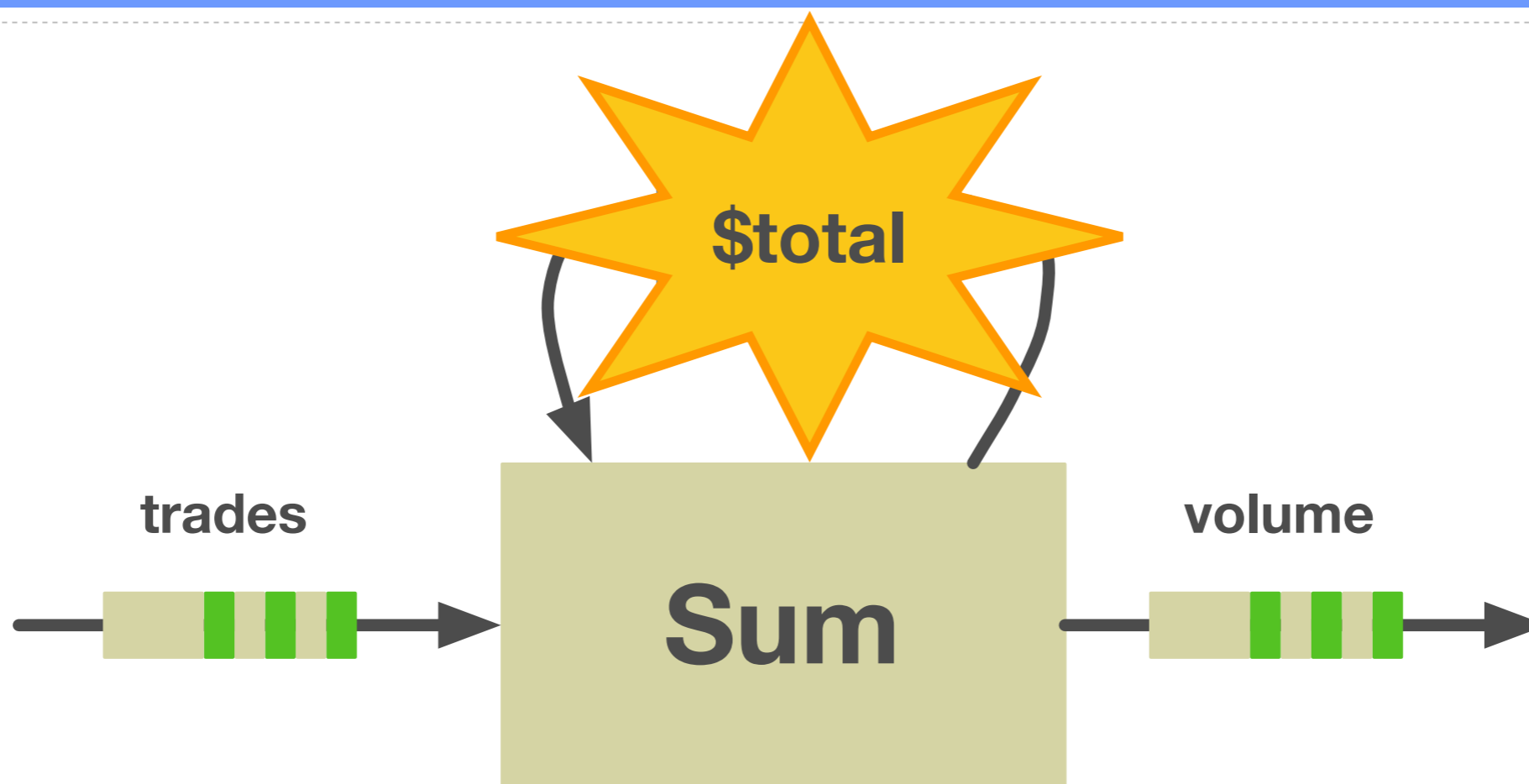
Queue Store



Q: The contents of the queues



Variable Store

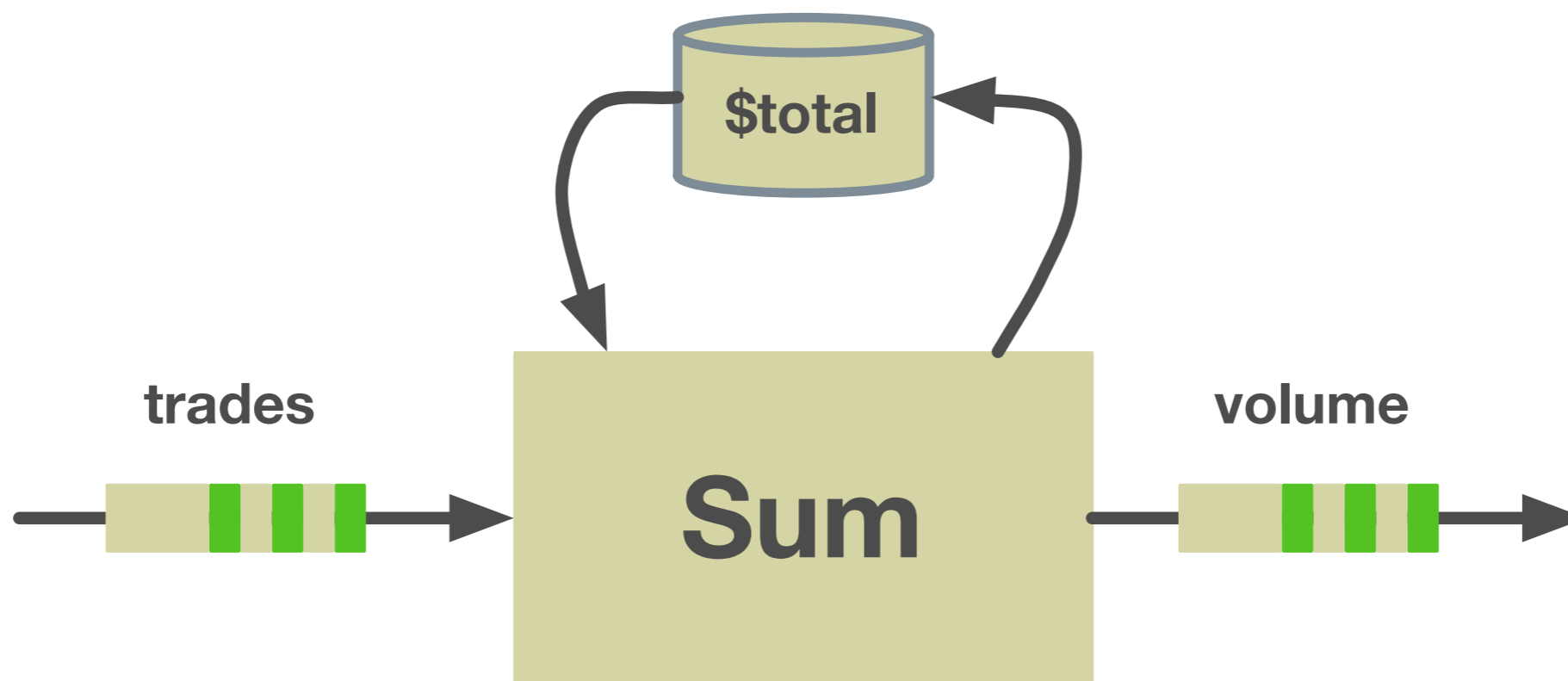


V: The contents of the variables



Brooklet

Operational Semantics



$$F \vdash \langle Q, V \rangle \rightarrow \langle Q', V' \rangle$$



Complete Calculus

Brooklet syntax:

$P_b ::= out\ in\ \bar{op}$	<i>Brooklet program</i>
$out ::= output\ \bar{q};$	<i>Output declaration</i>
$in ::= input\ \bar{q};$	<i>Input declaration</i>
$op ::= (\bar{q}, \bar{v}) \leftarrow f(\bar{q}, \bar{v});$	<i>Operator</i>
$q ::= id$	<i>Queue identifier</i>
$v ::= \$id$	<i>Variable identifier</i>
$f ::= id$	<i>Function identifier</i>

Brooklet example: IBM market maker.

```

output result;
input bids, asks;
(ibmBids) ← SelectIBM(bids);
(ibmAsks) ← SelectIBM(asks);
($lastAsk) ← Window(ibmAsks);
(ibmSales) ← SaleJoin(ibmBids, $lastAsk);
(result, $cnt) ← Count(ibmSales, $cnt);

```

Brooklet semantics: $F_b \vdash \langle V, Q \rangle \longrightarrow \langle V', Q' \rangle$

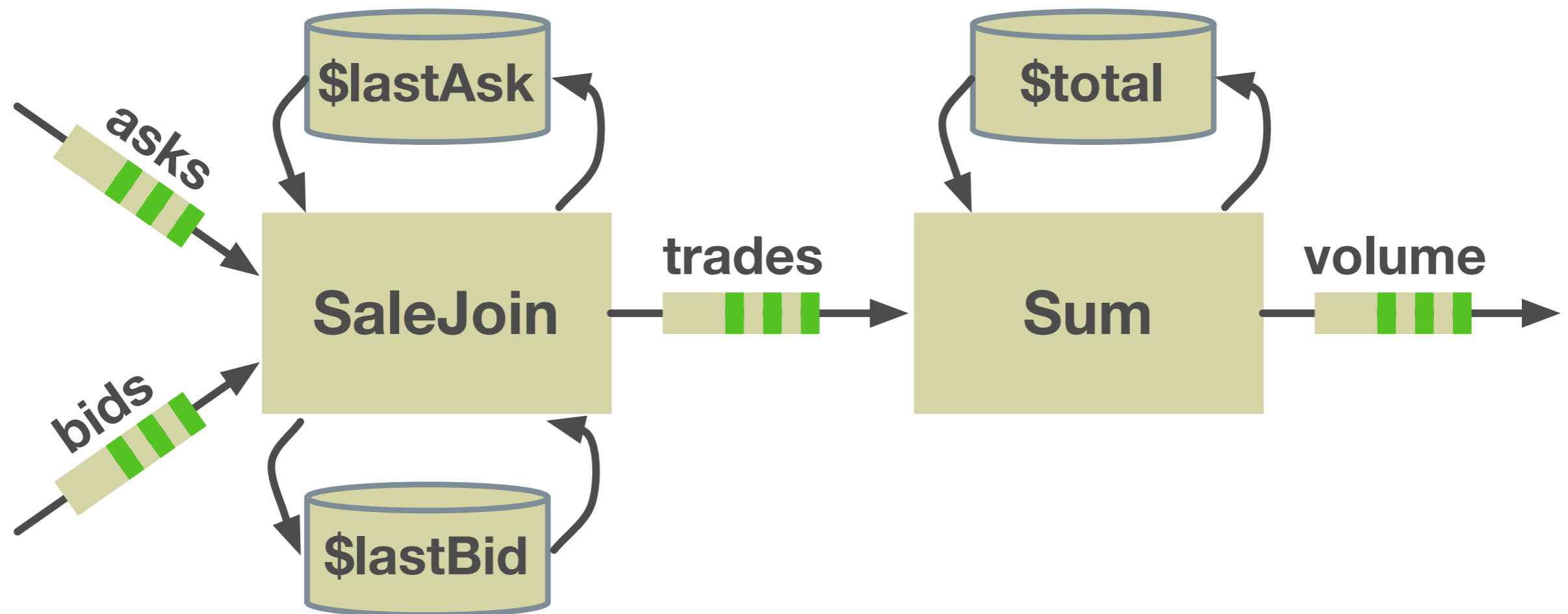
$$\begin{array}{l}
 d, b = Q(q_i) \\
 op = (_, _) \leftarrow f(\bar{q}, \bar{v}); \\
 (\bar{b}', \bar{d}') = F_b(f)(d, i, V(\bar{v})) \\
 V' = updateV(op, V, \bar{d}') \\
 Q' = updateQ(op, Q, q_i, \bar{b}') \\
 \hline
 F_b \vdash \langle V, Q \rangle \longrightarrow \langle V', Q' \rangle
 \end{array}
 \quad \text{(E-FIREQUEUE)}$$

$$\begin{array}{l}
 op = (_, \bar{v}) \leftarrow f(_, _); \\
 \hline
 updateV(op, V, \bar{d}) = [\bar{v} \mapsto \bar{d}]V
 \end{array}
 \quad \text{(E-UPDATEV)}$$

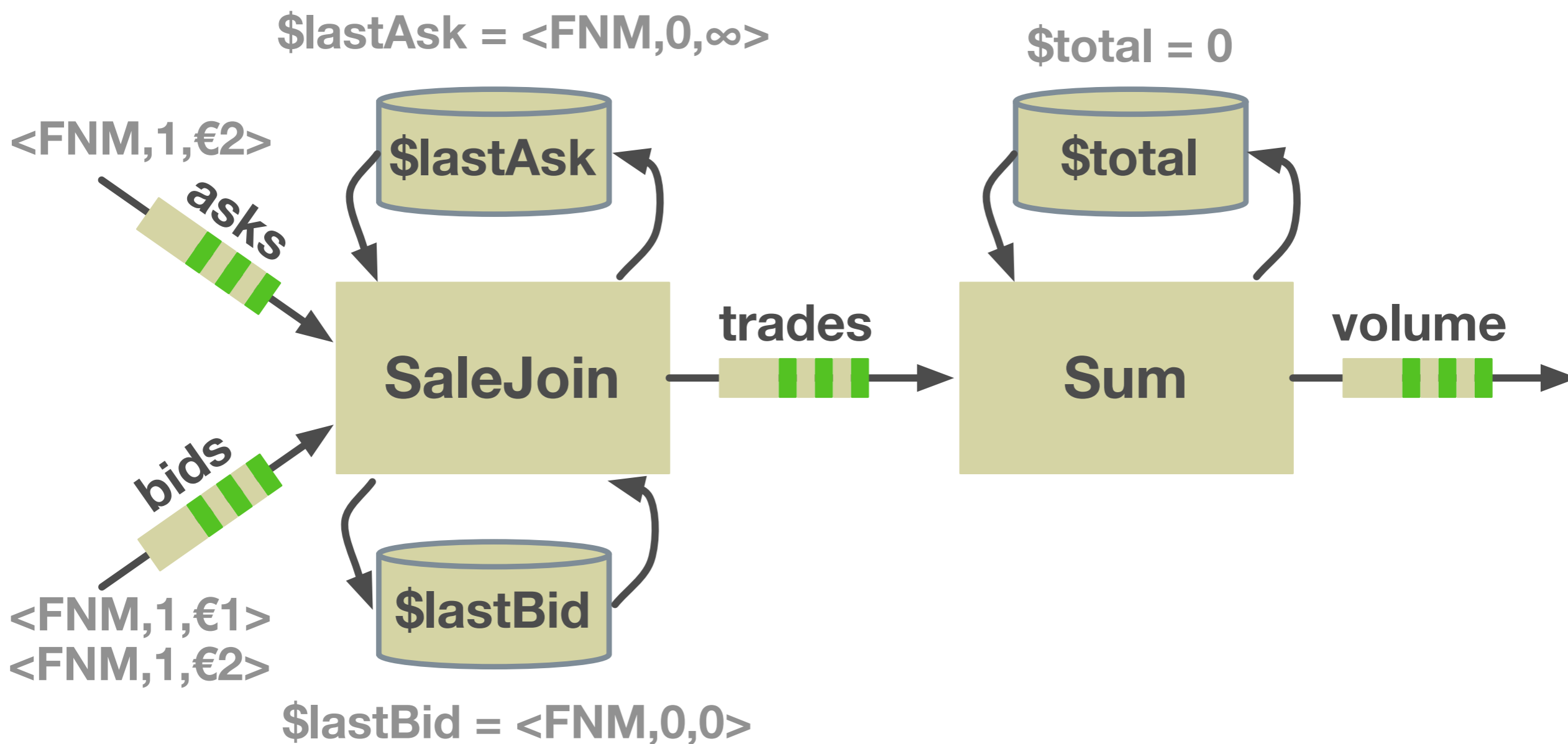
$$\begin{array}{l}
 op = (\bar{q}, _) \leftarrow f(_, _); \\
 d_f, b_f = Q(q_f) \\
 Q' = [q_f \mapsto b_f]Q \\
 Q'' = [\forall q_i \in \bar{q} : q_i \mapsto Q(q_i), b_i]Q' \\
 \hline
 updateQ(op, Q, q_f, \bar{b}) = Q''
 \end{array}
 \quad \text{(E-UPDATEQ)}$$



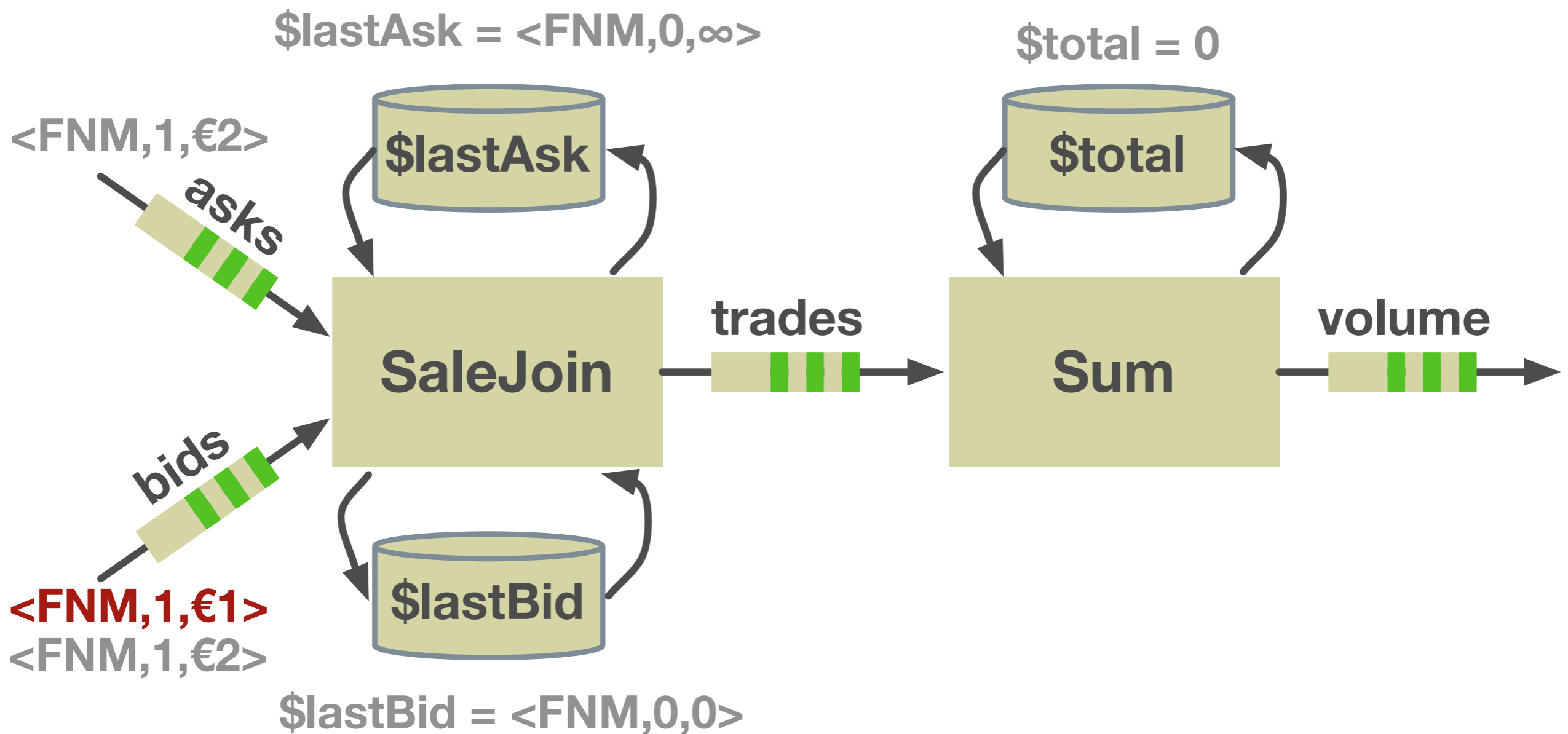
Example: A Fannie Mae Bid/Ask Join



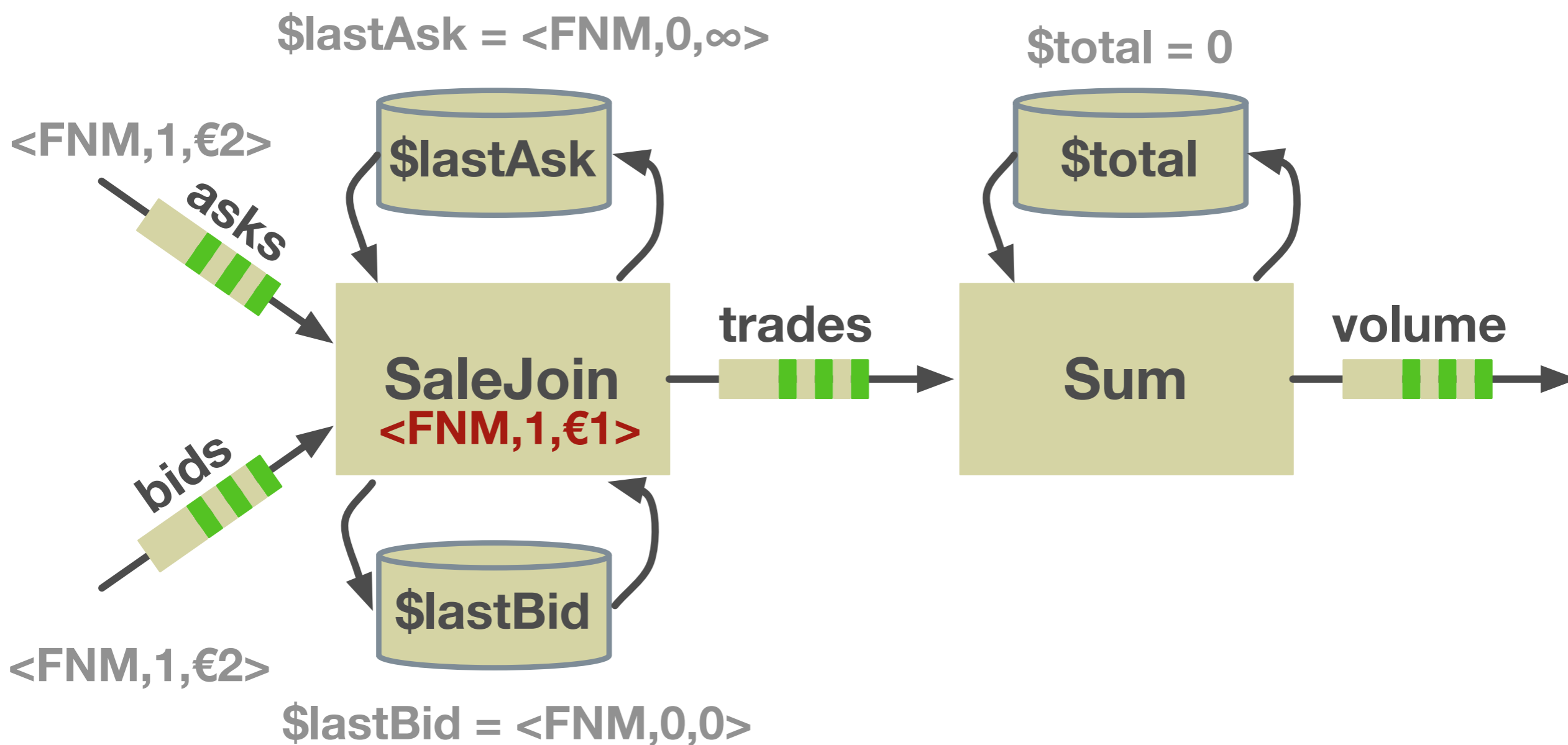
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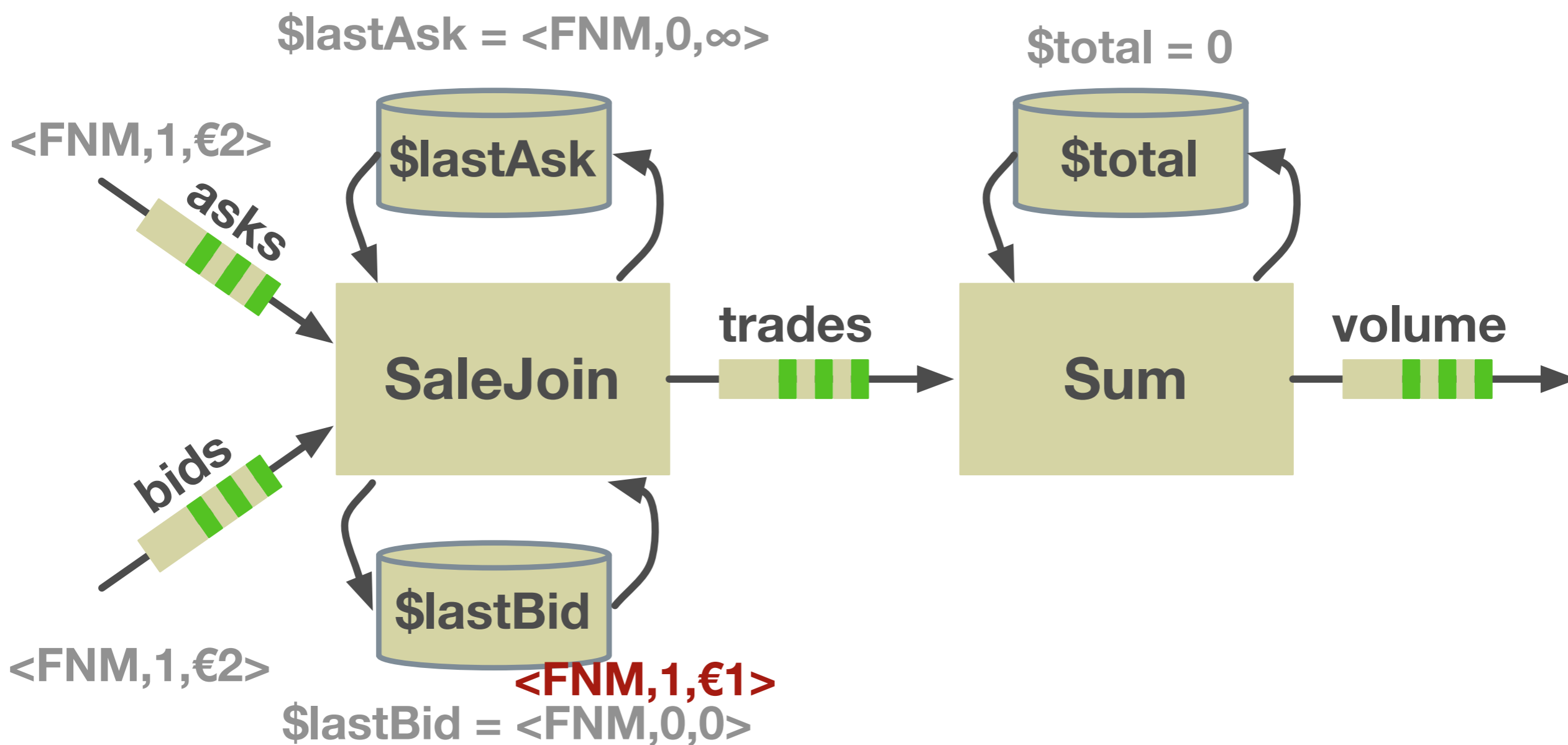
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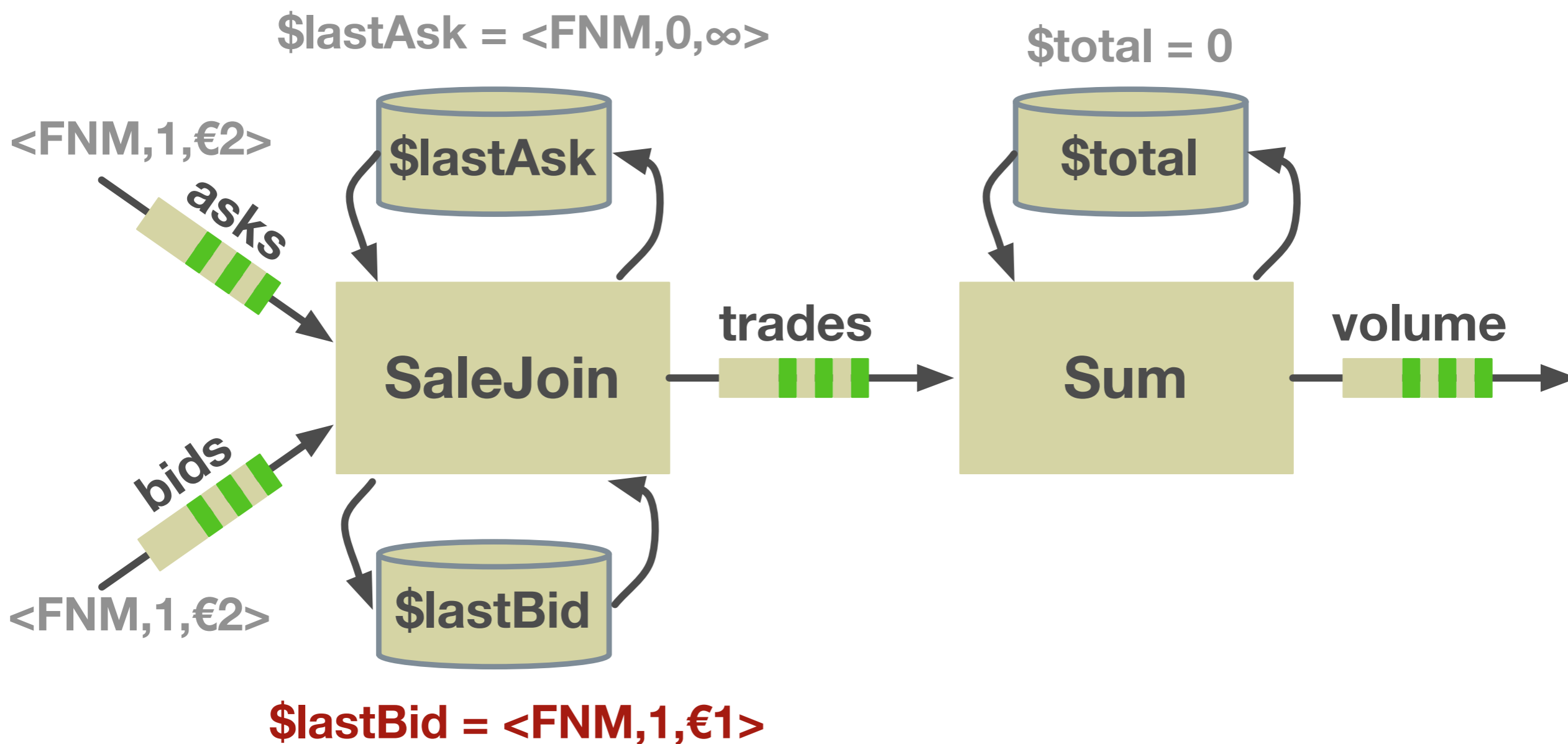
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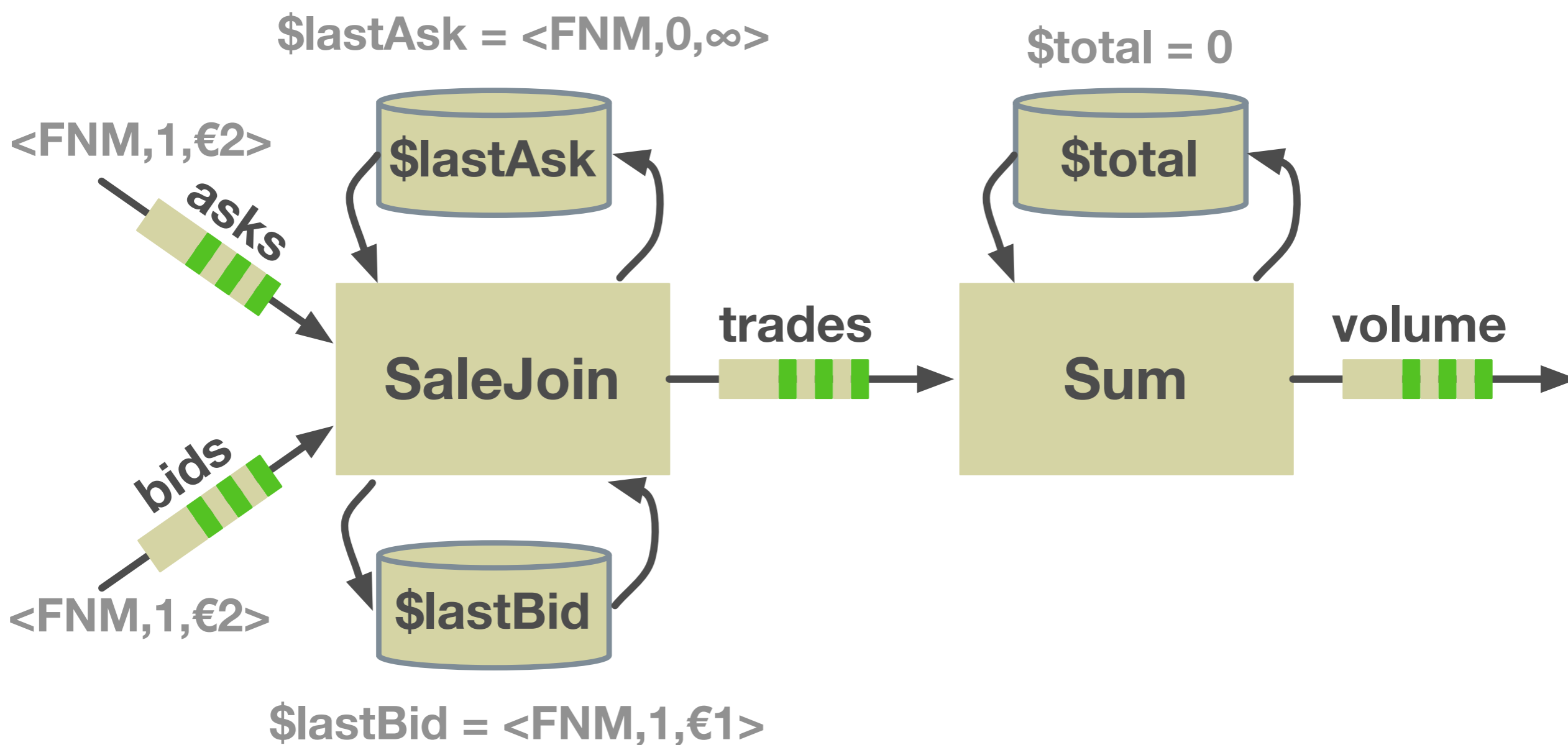
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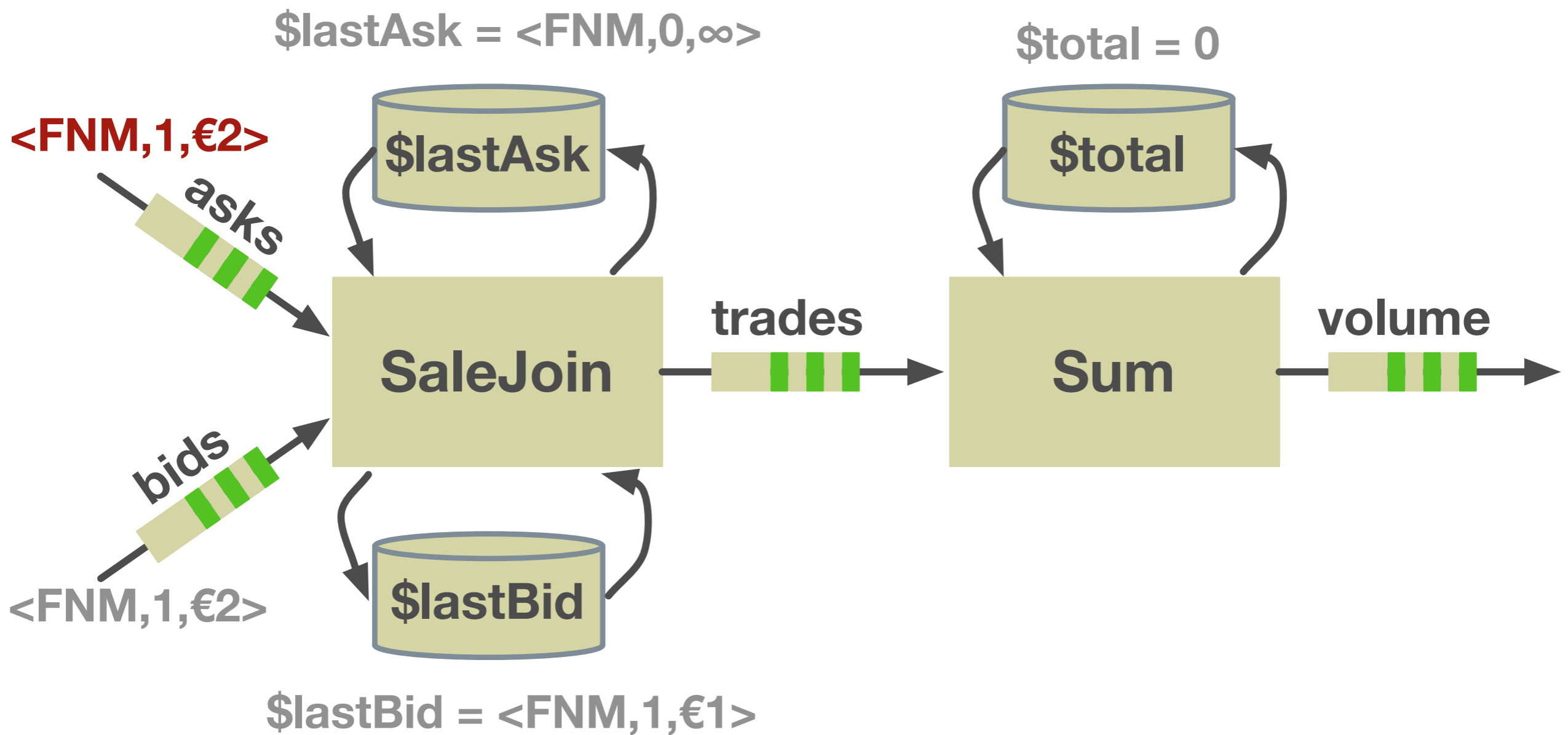
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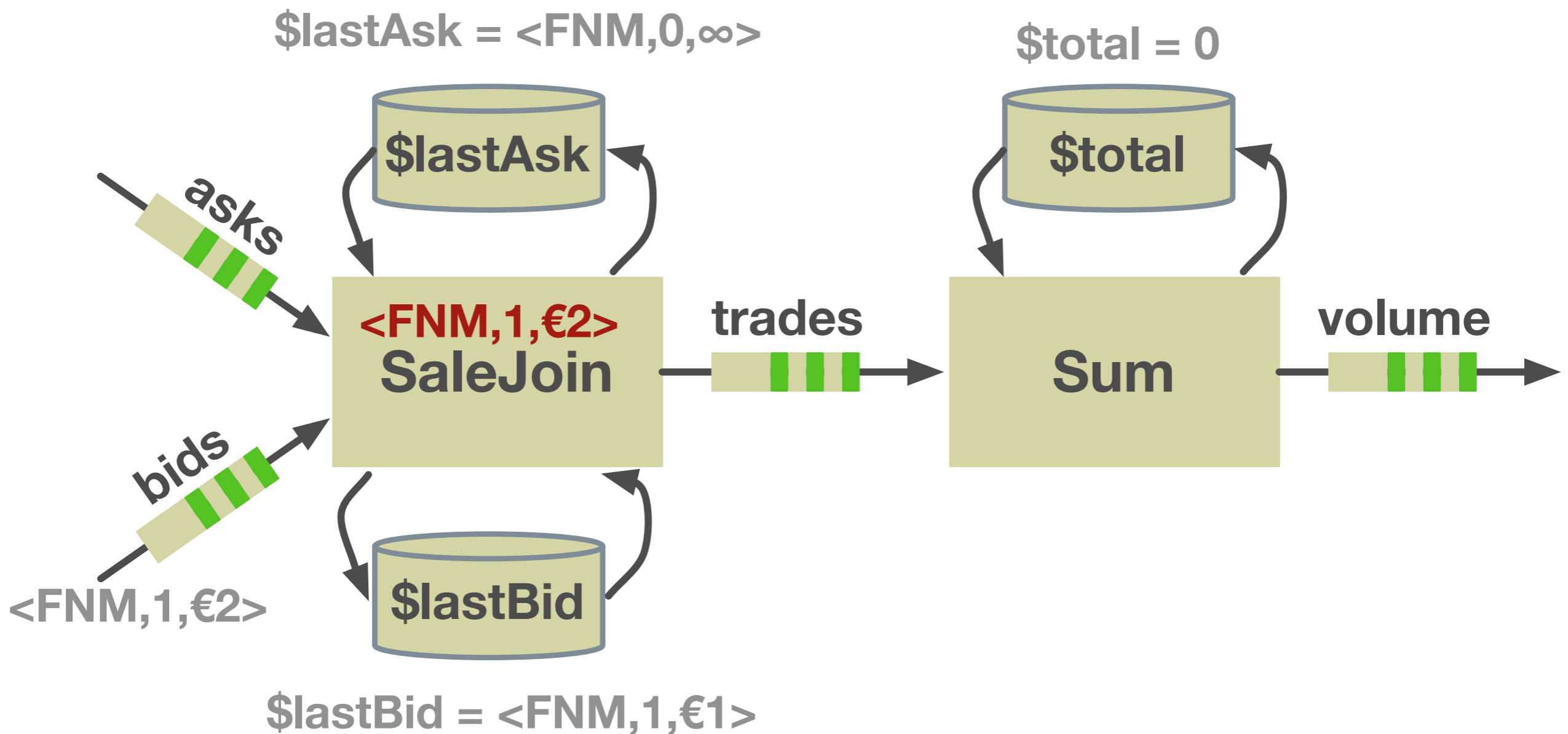
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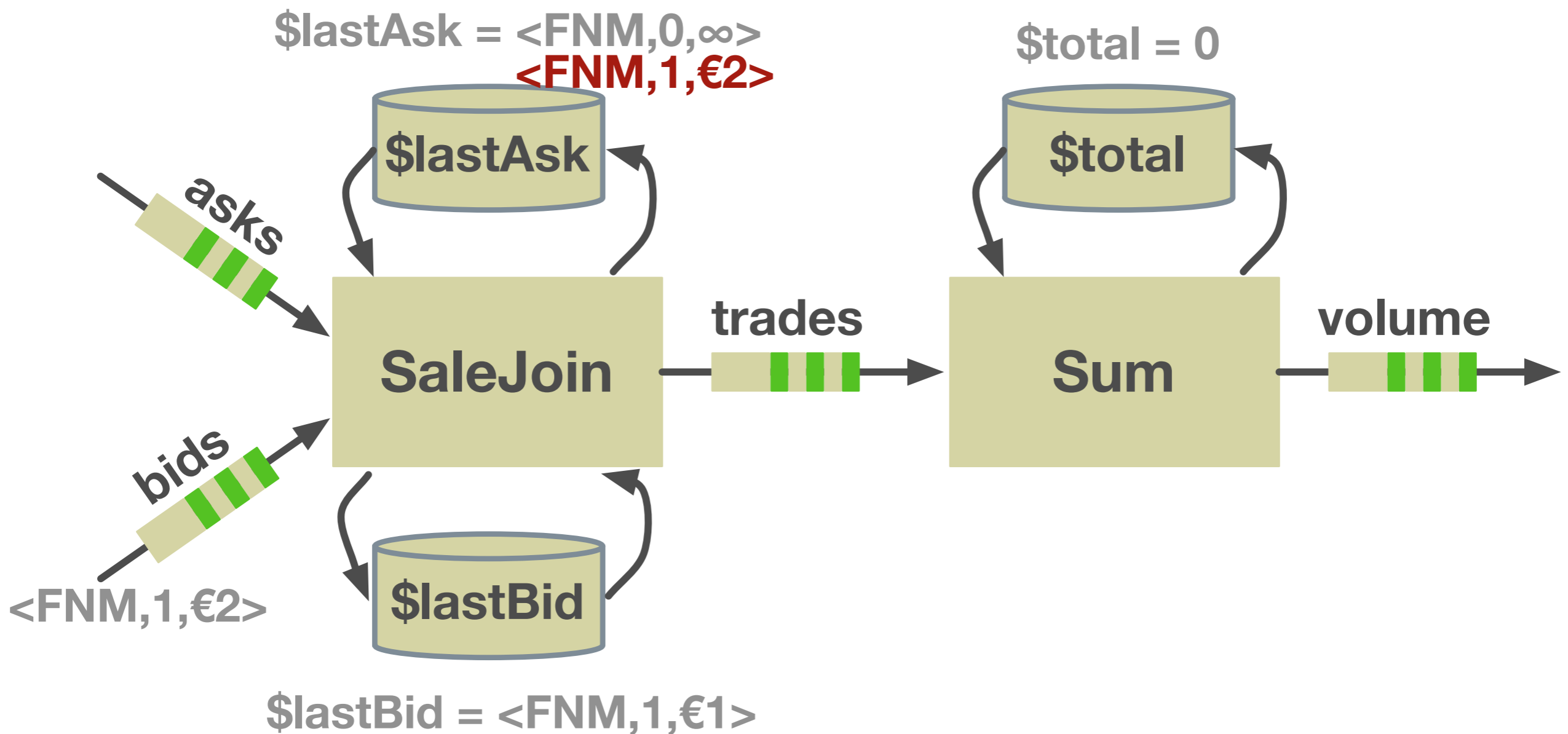
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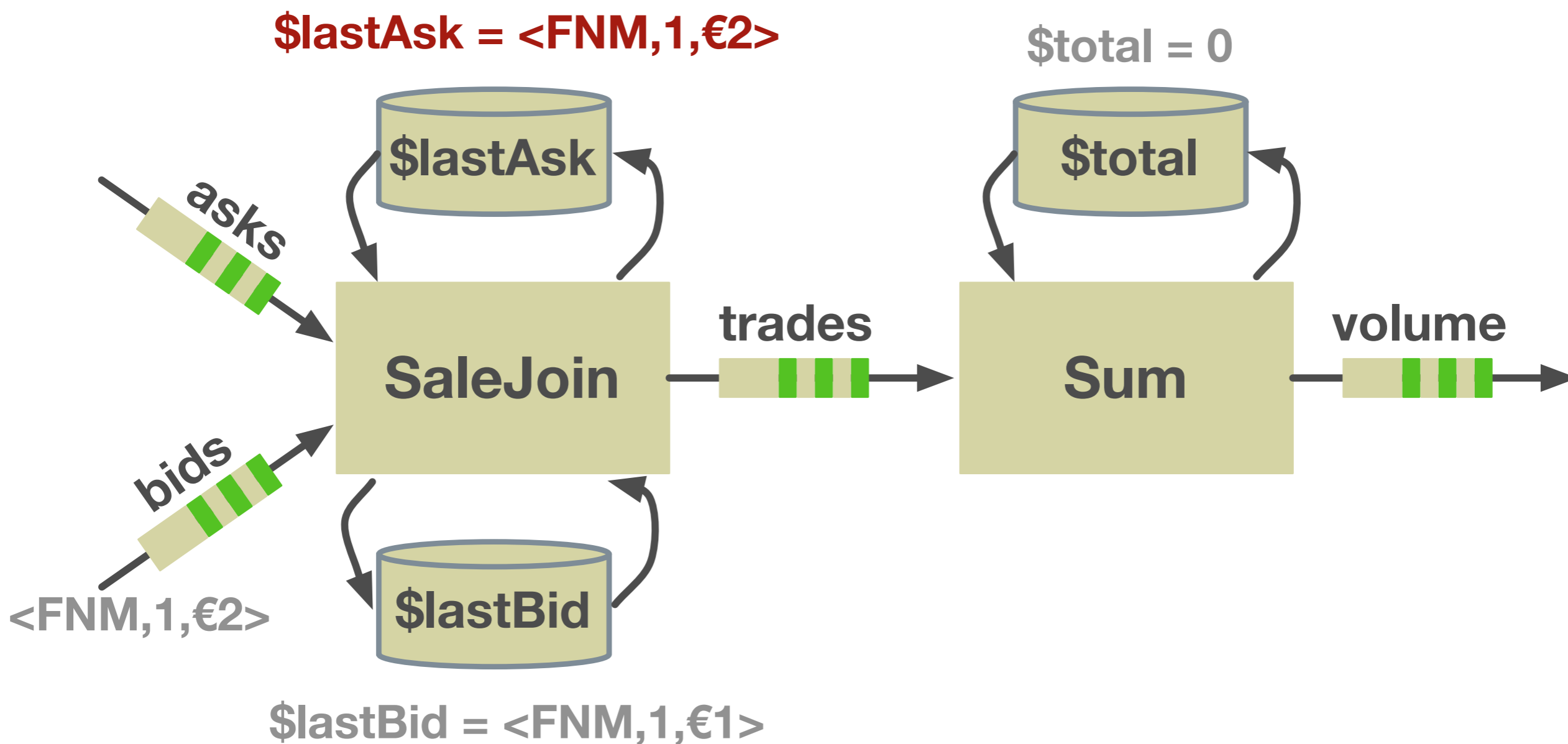
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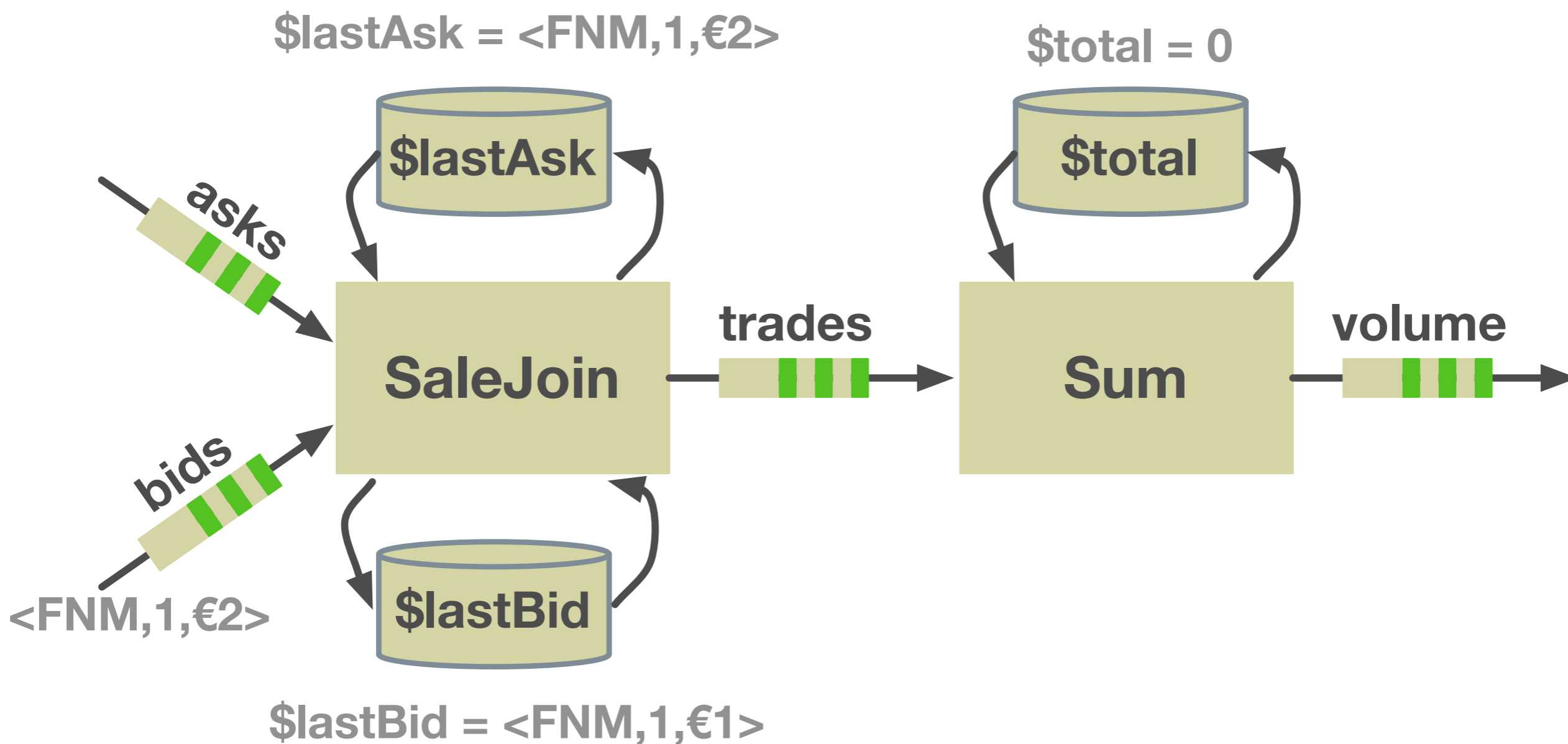
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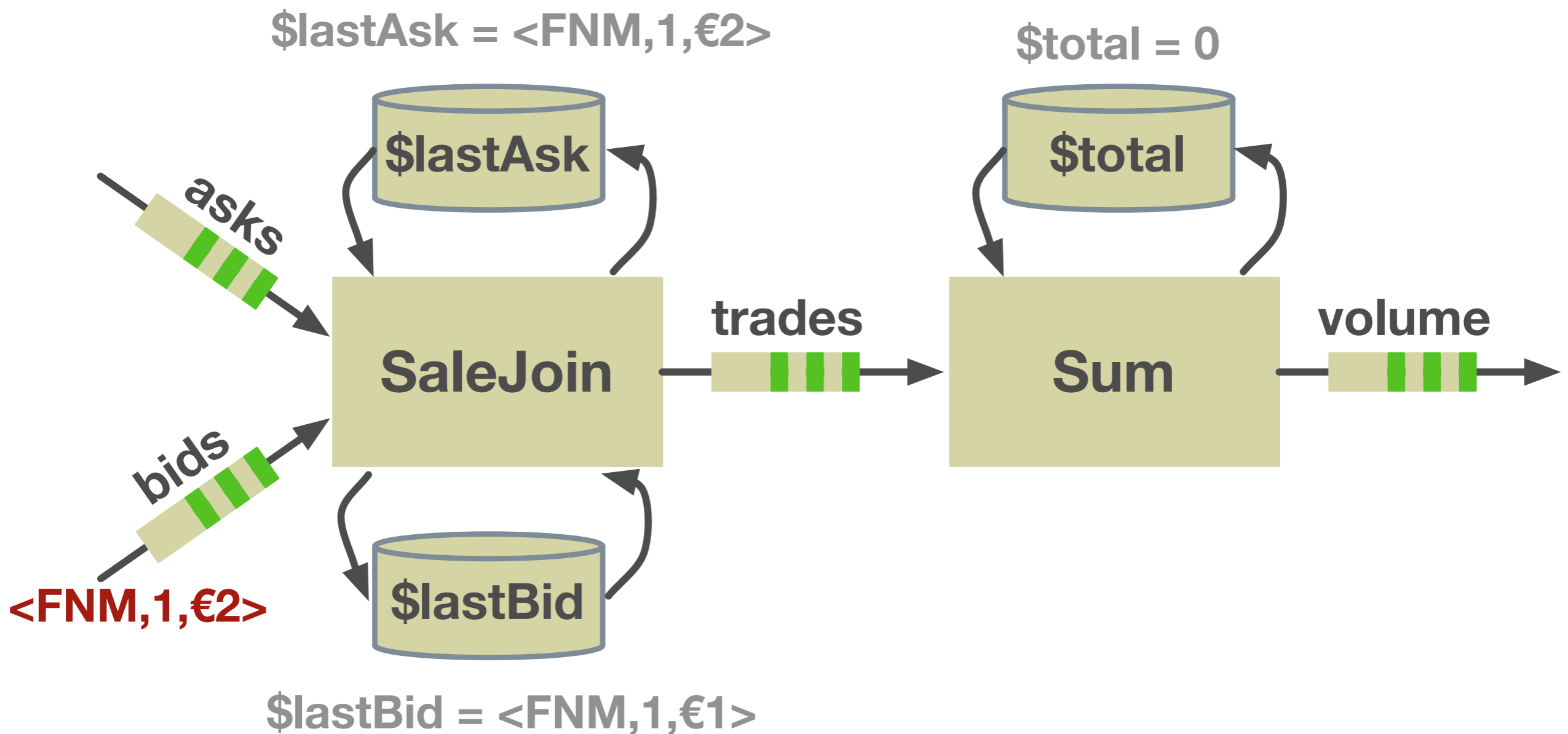
Example: A Fannie Mae Bid/Ask Join



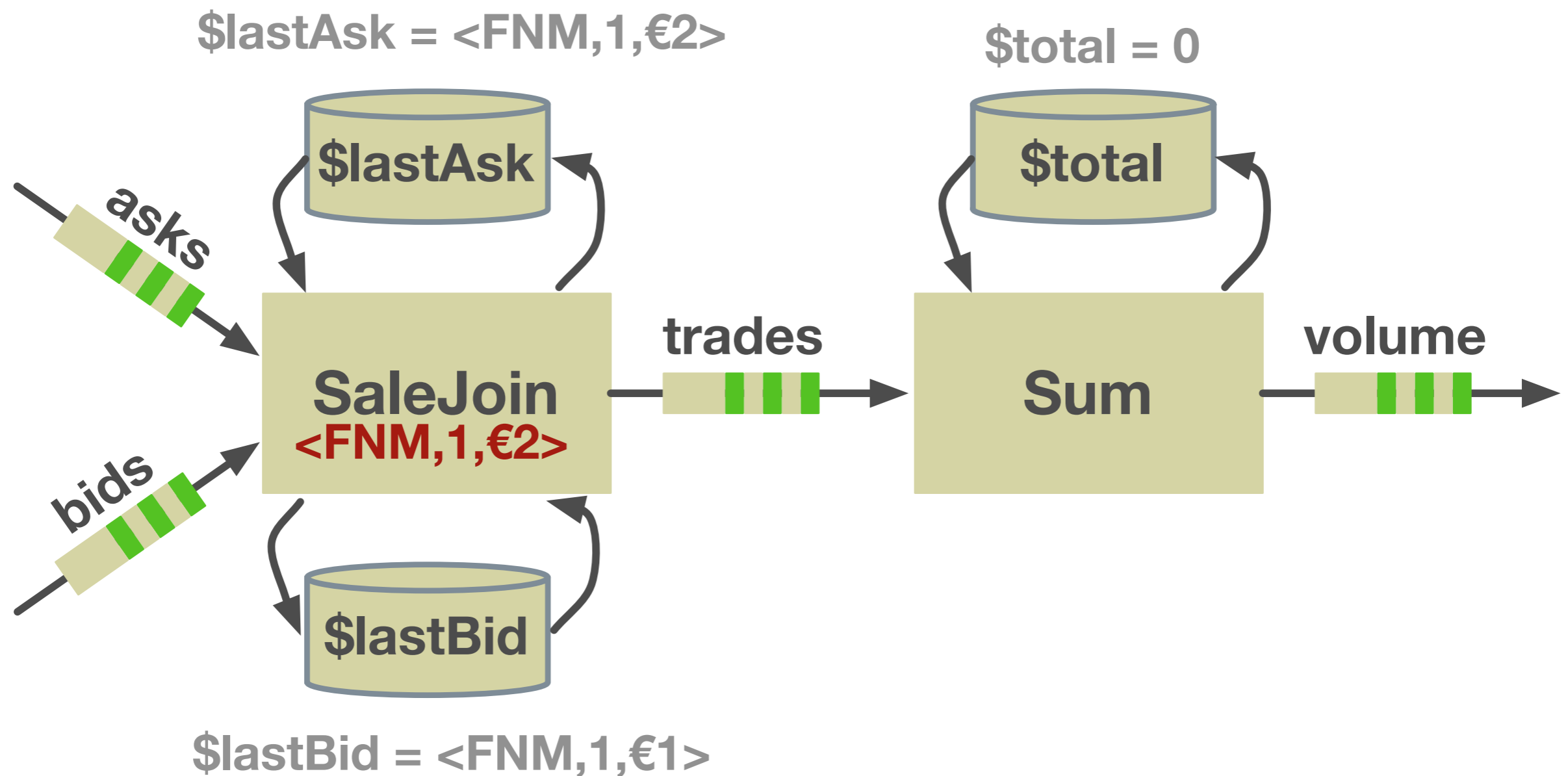
Example: A Fannie Mae Bid/Ask Join



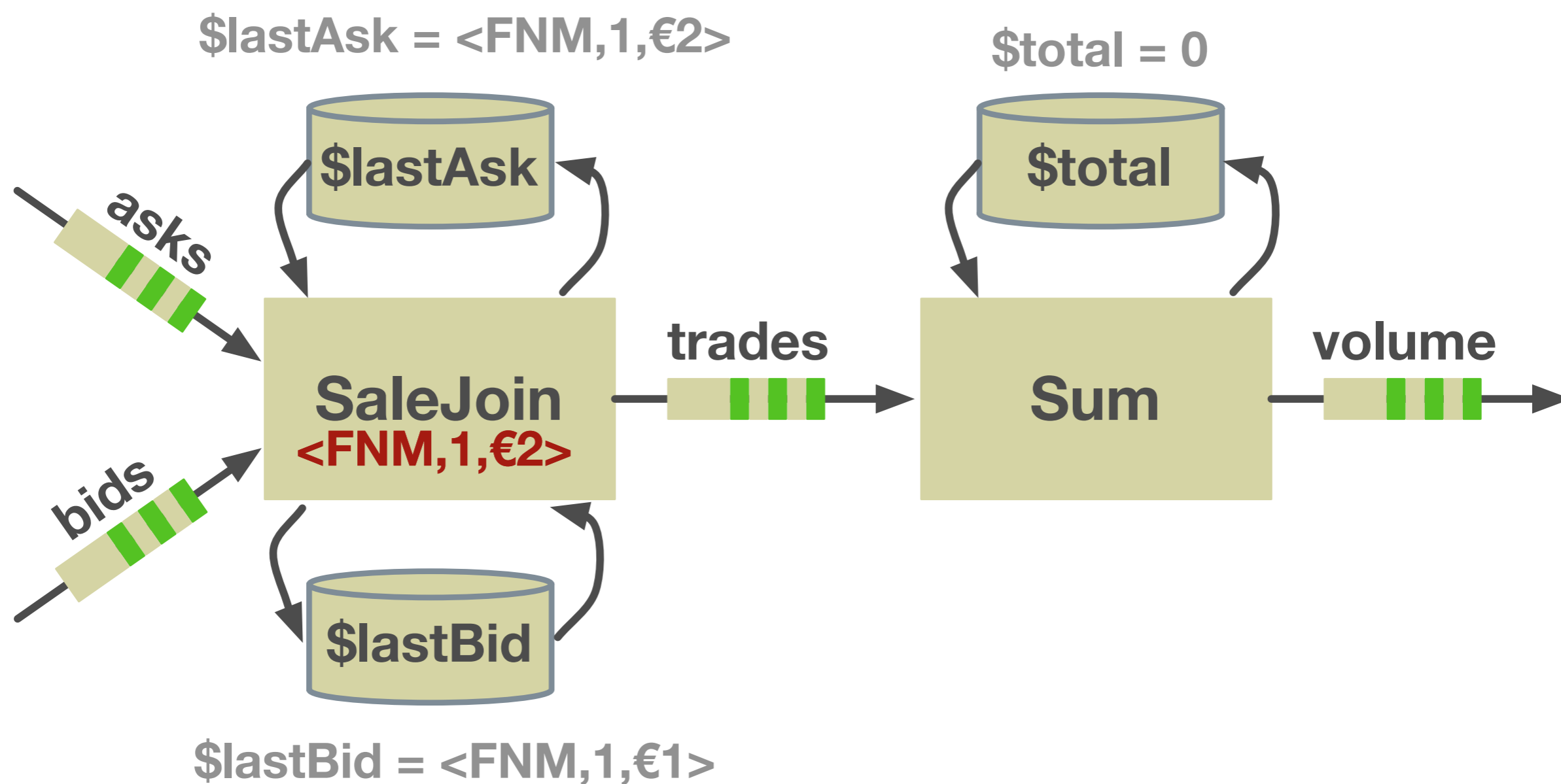
Example: A Fannie Mae Bid/Ask Join



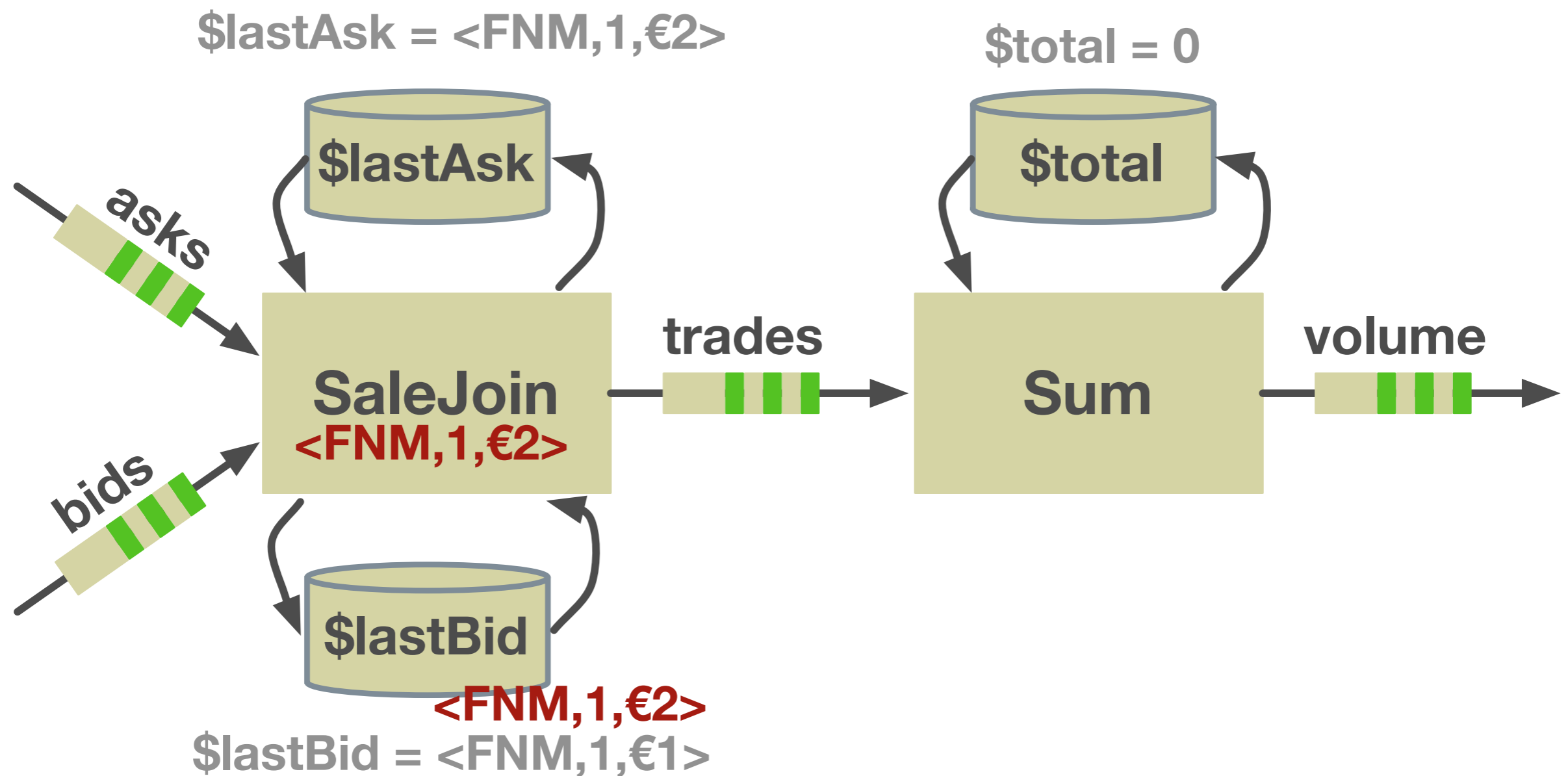
Example: A Fannie Mae Bid/Ask Join



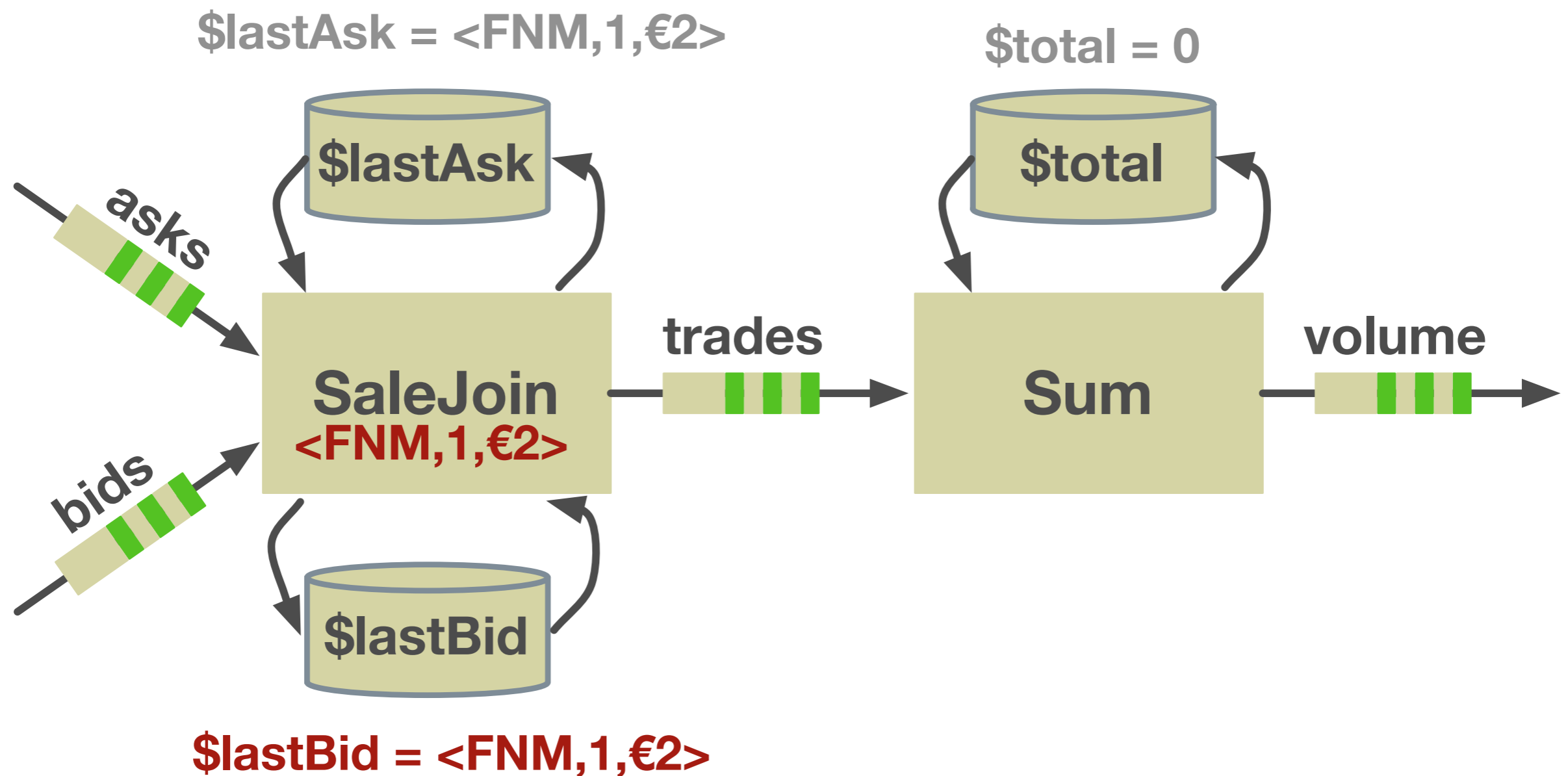
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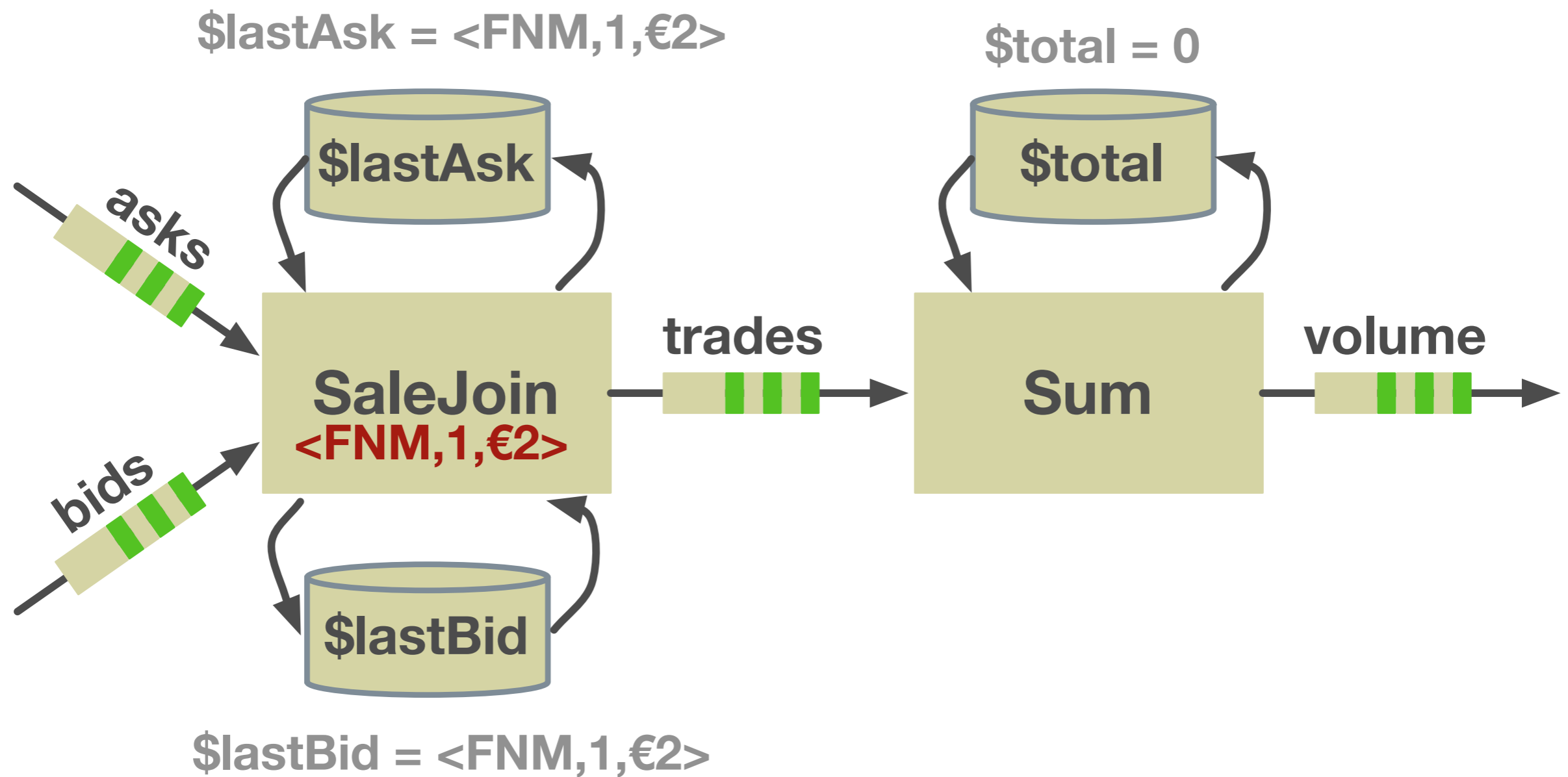
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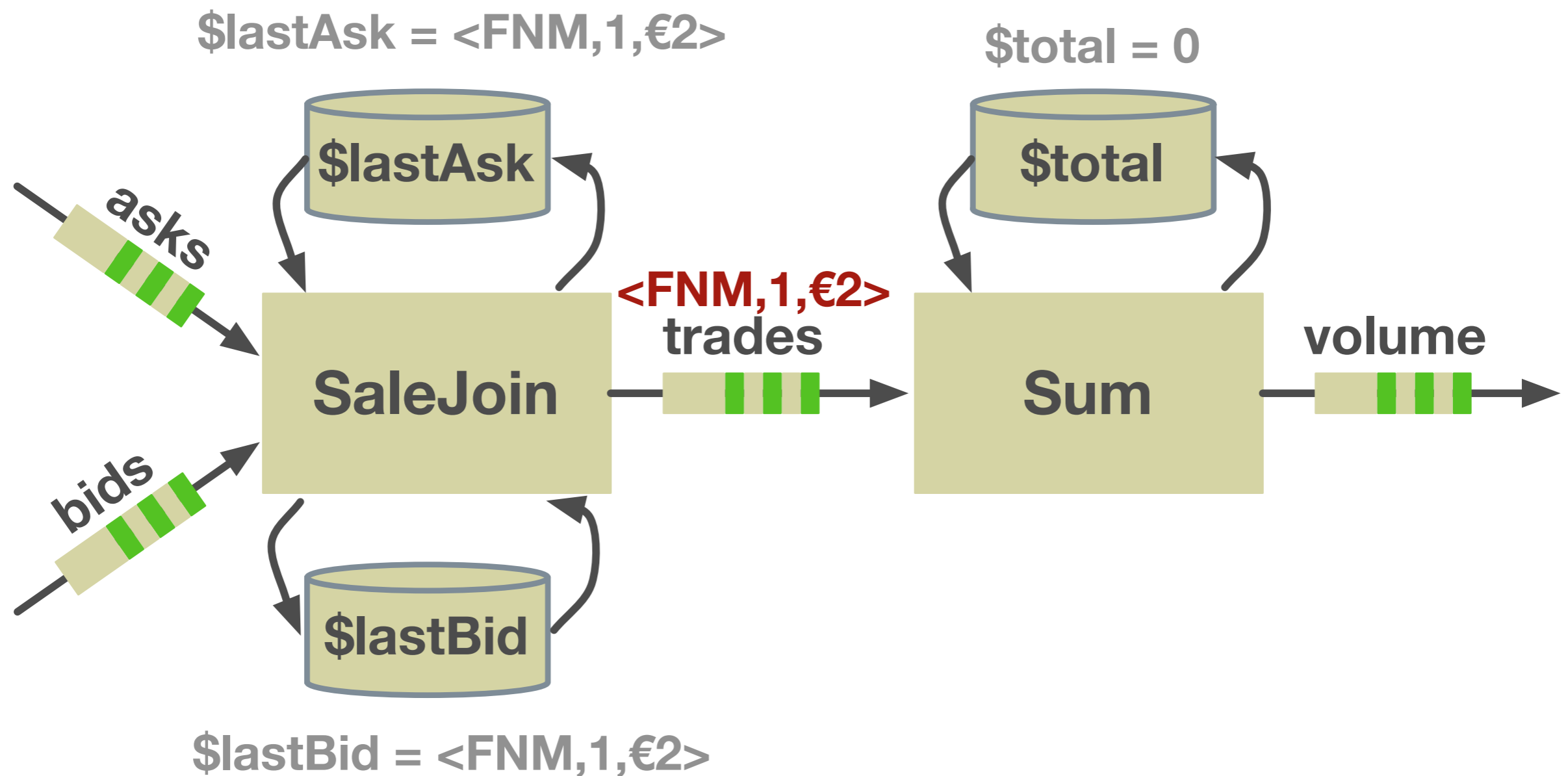
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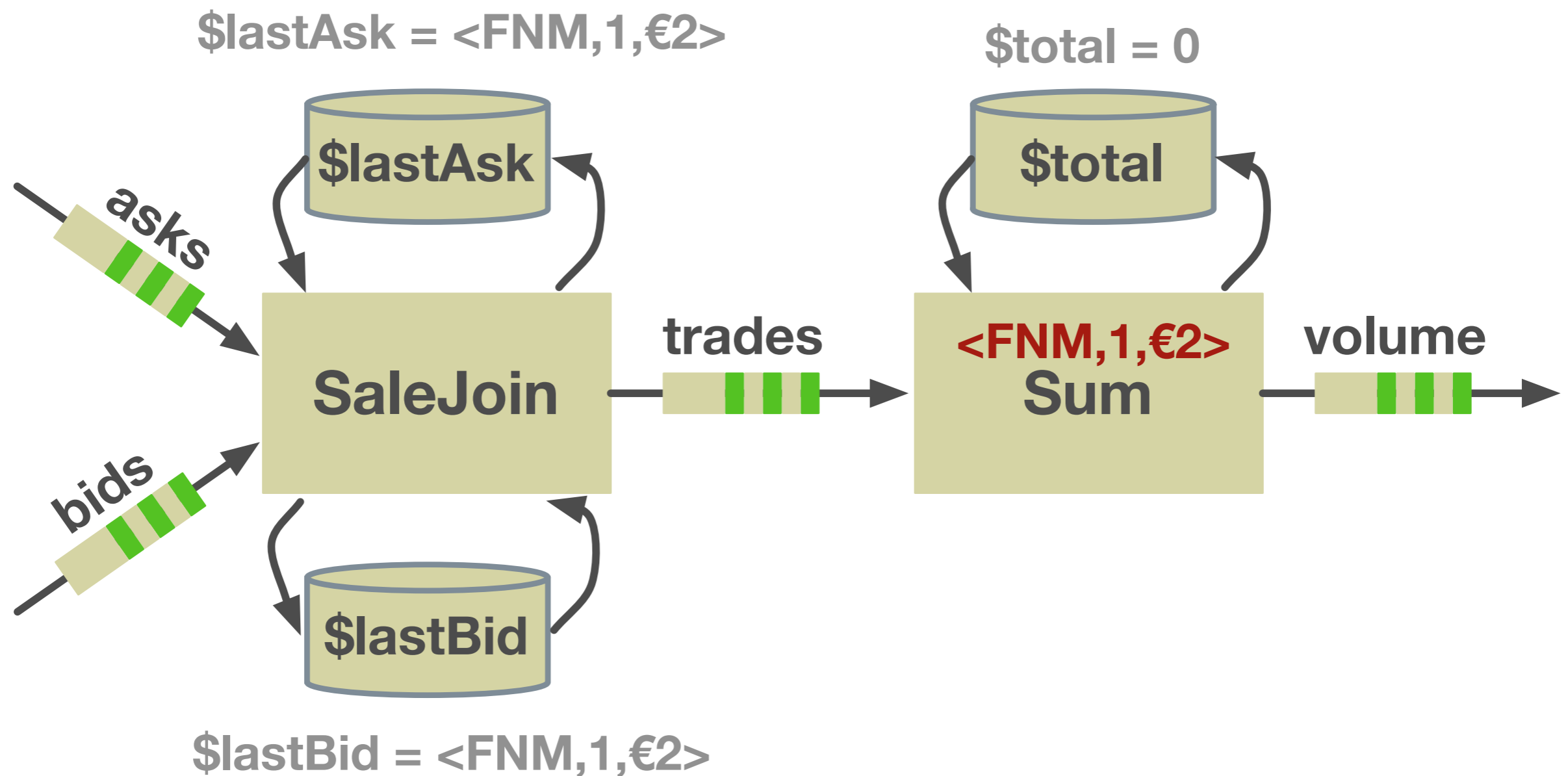
Example: A Fannie Mae Bid/Ask Join



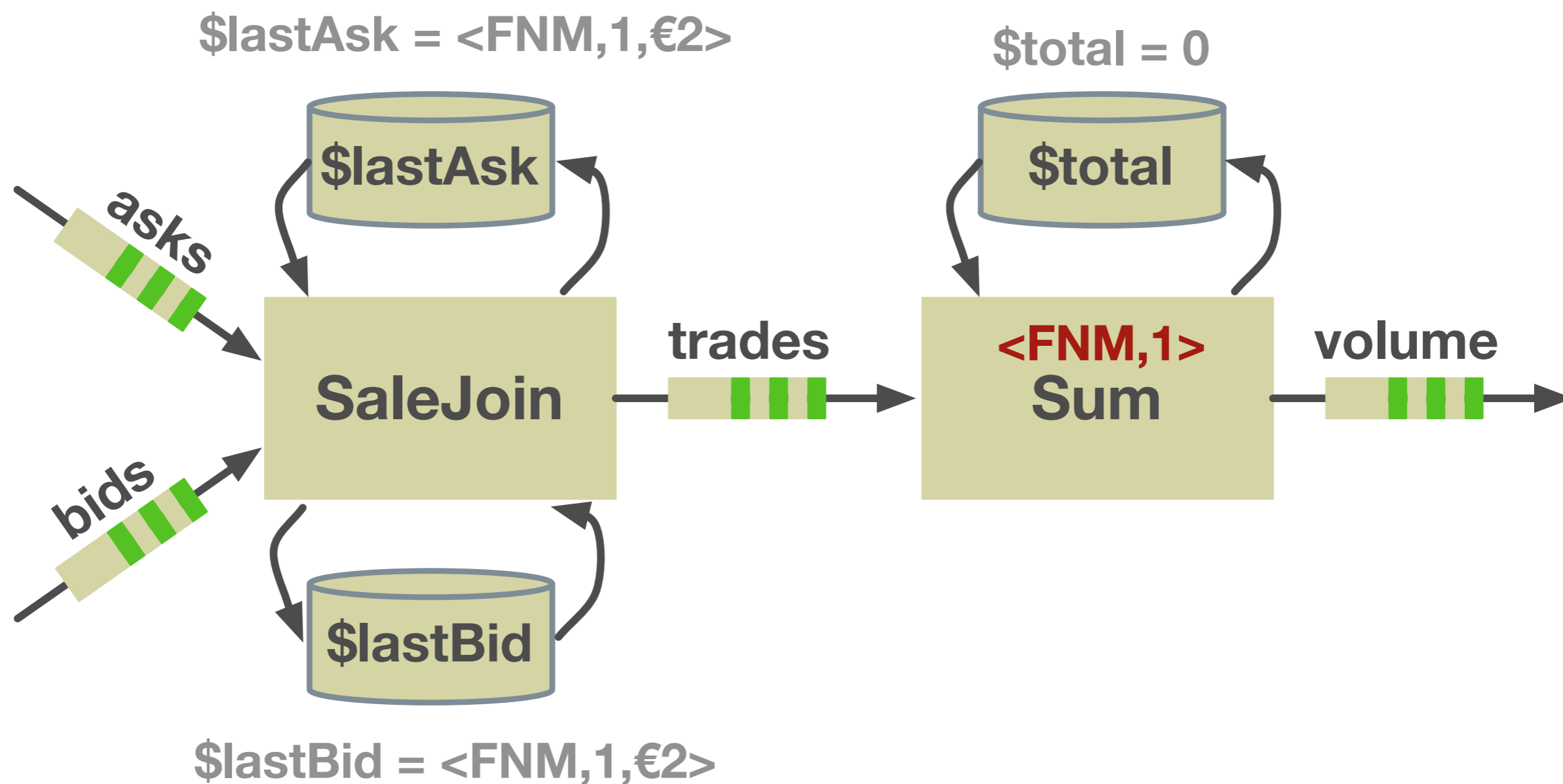
Example: A Fannie Mae Bid/Ask Join



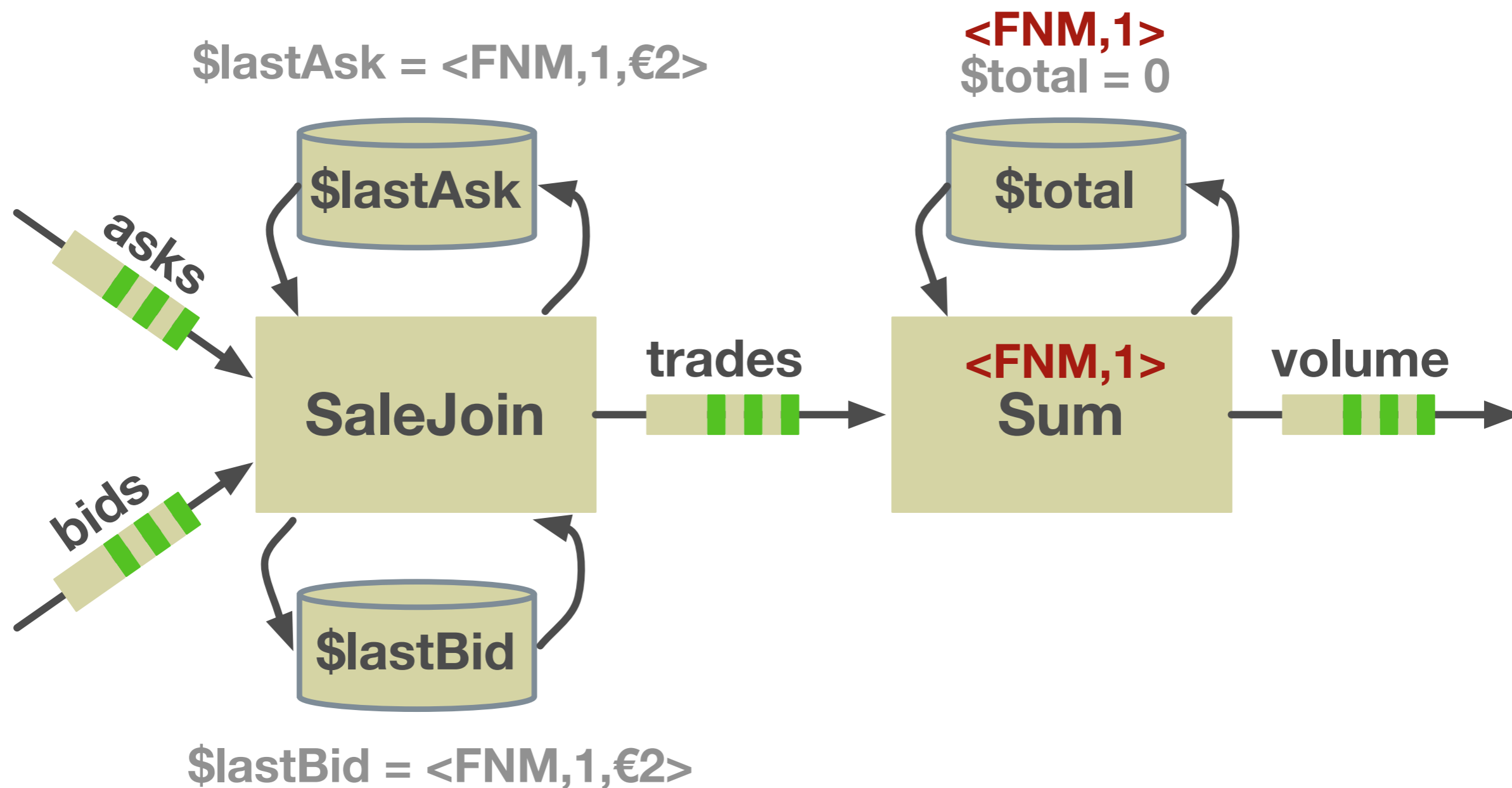
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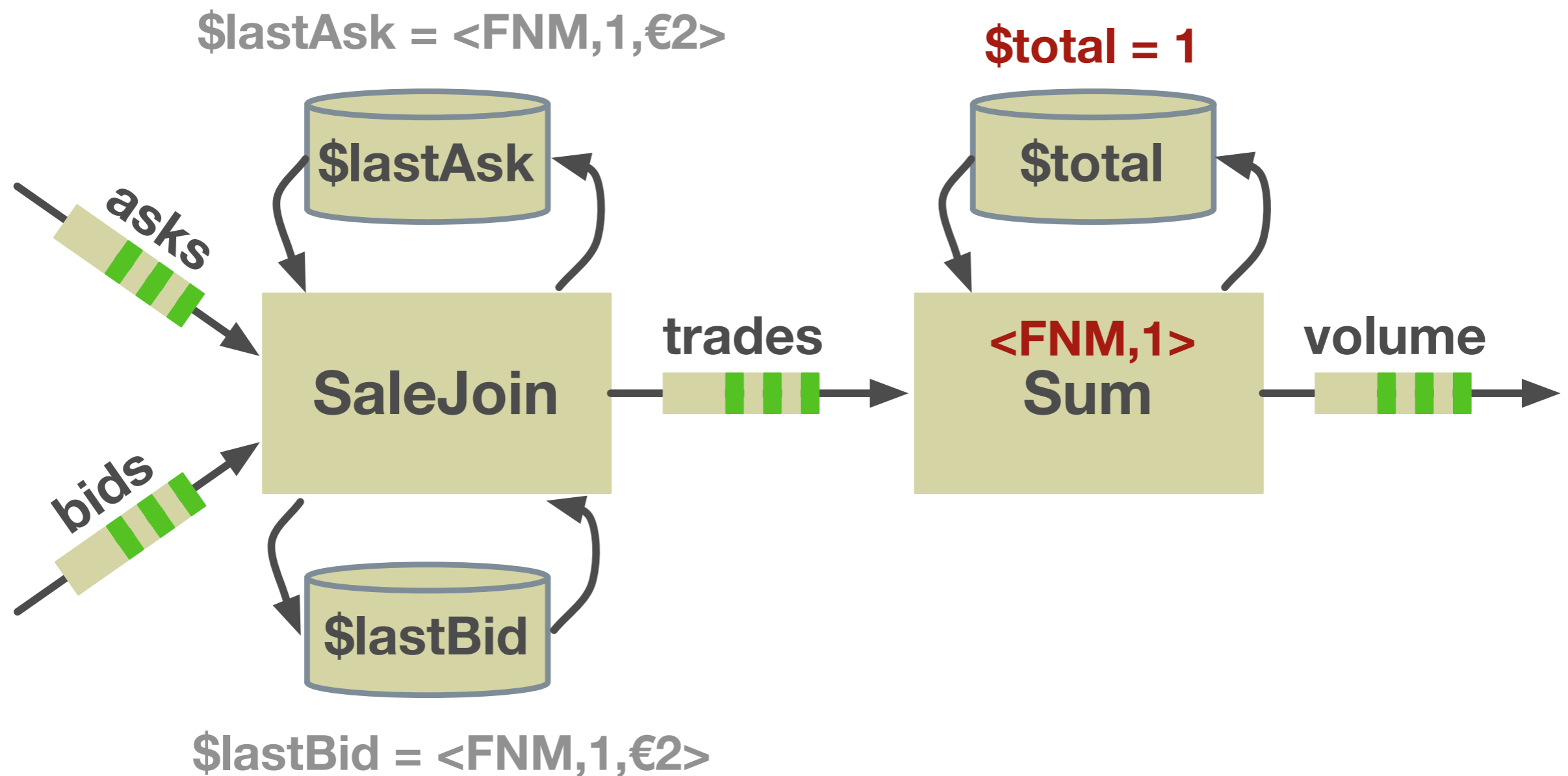
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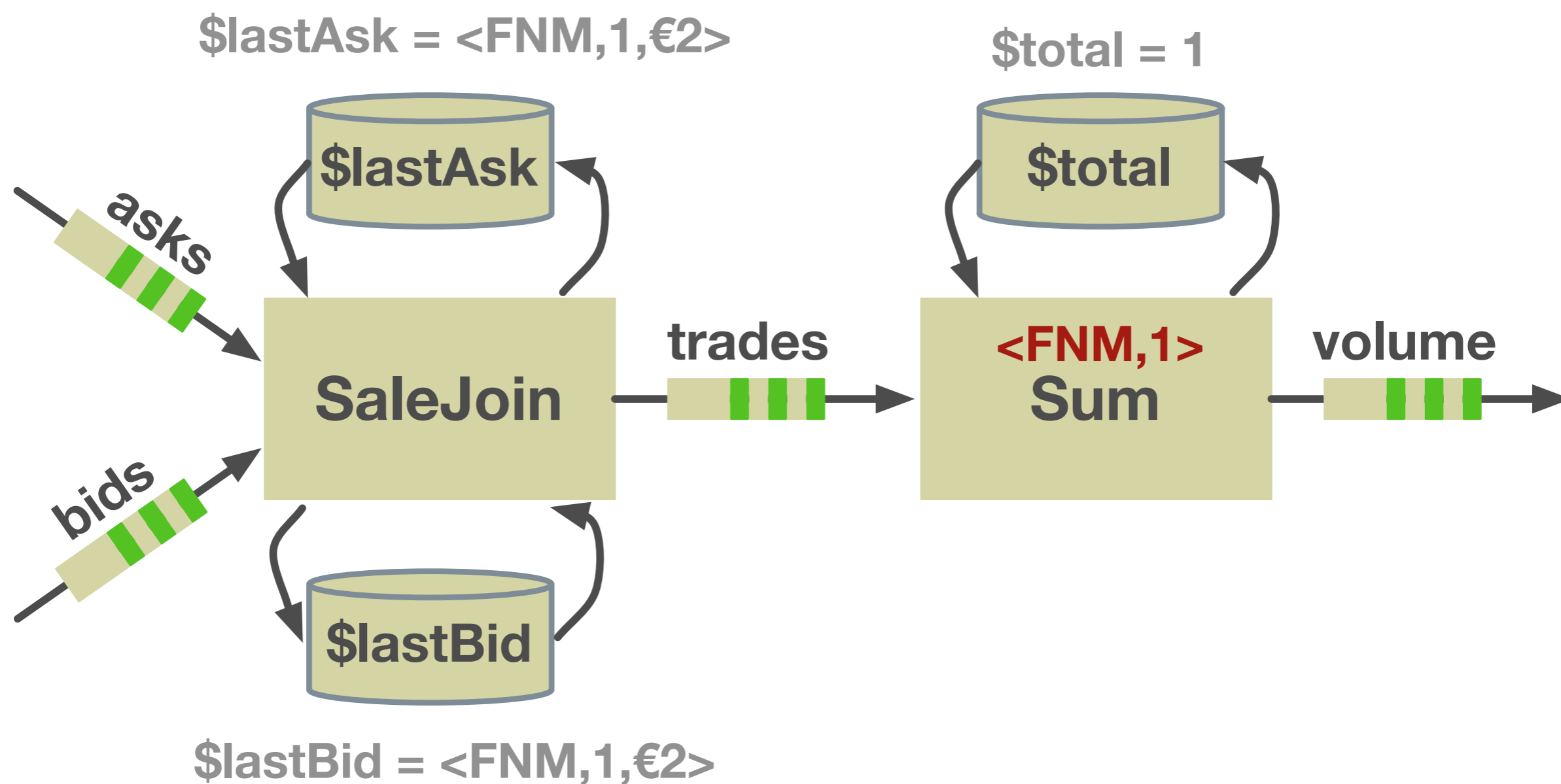
Example: A Fannie Mae Bid/Ask Join



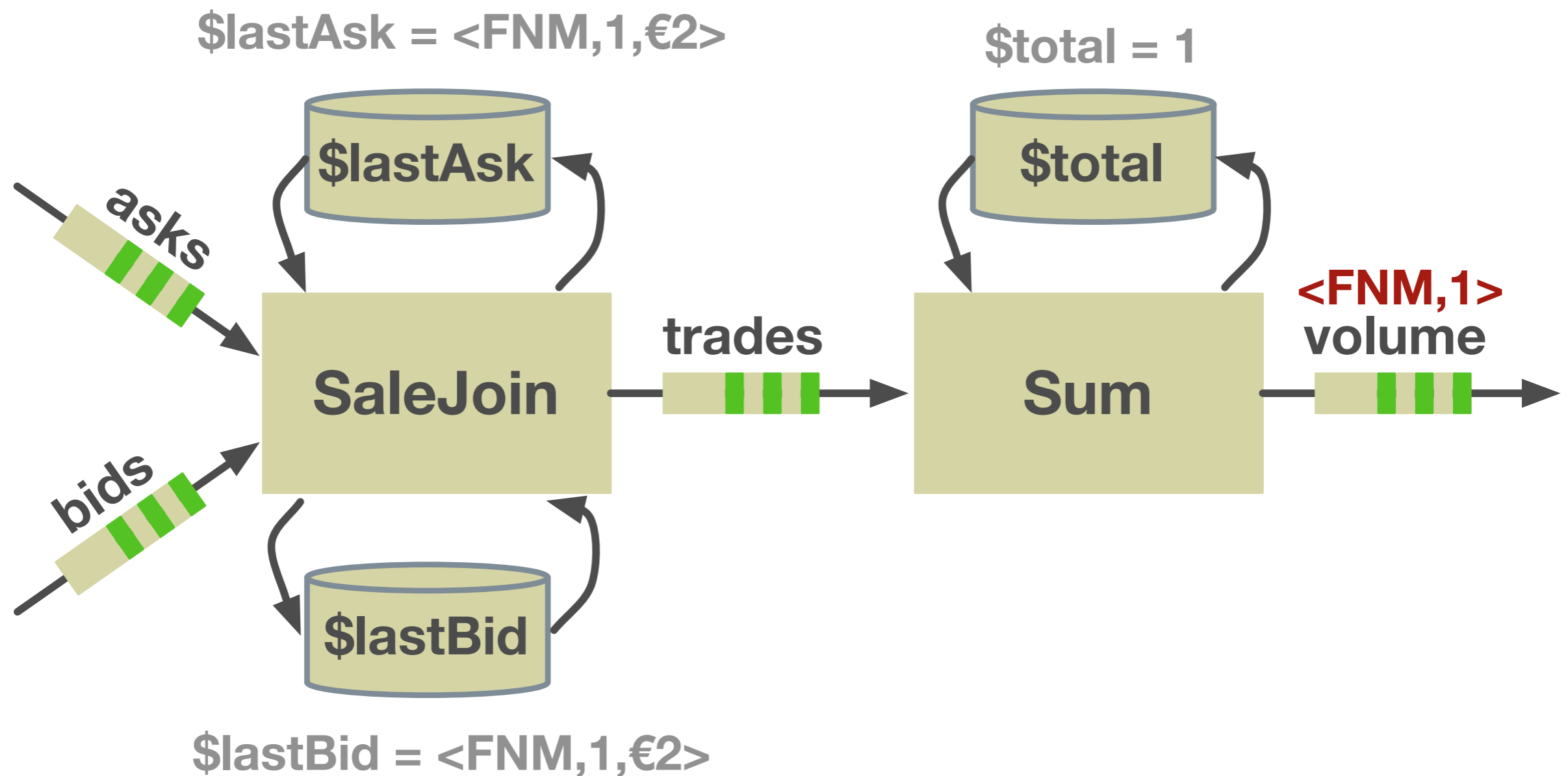
Example: A Fannie Mae Bid/Ask Join



Example: A Fannie Mae Bid/Ask Join



Example: A Fannie Mae Bid/Ask Join

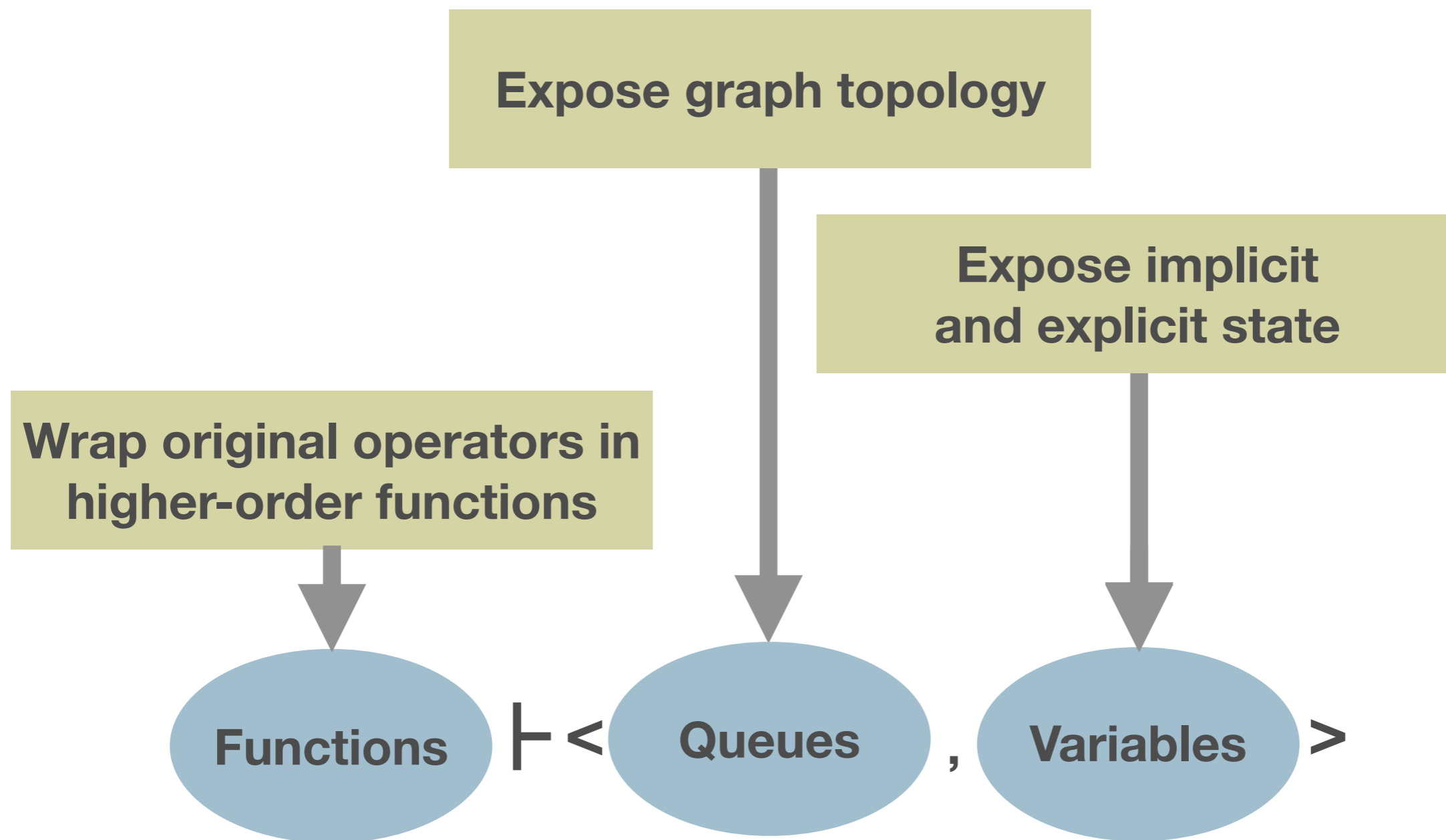


Translations

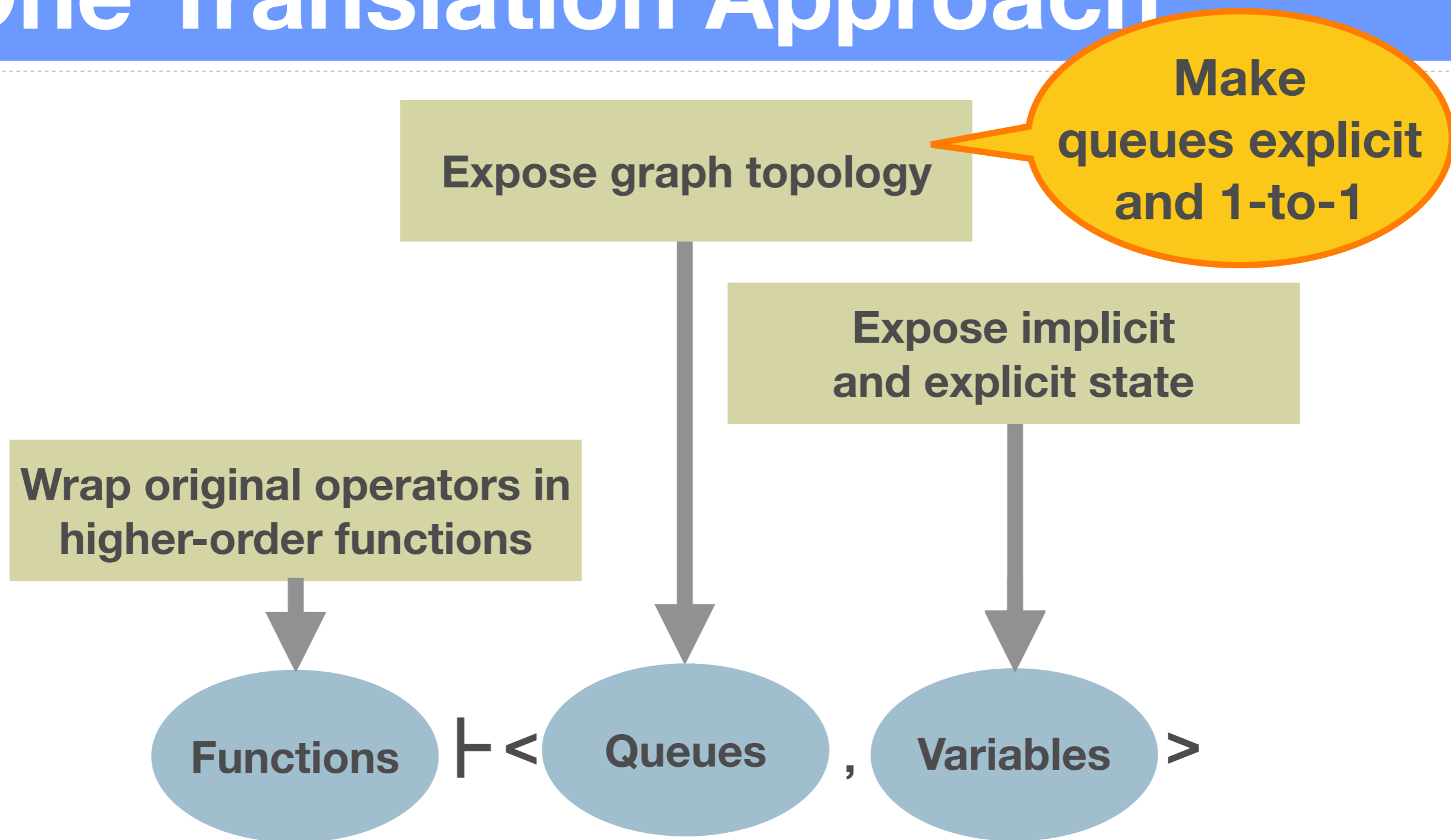
**Demonstrating Brooklet's generality
by translating three rather diverse streaming languages**



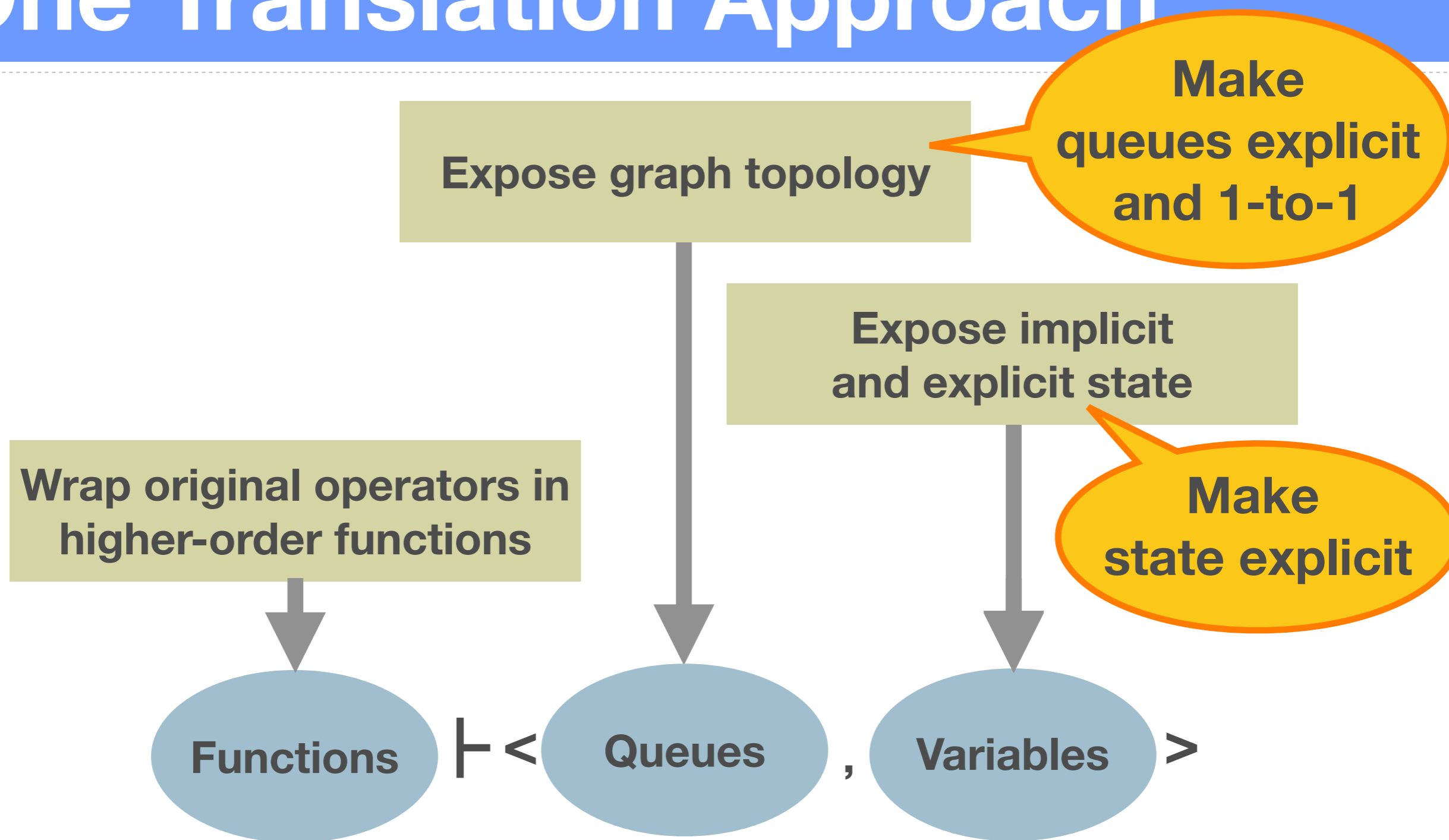
CQL, StreamIt, Sawzall: One Translation Approach



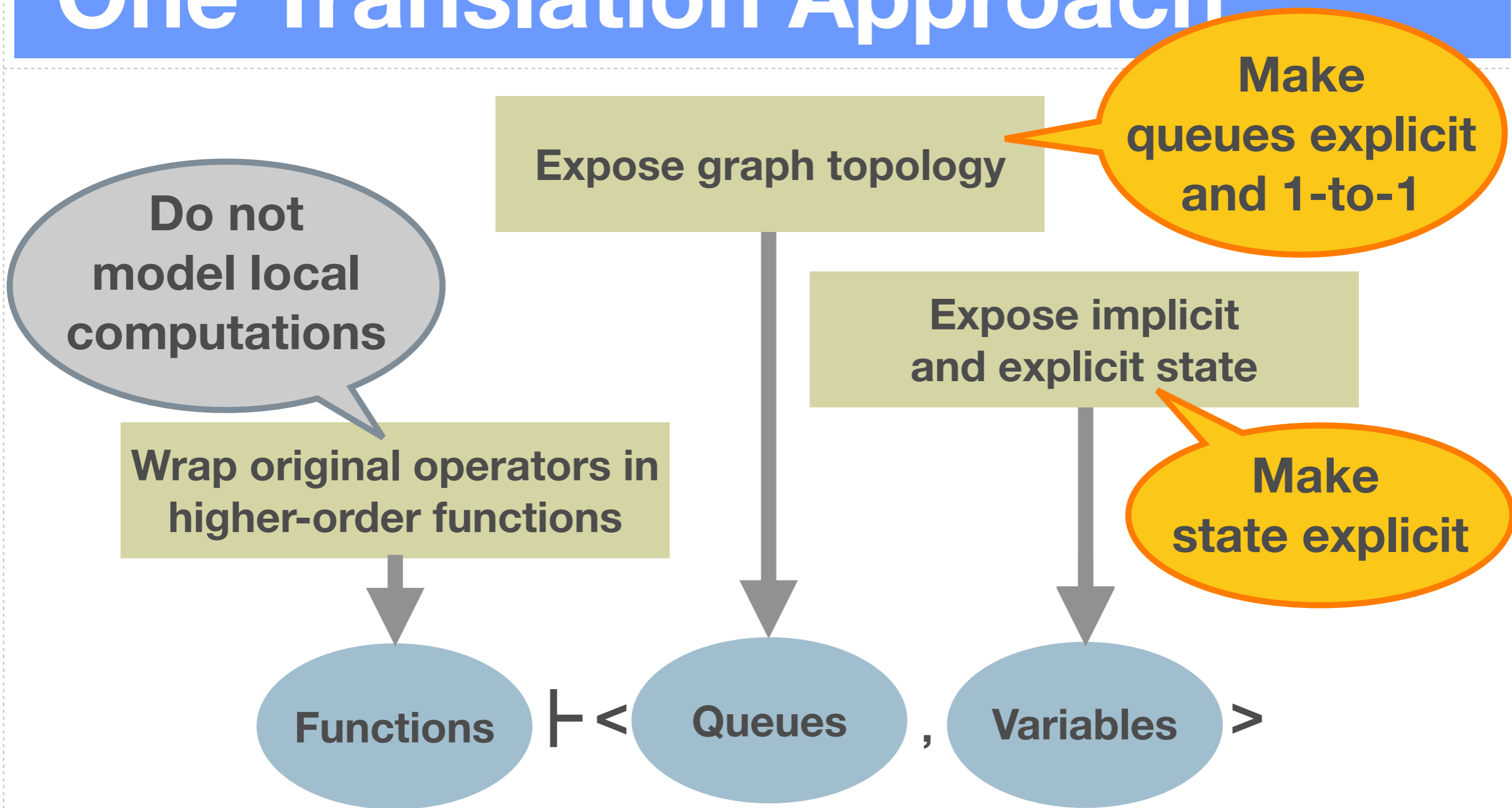
CQL, StreamIt, Sawzall: One Translation Approach



CQL, StreamIt, Sawzall: One Translation Approach



CQL, StreamIt, Sawzall: One Translation Approach

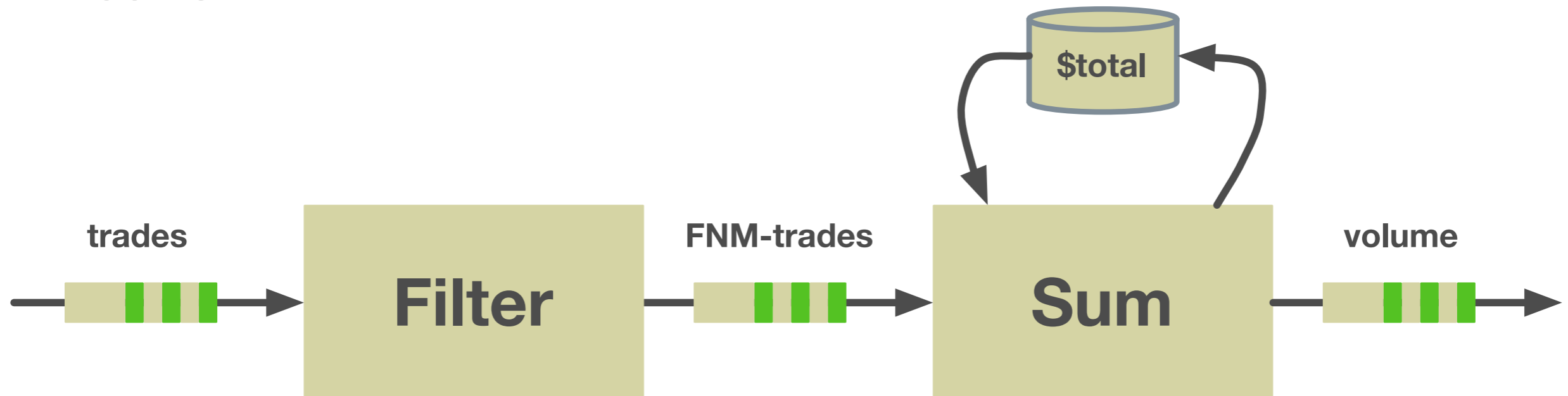


Example: CQL to Brooklet

*select Sum(shares) from trades
where trades.ticker = "FNM"*

CQL

Brooklet

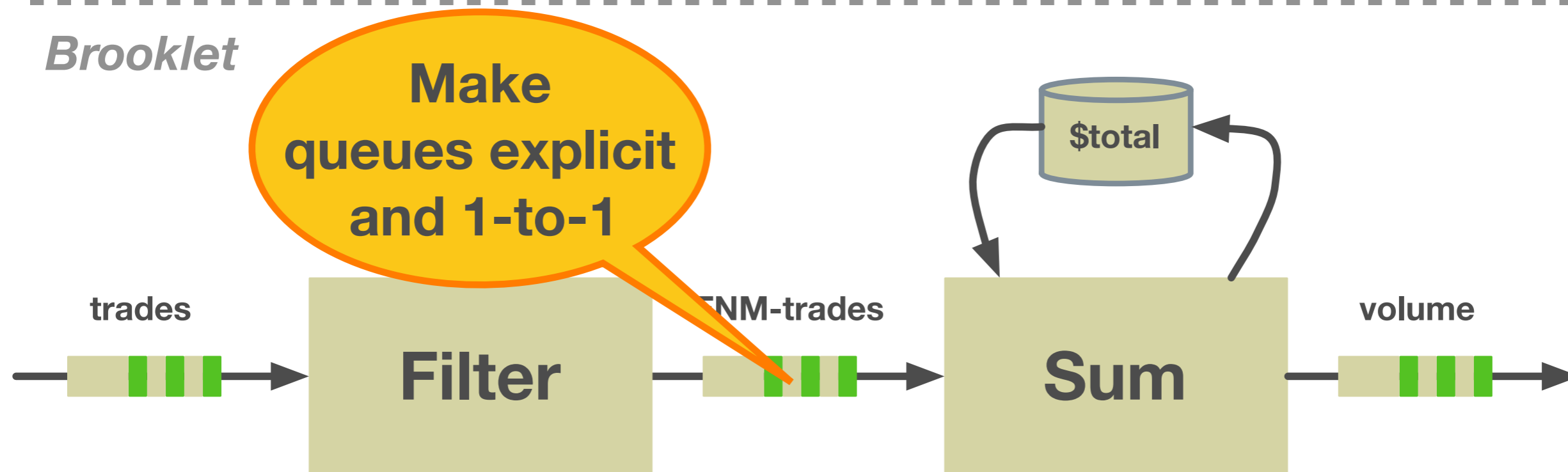


Example: CQL to Brooklet

*select Sum(shares) from trades
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CQL

Brooklet

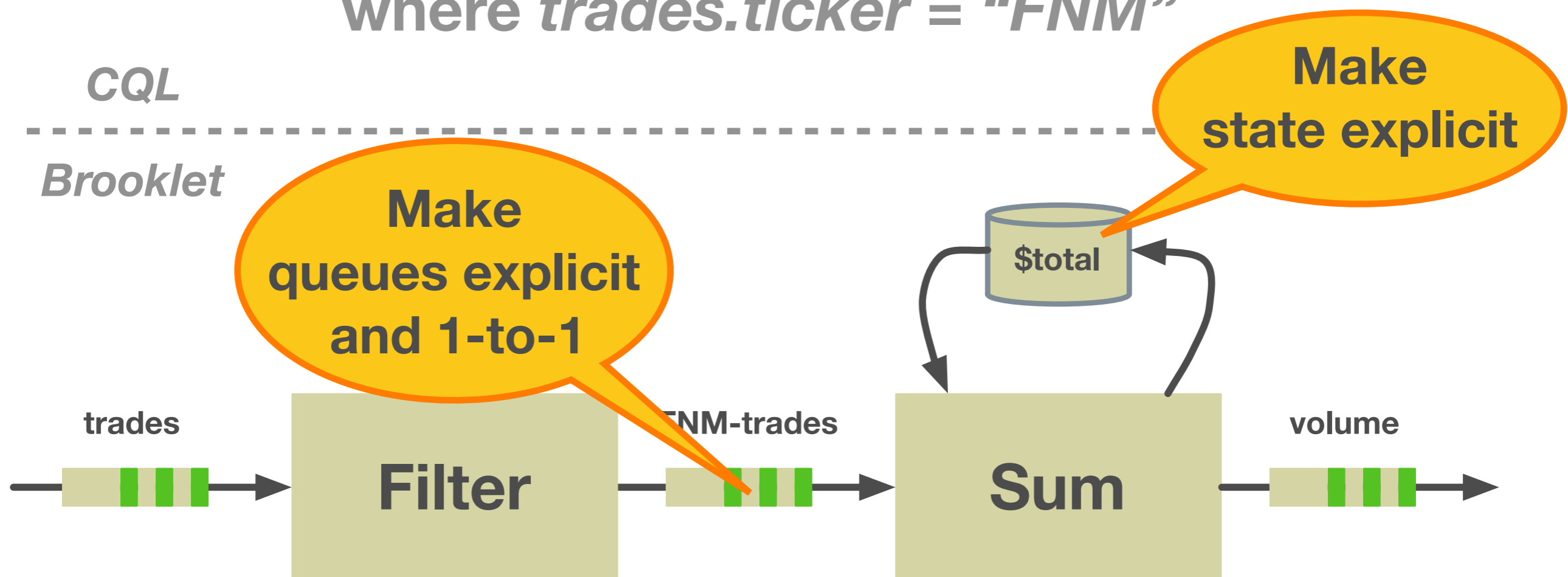


Example: CQL to Brooklet

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CQL

Brooklet

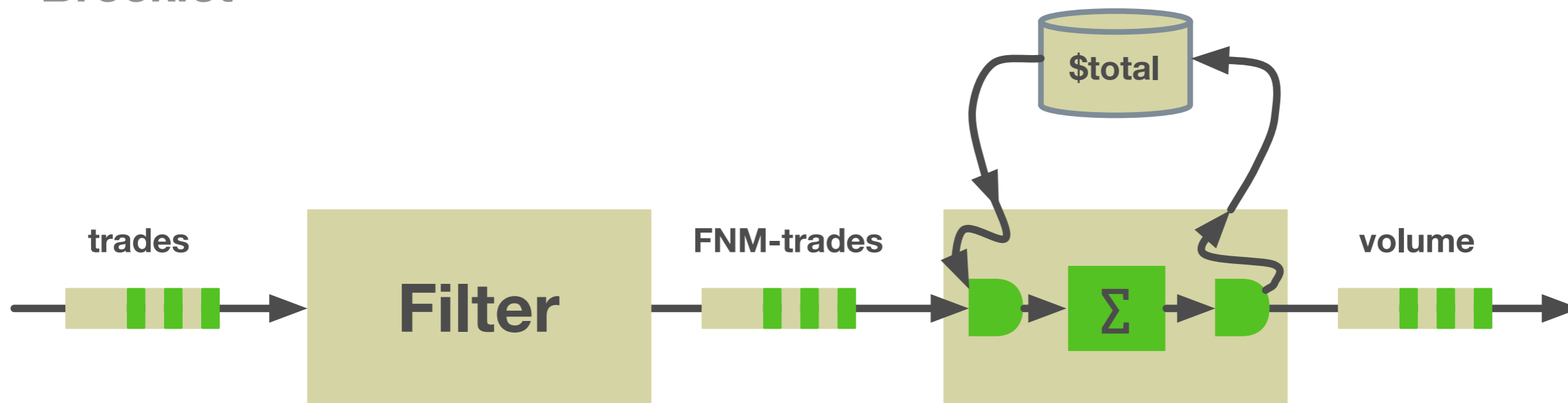


Example: CQL to Brooklet

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CQL

Brooklet

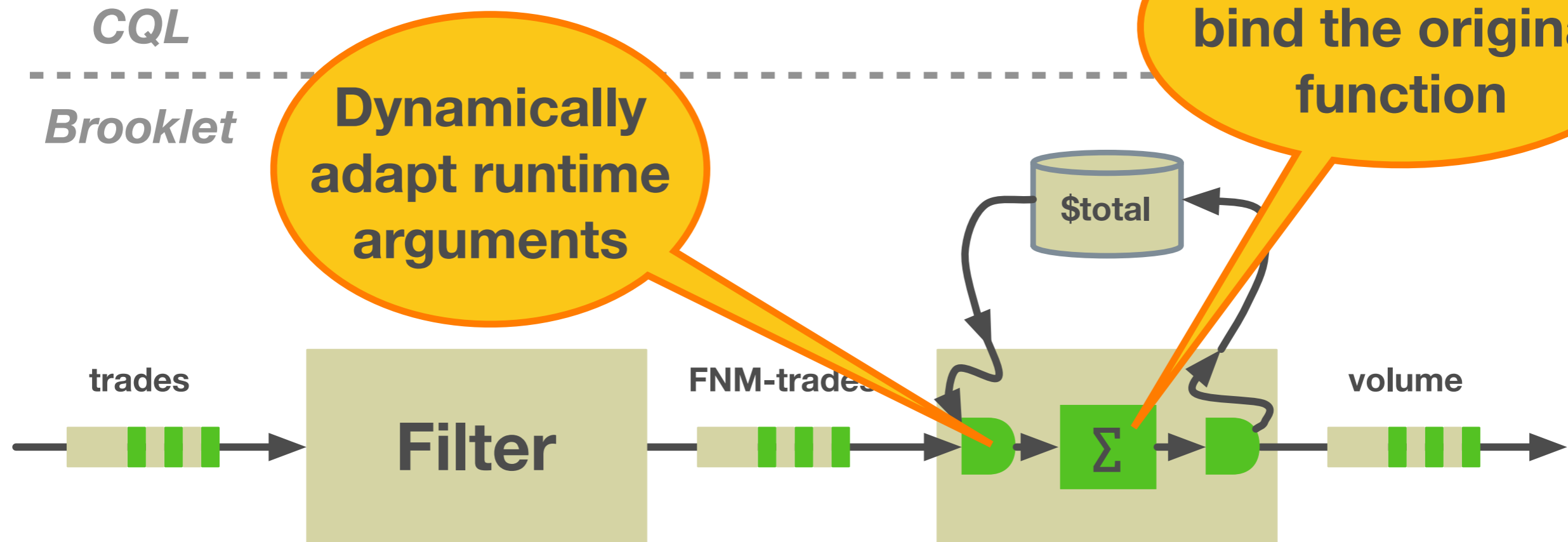


Example: CQL to Brooklet

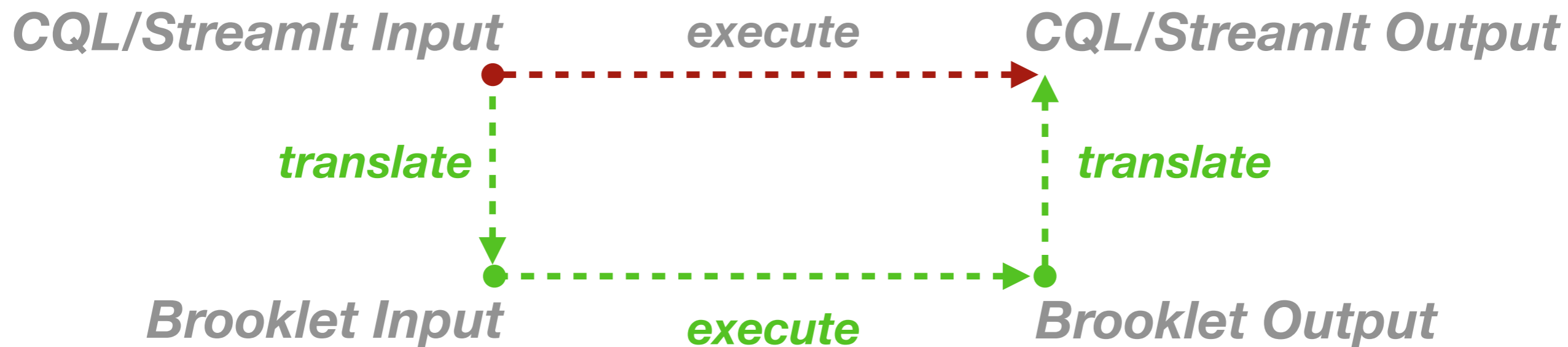
*select Sum(shares) from trades
where trades.ticker = "FNM"*

CQL

Brooklet



Translation Correctness Theorem



- ❏ Results under CQL and StreamIt semantics are the same as the results under Brooklet semantics after translation
- ❏ First formal semantics for Sawzall

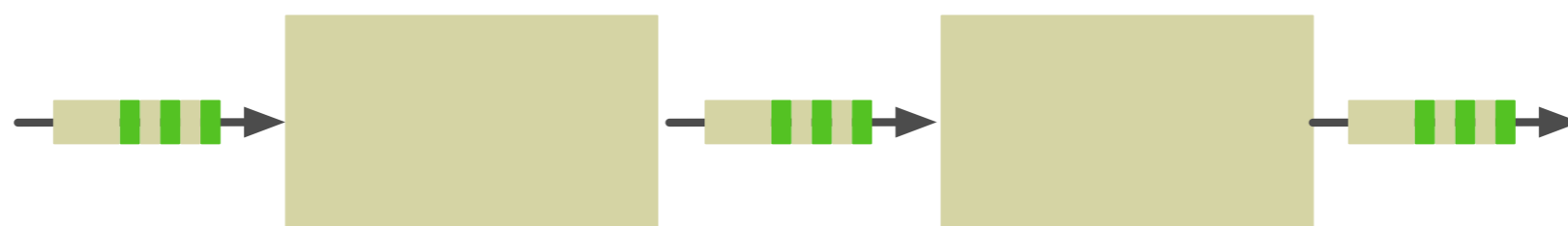


Optimizations

**Demonstrating Brooklet's utility
by realizing three essential optimizations**



Operator Fusion: Eliminate Queueing Delays



before



after



**Look for connected operators,
whose state isn't used anywhere else**



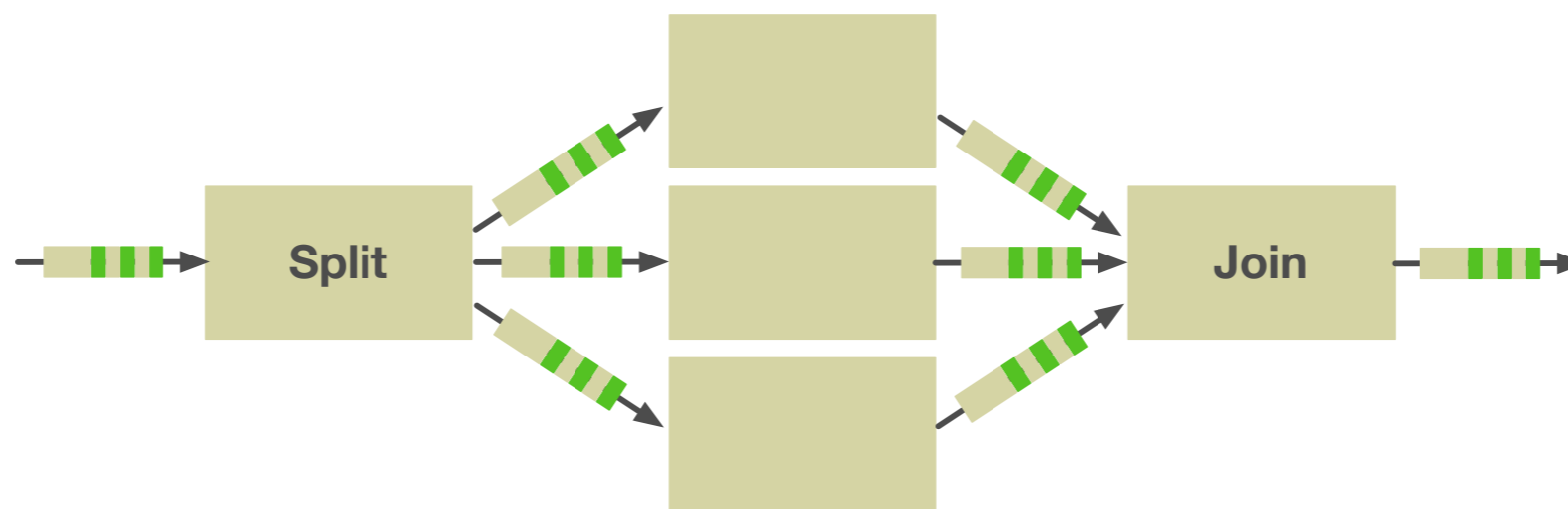
Operator Fission: Process More Data in Parallel



before



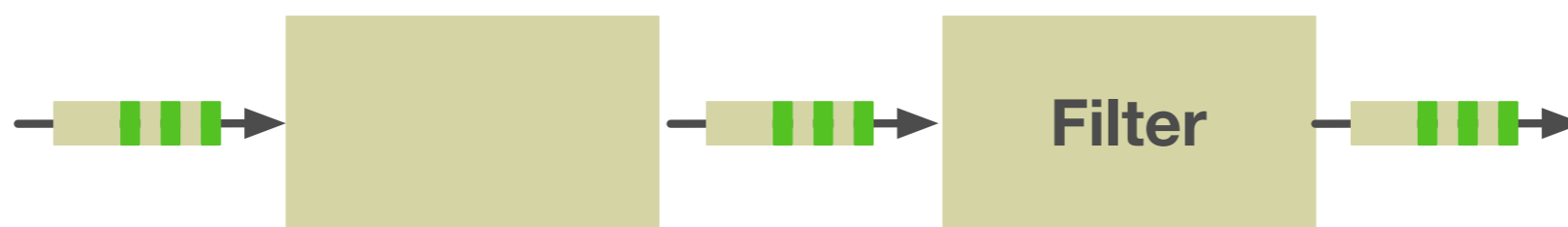
after



Look for stateless operators



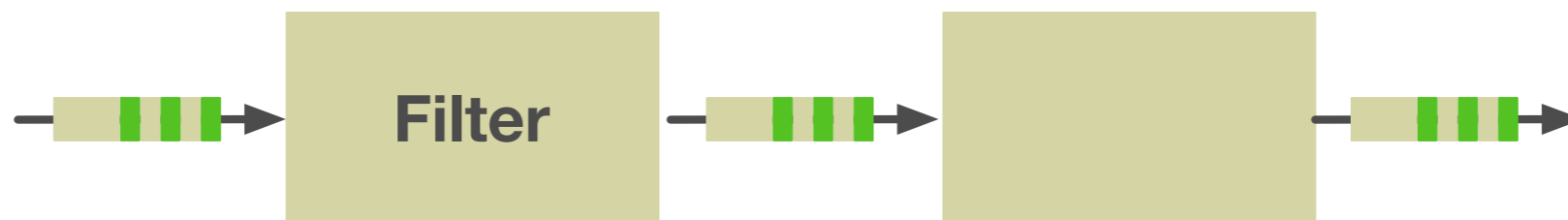
Operator Reordering: Filter Data Early



before



after



Look for operators whose read/write sets don't overlap [Ghelli et al., SIGMOD 08]



From a Calculus to an Intermediate Language

The River Intermediate Language



An Intermediate Language for Stream Processing

❖ Benefits of a VEE/IL are well known

- ❖ Increase portability, share optimizations, etc.

❖ Streaming needs its own IL

- ❖ Need to reason across machines, support different optimizations

❖ Brooklet serves as a solid foundation

- ❖ Challenge: How to bridge the gap between theory and practice?



Make Abstractions Concrete

Brooklet

Sequence of atomic steps

Pure functions, state threaded through invocations

Non-deterministic execution

Opaque functions

No physical platform, independent from runtime

Finite execution

River

Operators execute concurrently

Stateful functions, protected with automatic locking

Restricted execution with bounded queues, and back-pressure

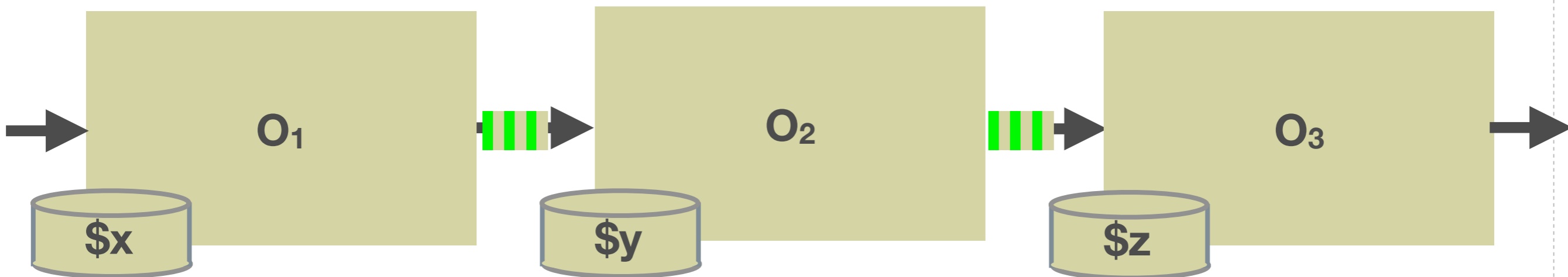
Function implementations

Abstract representation of runtime e.g. placement

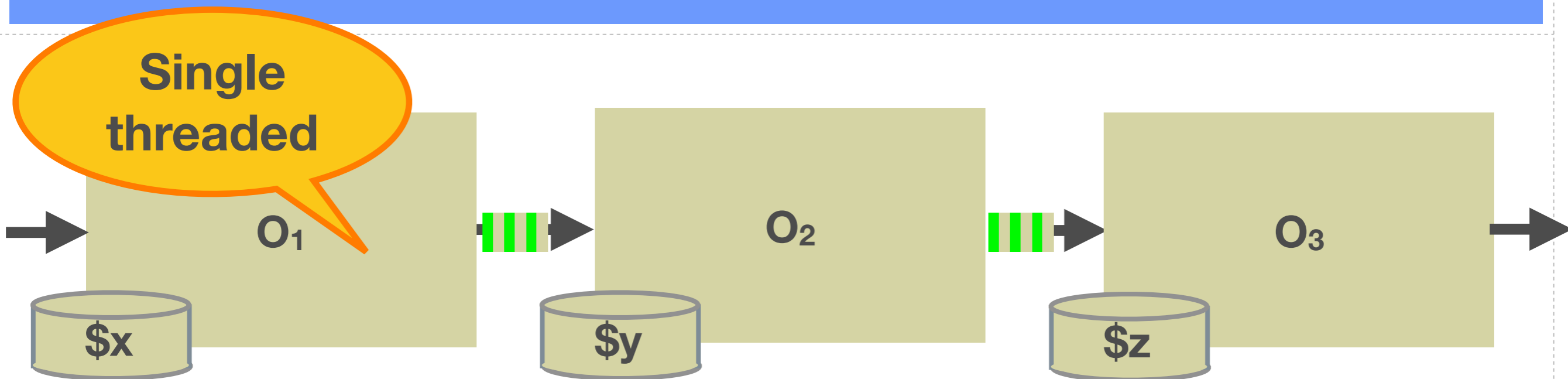
Indefinite execution



Concurrent Execution: No Shared State

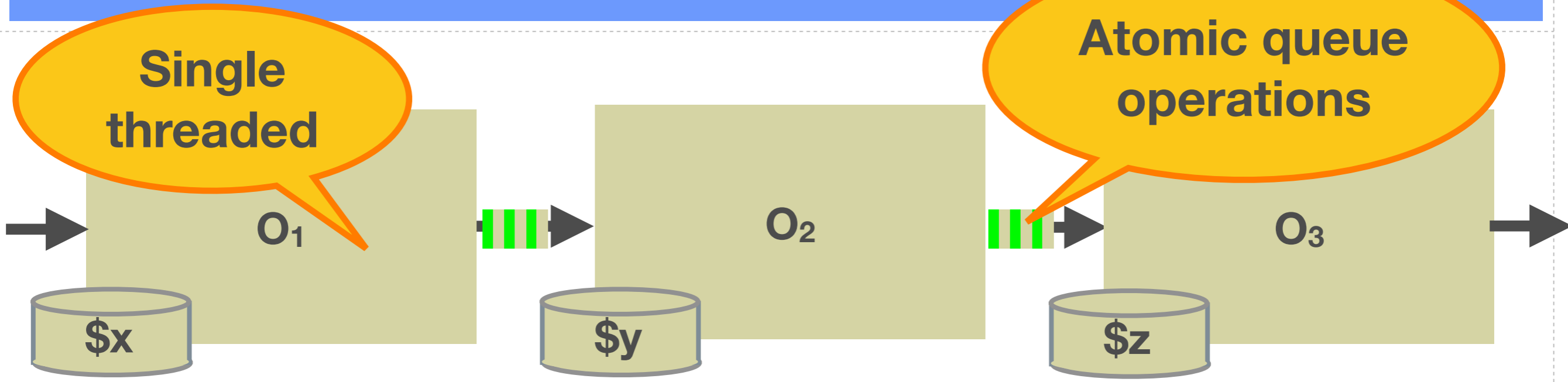


Concurrent Execution: No Shared State

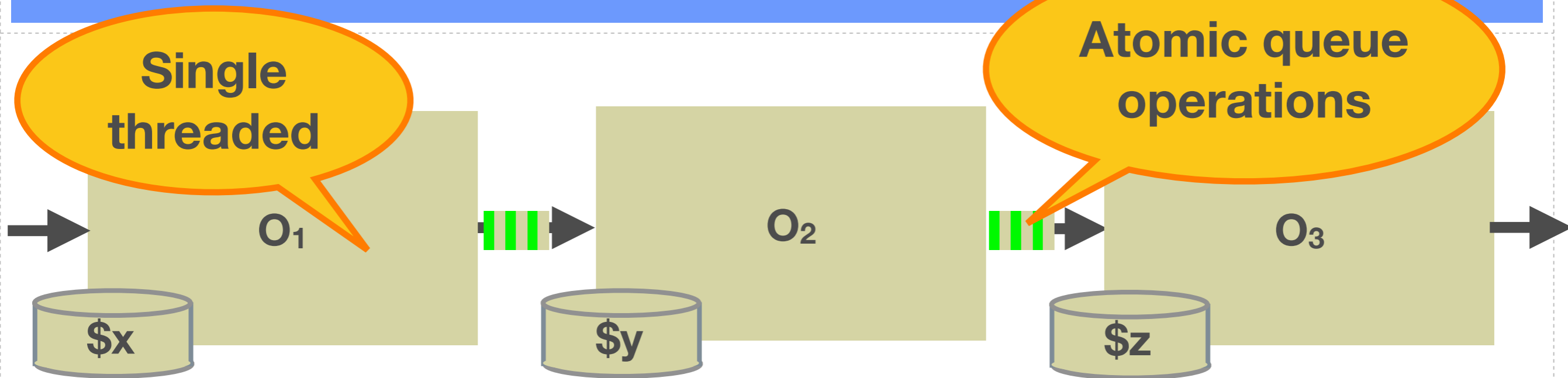




Concurrent Execution: No Shared State



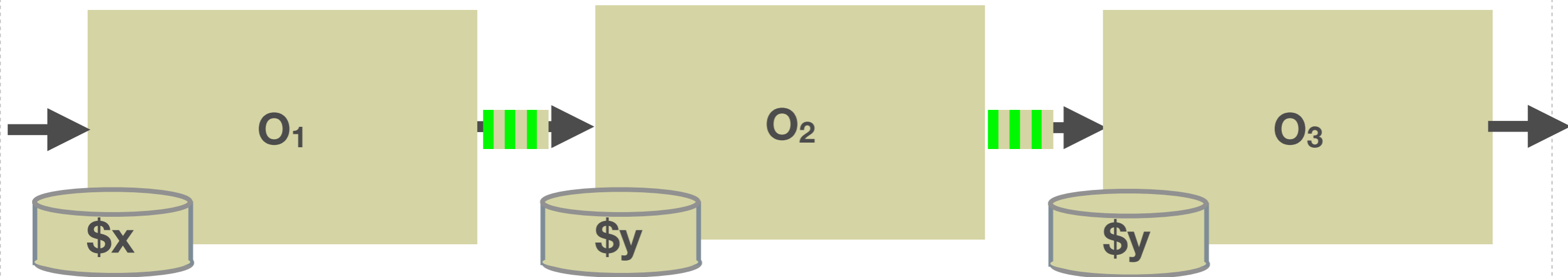
Concurrent Execution: No Shared State



- Brooklet operators fire one at a time
- River operators fire concurrently
- For both, data must be available



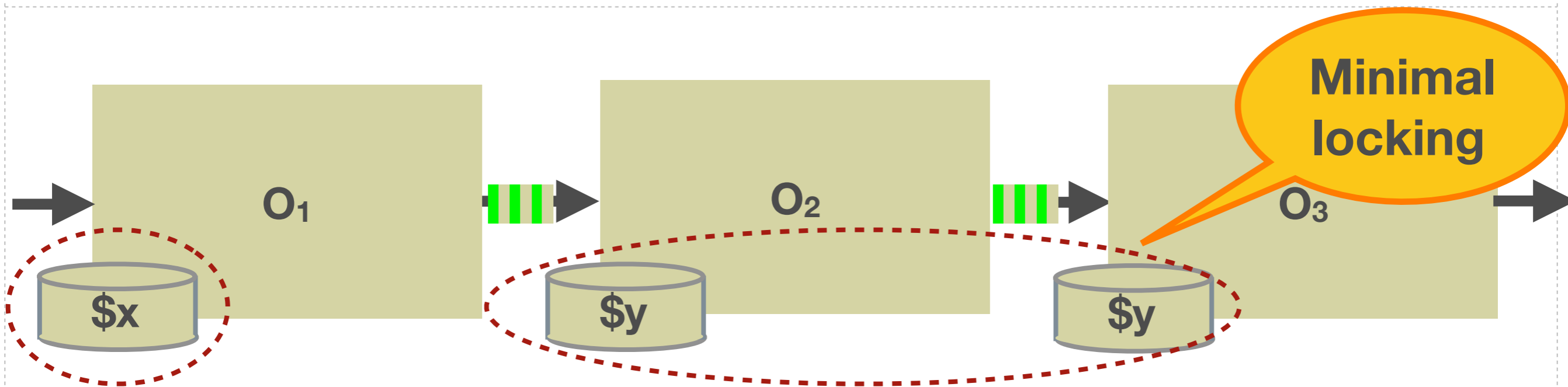
Concurrent Execution: With Shared State



- ⊞ Locks form equivalence classes over shared variables
- ⊞ Every shared variable is protected by one lock
- ⊞ Shared variables in the same class protected by same lock
- ⊞ Locks acquired/released in standard order



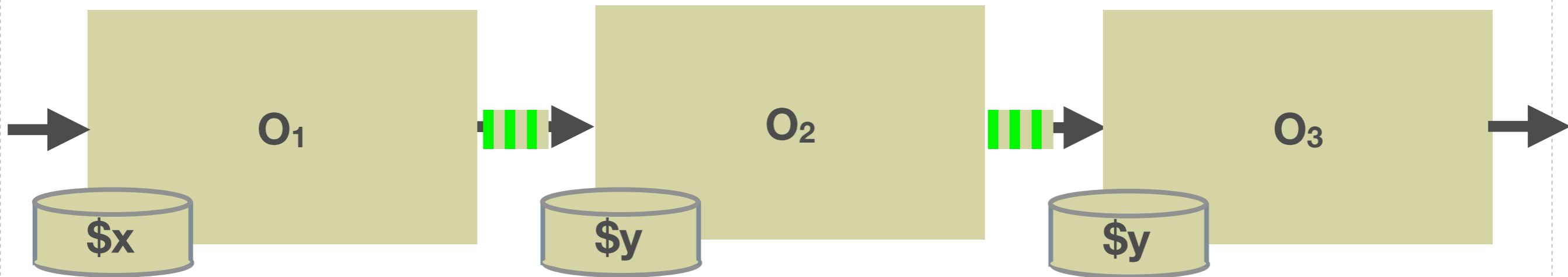
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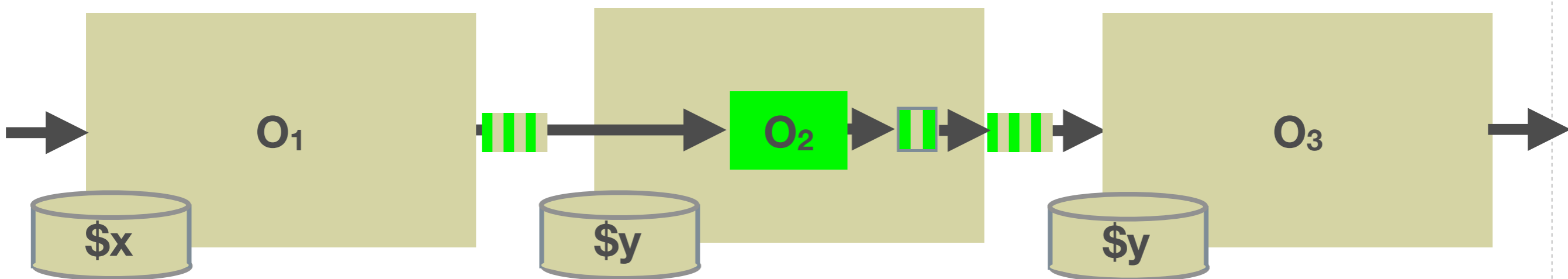
Restricted Execution: Bounded Queues



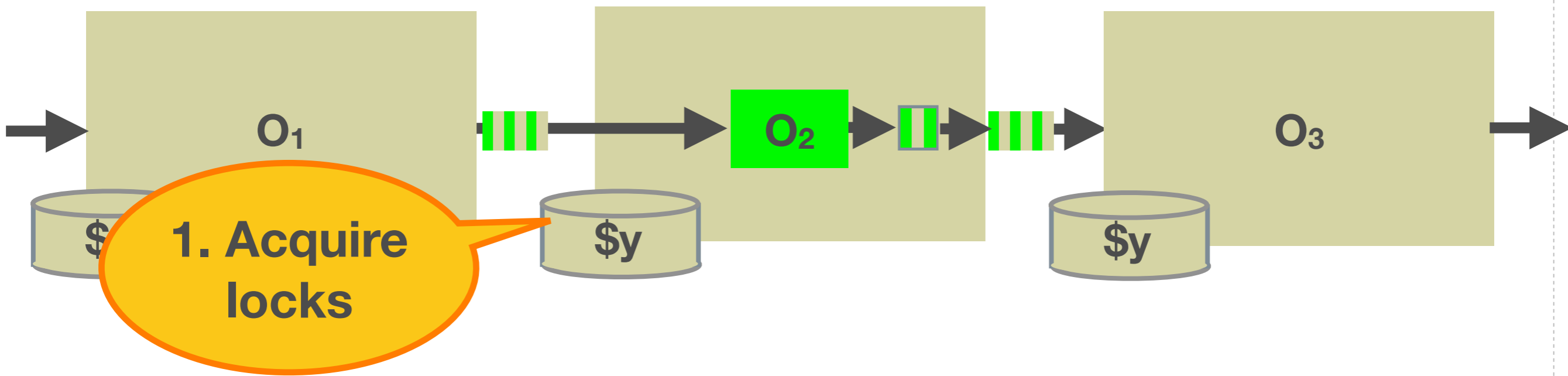
- ❖ Naïve approach: block when output queue is full
- ❖ If O_2 holds the lock on $\$x$ and blocks, O_3 cannot execute
- ❖ **Deadlock!**



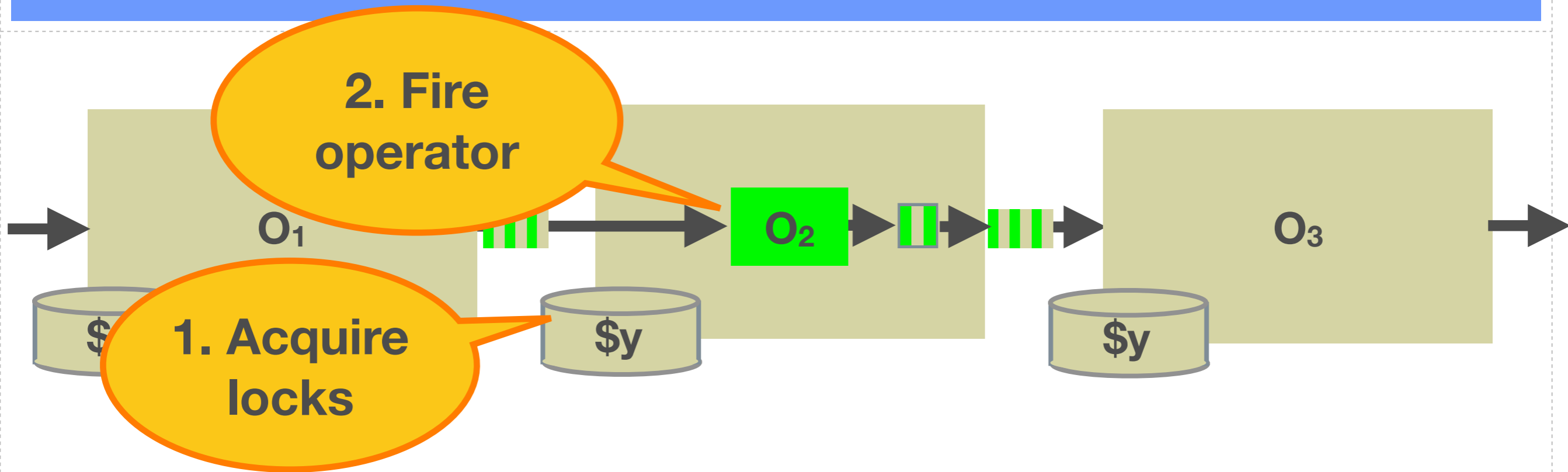
Restricted Execution: Safe Back-Pressure



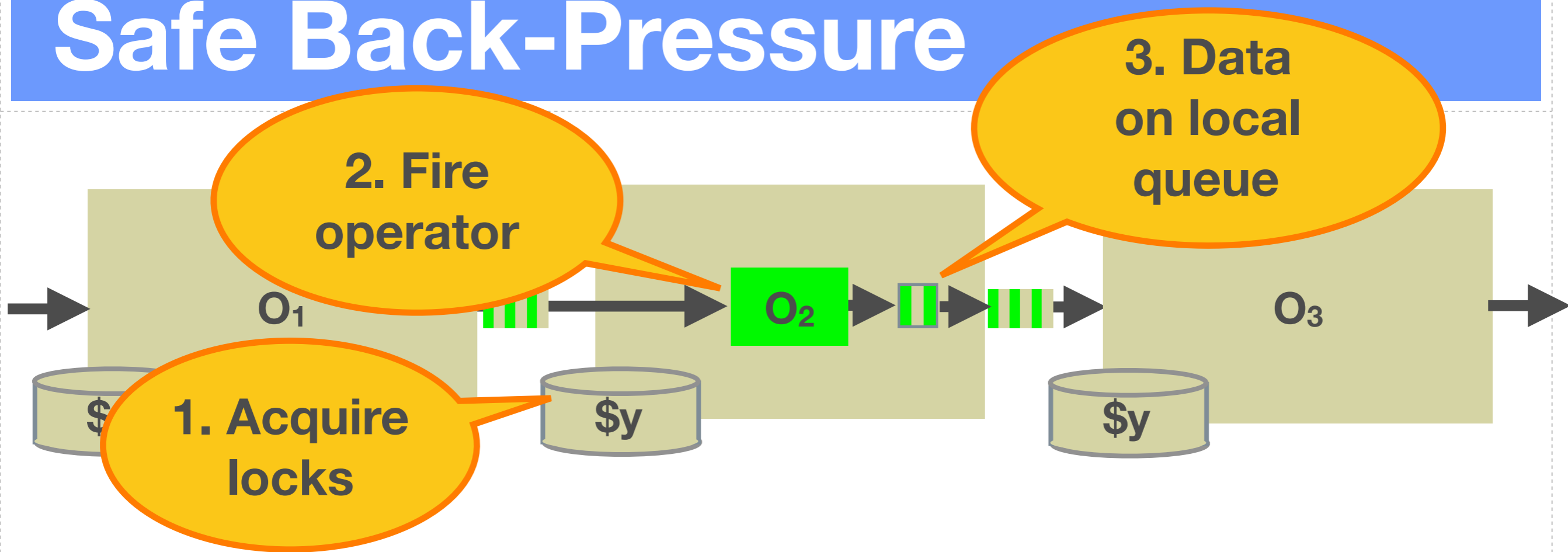
Restricted Execution: Safe Back-Pressure



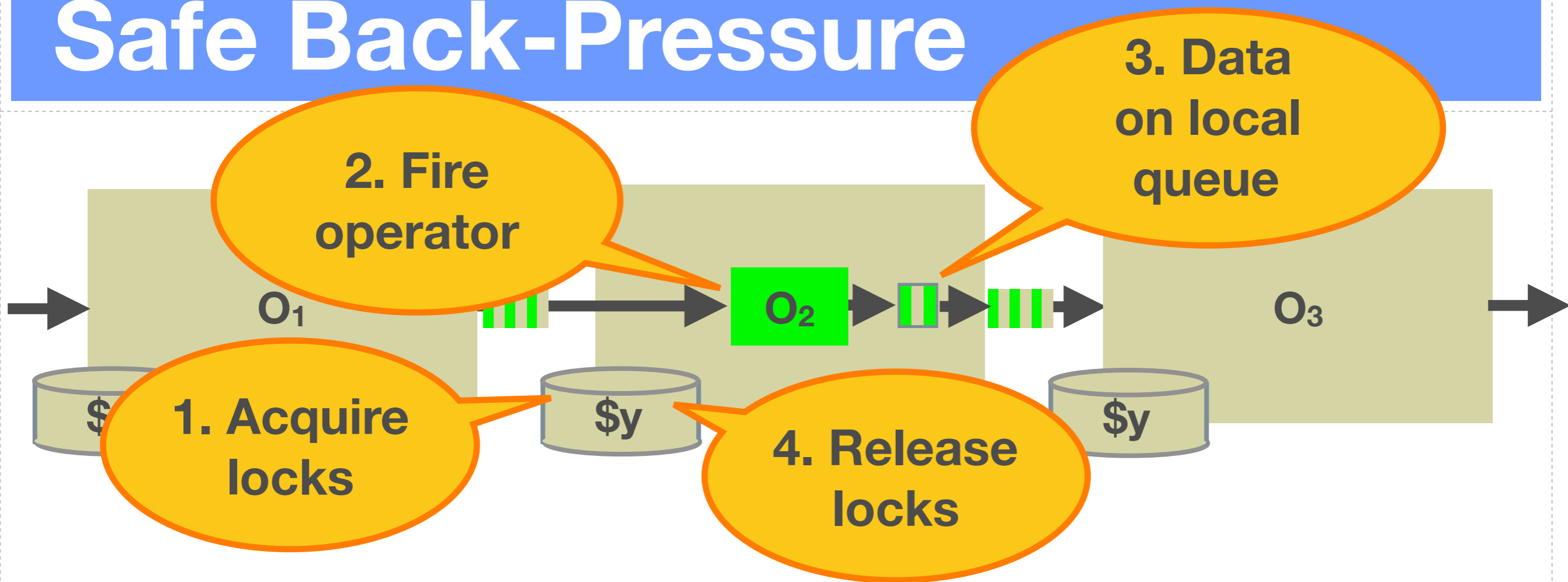
Restricted Execution: Safe Back-Pressure



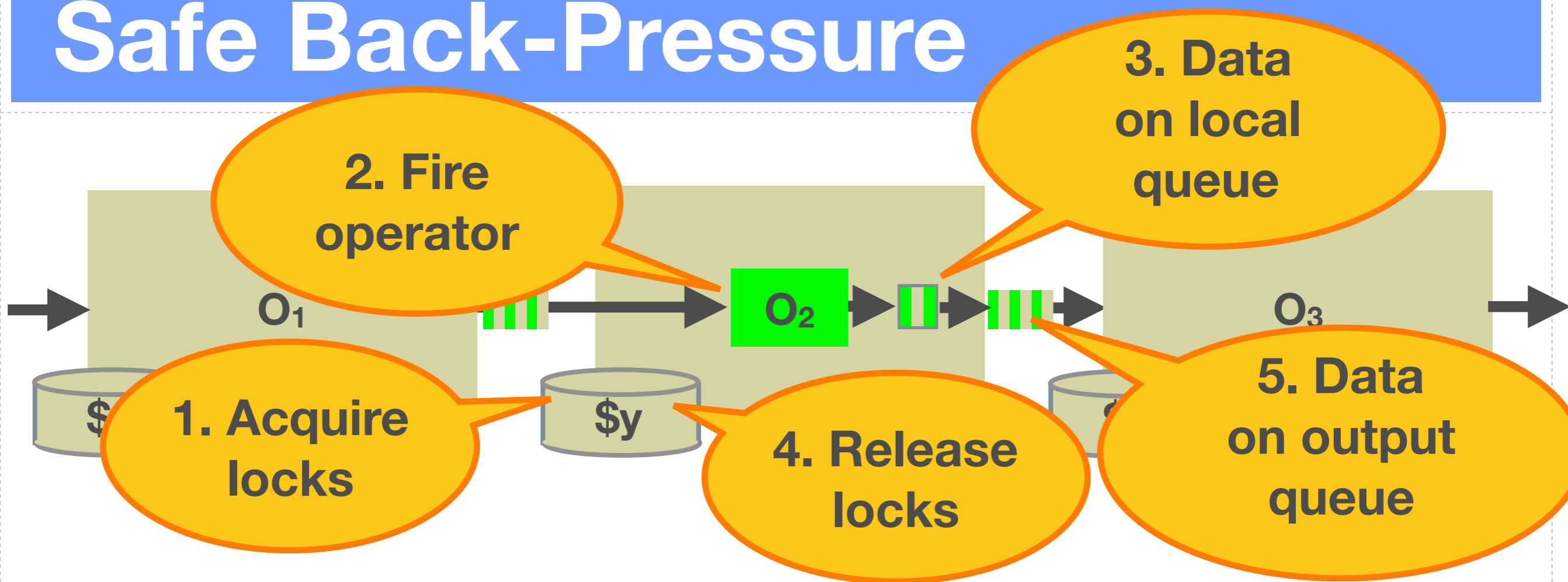
Restricted Execution: Safe Back-Pressure



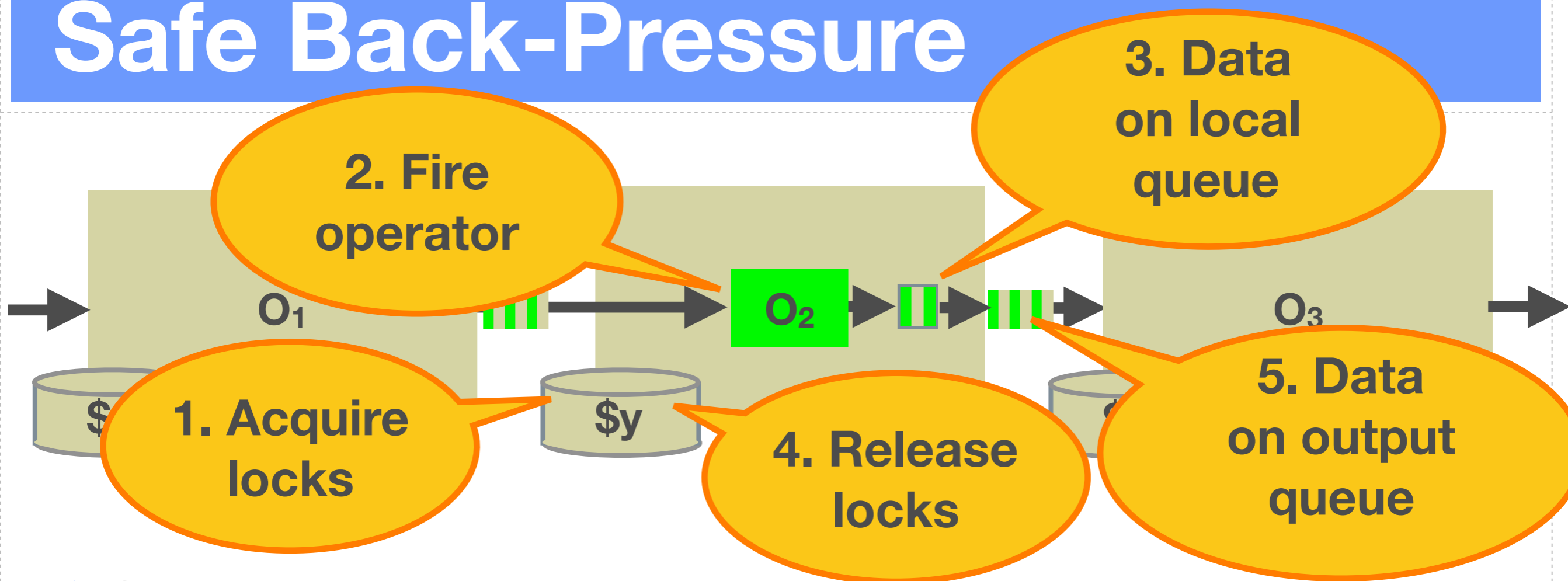
Restricted Execution: Safe Back-Pressure



Restricted Execution: Safe Back-Pressure



Restricted Execution: Safe Back-Pressure



Only step 5 can block

Locks have already been released, so O_3 can execute

Even if downstream is full, there is no deadlock



Applications of an Intermediate Language

- ❖ **Must make language development economic**
 - ❖ Implementation language, language modules, operator templates
- ❖ **Must support a broad range of optimizations**
 - ❖ Annotations provide additional information between source and IL



Function Implementations and Translations

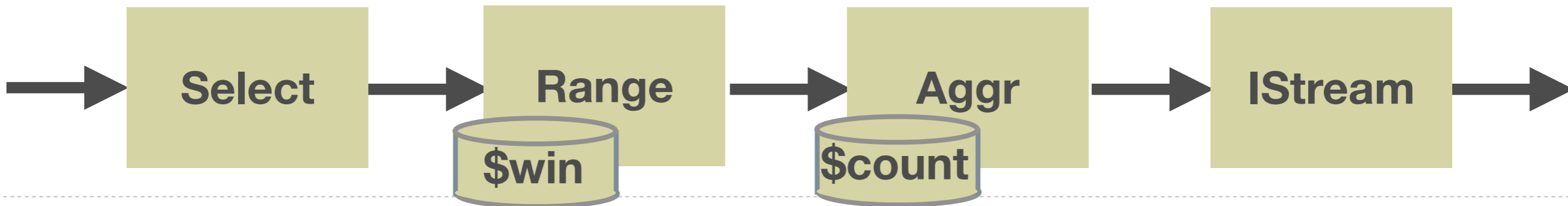
```
logs : {origin : string; target : string} stream;
hits : {origin : string; count : int} stream =
  select istream(origin, count(origin))
  from logs [range 300]
  where origin != target
```

Pre-existing operator templates

Bag.filter (fun x -> #expr)

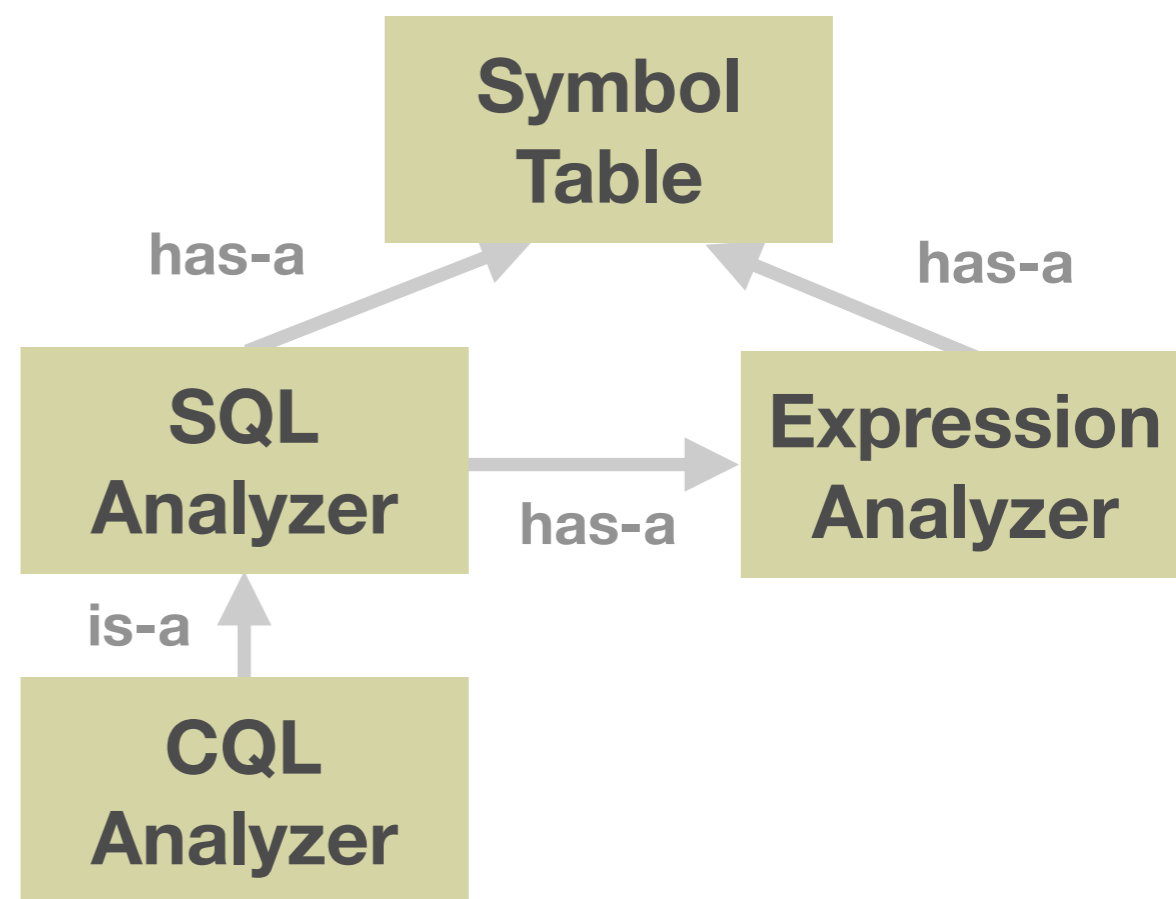
Expose operators, communication, and state

Bag.filter (fun x -> origin != target)



Translations with Modules

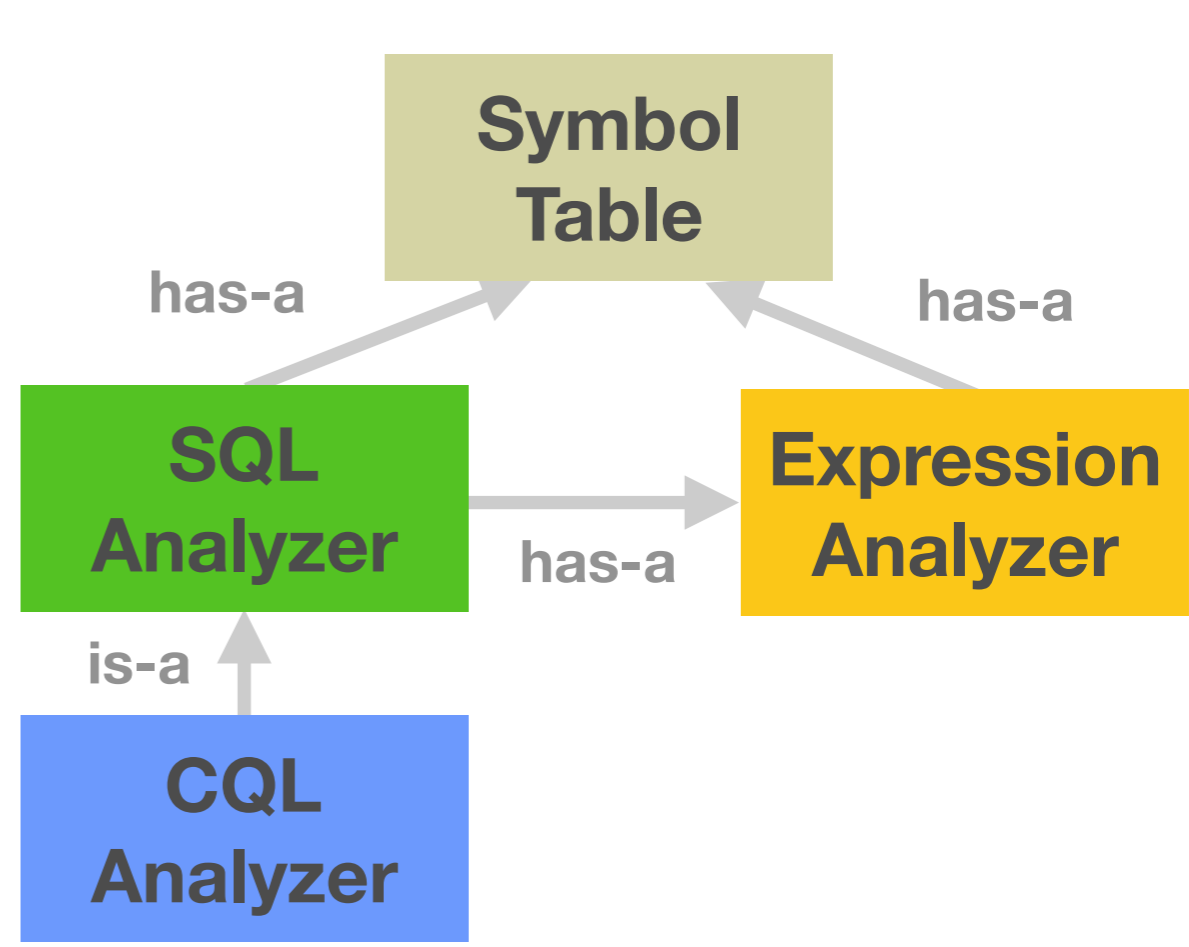
```
select istream(*)
from quotes[now], history
where quotes.ask <= history.low
and quotes.ticker = history.ticker
```



Translations with Modules

```

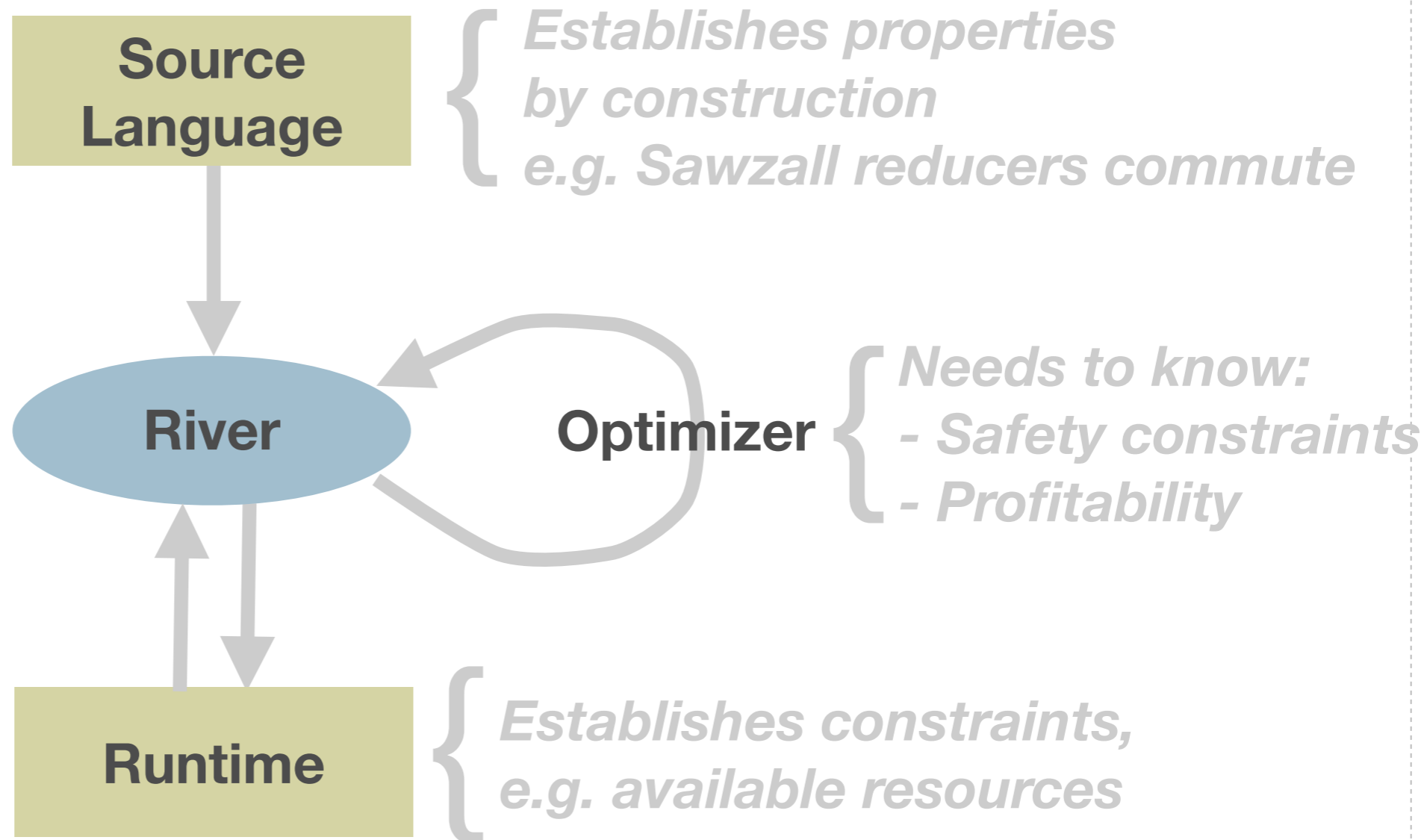
select istream(*)
from quotes[now], history
where quotes.ask <= history.low
and quotes.ticker = history.ticker
    
```



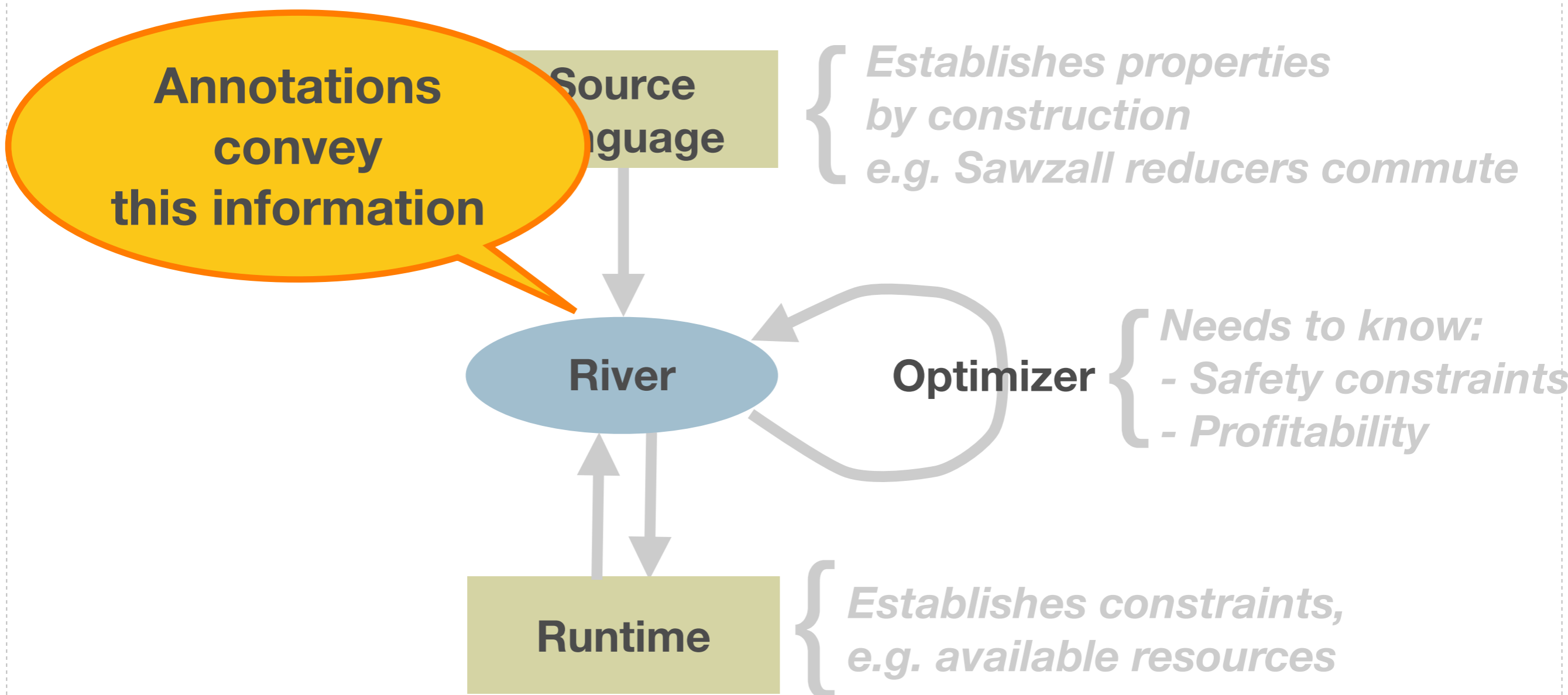
CQL = SQL + Streaming + Expressions



Optimization Support: Extensible Annotations



Optimization Support: Extensible Annotations



Optimization Support: Extensible Annotations

Annotations
convey
this information

Separate policy
from mechanism

Source
Language

{ Establishes properties
by construction
e.g. Sawzall reducers commute

River

Optimizer

{ Needs to know:
- Safety constraints
- Profitability

Runtime

{ Establishes constraints,
e.g. available resources



Optimization Support: Current Annotations

Annotation	Description	Optimization
@Fuse(ID)	Fuse operators with same ID in the same process	Fusion
@Parallel()	Perform fission on an operator	Fission
@Commutative()	An operator's function is commutative	Fission
@Keys(k ₁ ,...,k _n)	An operator's state is partitionable by the key fields k ₁ ,...,k _n	Fission
@Group(ID)	Place operators with same ID on the same machine	Placement



Evaluation

Four benchmark applications

 CQL Linear Road

 StreamIt FM Radio

 Sawzall Batch Web Log
Analyzer

 CQL Continuous Web Log
Analyzer

Three optimizations

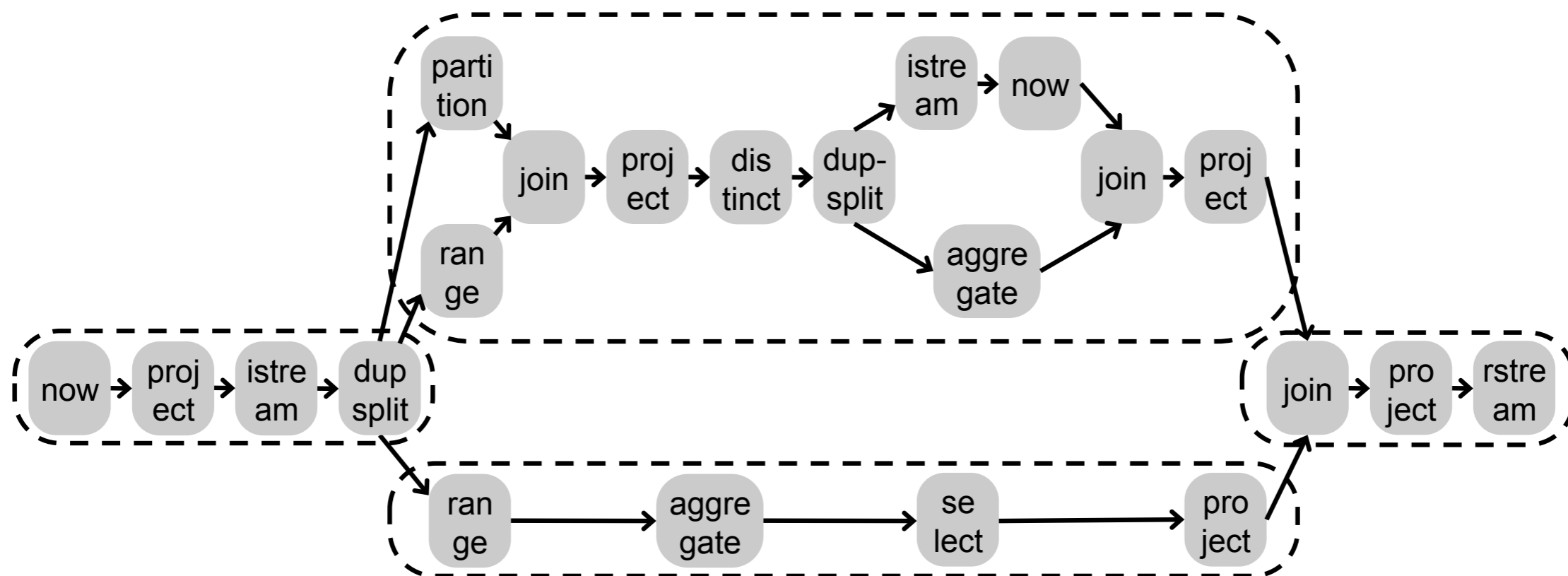
 Placement

 Fission

 Fusion



Distributed Linear Road

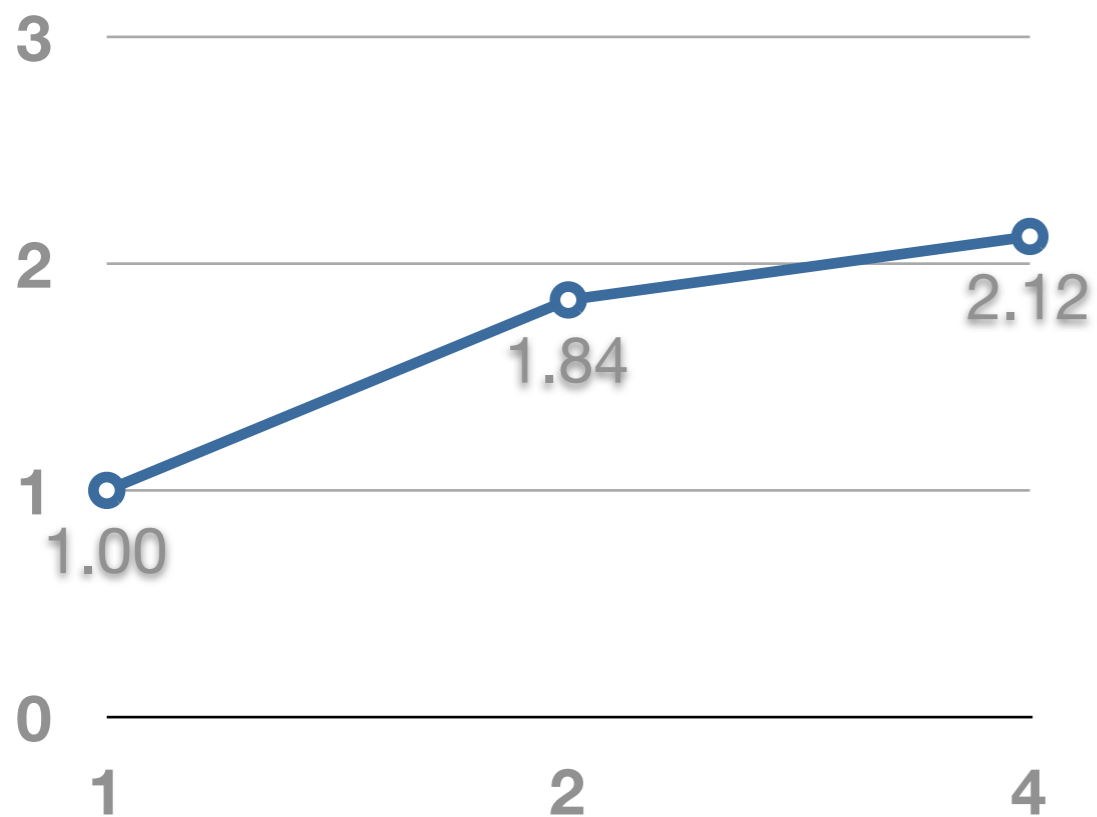


First distributed CQL implementation



CQL Parallelization Has Limited Effect

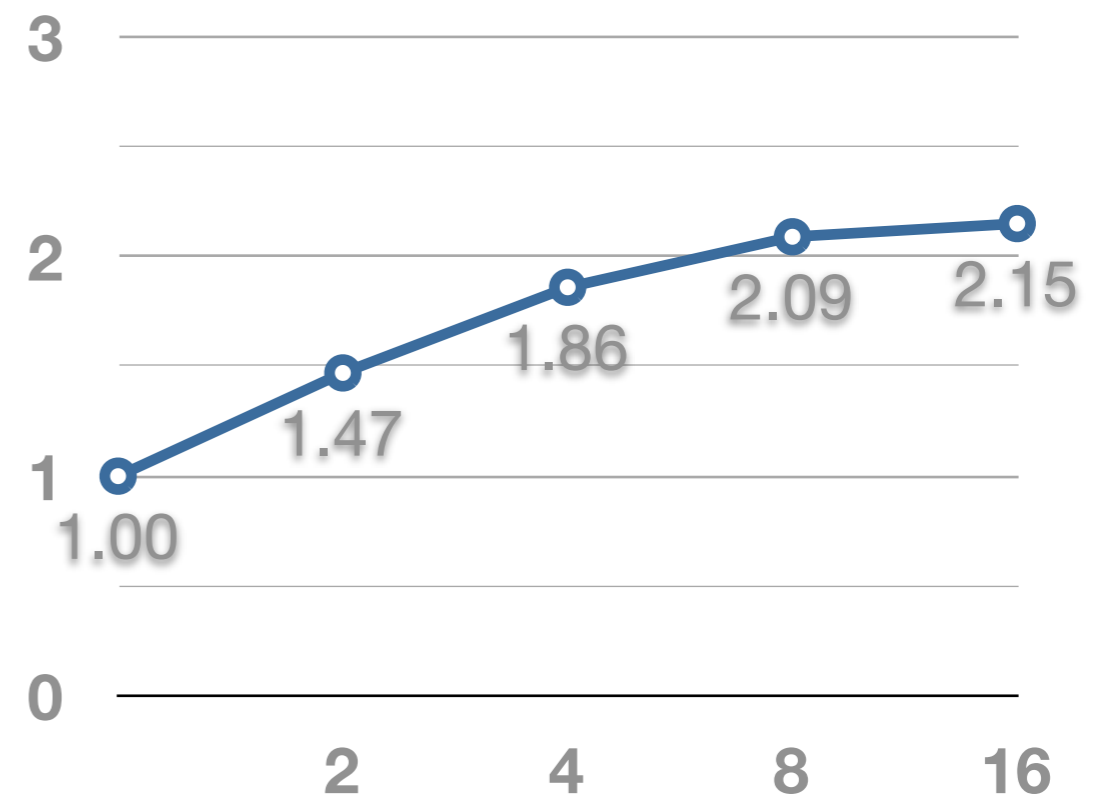
Linear Road Speedup



2.12x speedup on 4 machines

Limited task and pipeline parallelism

CQL Log Analyzer Speedup

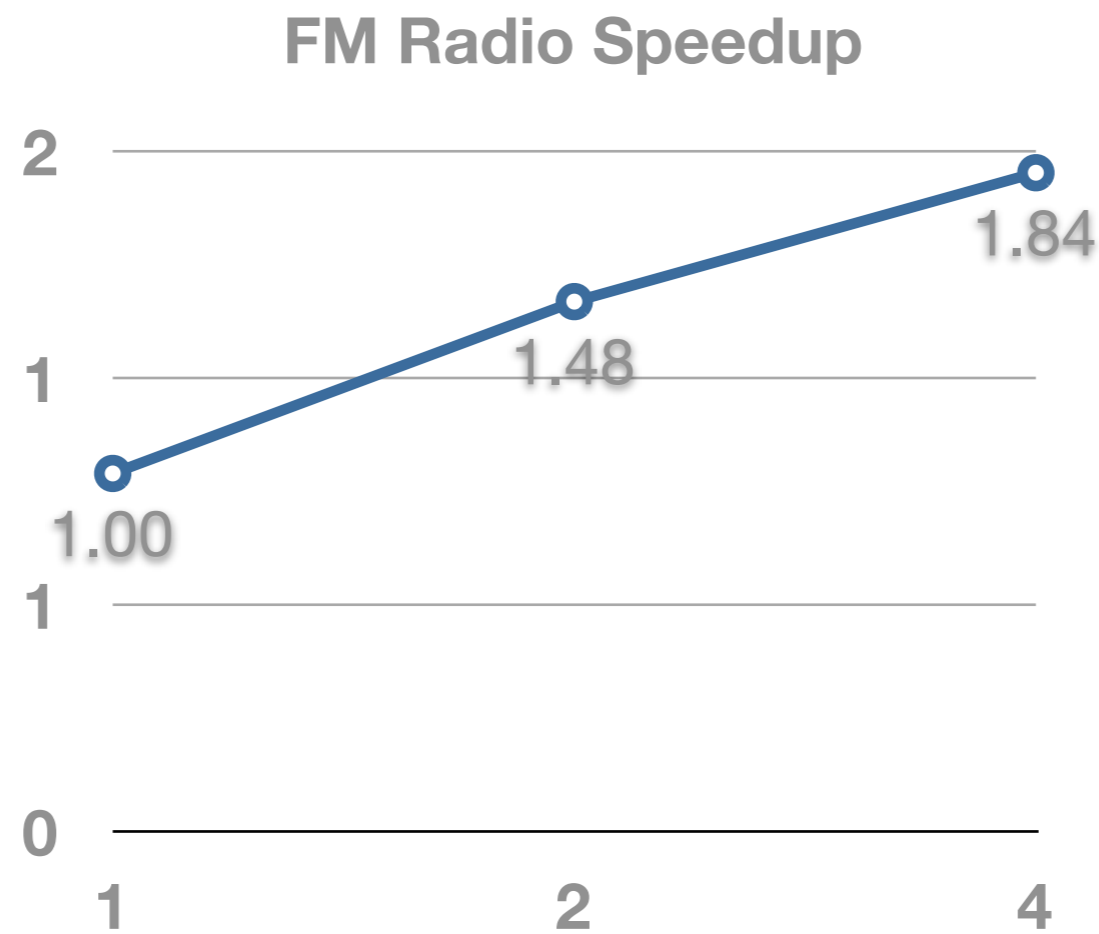


2.15x speedup on 16 machines

Synchronization is bottleneck



Reusable Optimizations

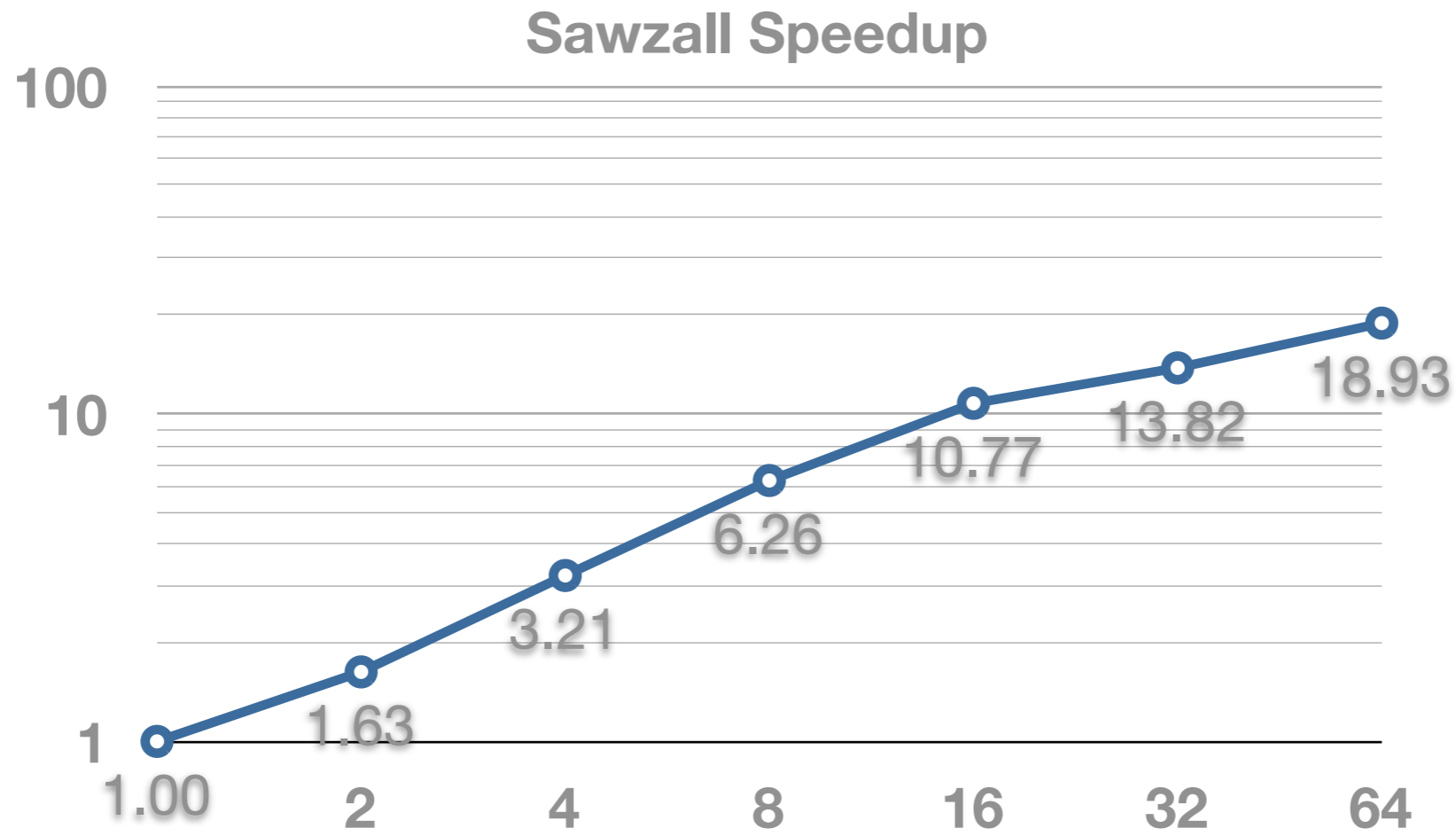


StreamIt FM Radio can re-use the placement optimization

1.84x speedup on 4 machines



MapReduce on River Scales (Almost) Linearly



Our Sawzall uses the same data-parallelism optimizer as CQL

10.77x speedup on 16 machines, 18.93x speedup on 64 cores

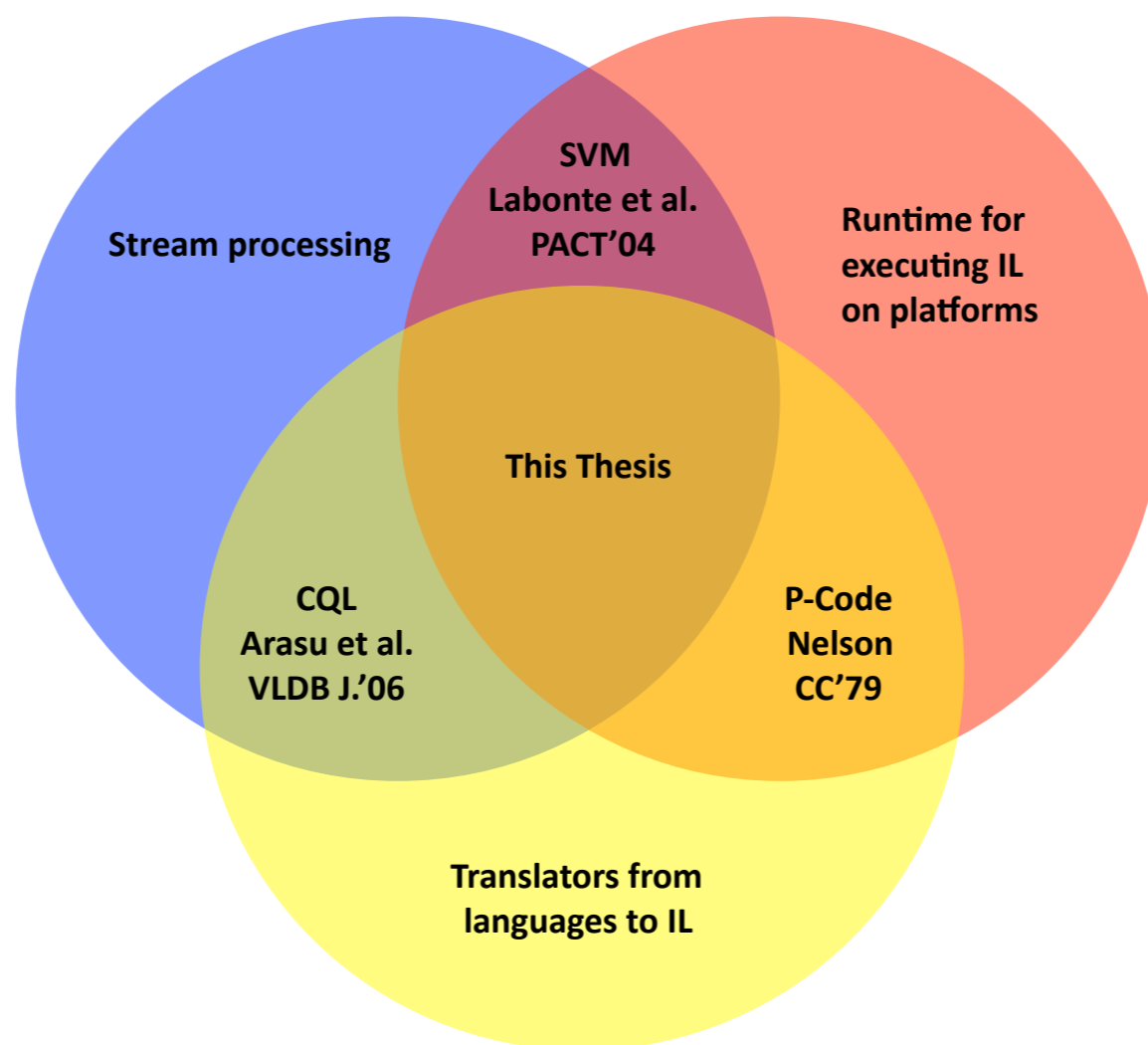




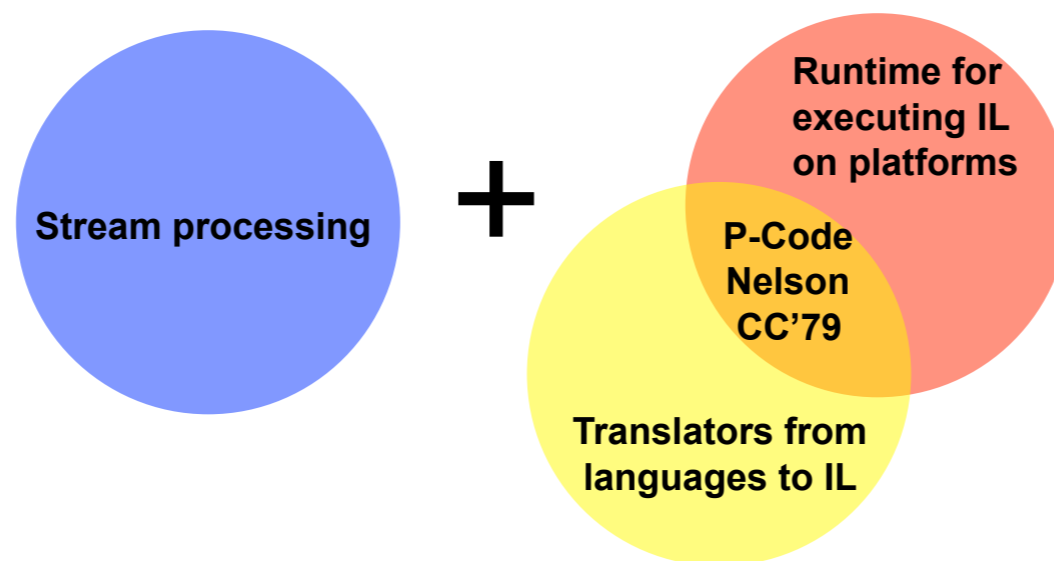
Related Work



Related Work



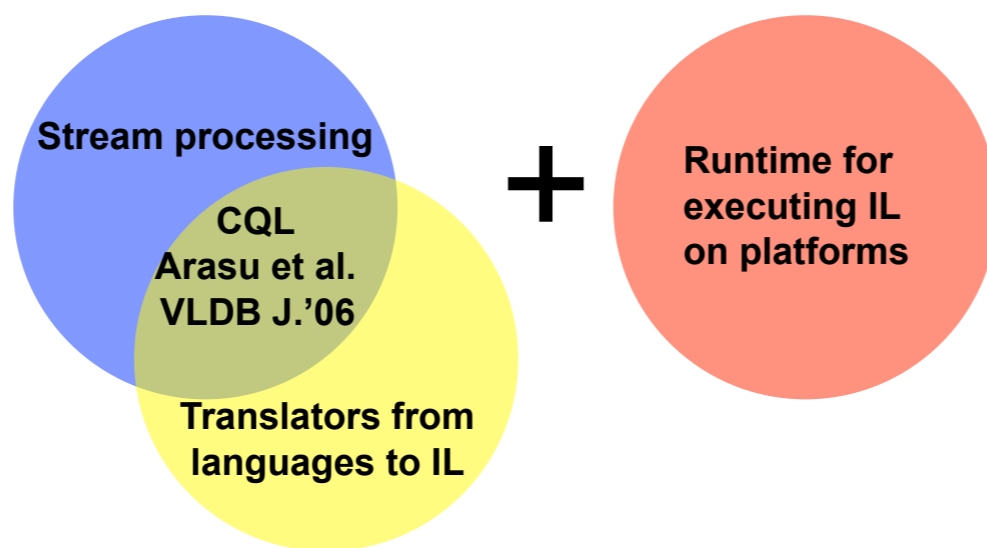
Comparison to Traditional ILs



Traditional IL	River IL
For Pascal, Java, C#	For StreamSQL, Sawzall, StreamIt
IL is lower-level	IL for explicit streaming topology
Data at rest (registers)	Data in motion (queues)
Instructions that run in a sequence, one after the other	Functions that run in parallel, continuously



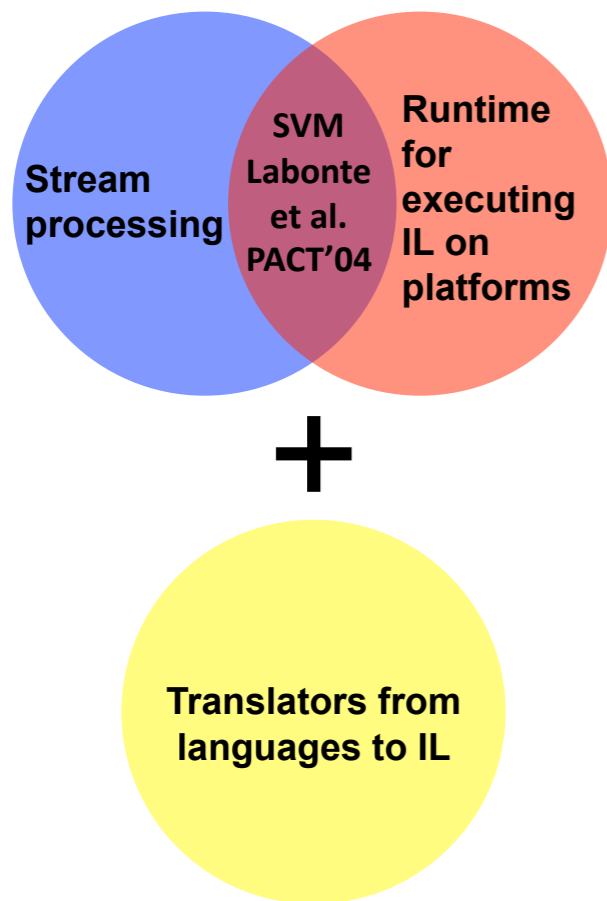
Comparison to CQL



CQL	River IL
Described in terms of SRA (stream-relational algebra)	Uses more general streaming IL (not restricted to relational)
Inter-dependent with a single runtime	Virtual, independent of any particular runtime



Comparison to SVM



SVN	River IL
Missing translators from any language	Translation by recursion over syntax, making state explicit, encapsulating computation in functions
Synchronous, assumes centralized controller	Asynchronous, no centralized controller
Assumes machine model with shared memory and CPUs	Abstracts away streaming runtime (may even be a distributed cluster)





Conclusions



Limitations

Component	Limitations or Future Work
Optimizations Catalog	Interaction of optimizations, compiler analysis, standard benchmarks
Brooklet	Relationship to other calculi, time constraints, more optimizations, dynamism
River	Support for dynamism, performance, design of new languages



Conclusion

- Stream processing is crucial, and needs software infrastructure
 - Identify requirements with a catalog of optimizations
 - Provide a formal foundation with a calculus
 - Design a practical IL with a rigorous semantics
- Overall this work:
 - Enables further advances in language and optimizations design
 - Encourages innovation in stream processing





CQL Translation Rules

CQL program translation: $\llbracket F_c, P_c \rrbracket_c^p = \langle F_b, P_b \rangle$
 $\llbracket F_c, SName \rrbracket_c^p = \emptyset, \text{output } SName; \text{input } SName; \bullet$
 (T_c^p-SNAME)

$\llbracket F_c, RName \rrbracket_c^p = \emptyset, \text{output } RName; \text{input } RName; \bullet$
 (T_c^p-RNAME)

$F_b, \text{output } q_o; \text{input } \bar{q}; \overline{op} = \llbracket F_c, P_{cs} \rrbracket_c^p$
 $q'_o = \text{freshId}() \quad v = \text{freshId}()$
 $F'_b = [S2R \mapsto \text{wrapS2R}(F_c(S2R))]F_b$
 $\overline{op}' = \overline{op}, (q'_o, v) \leftarrow S2R(q_o, v);$

$\llbracket F_c, S2R(P_{cs}) \rrbracket_c^p = F'_b, \text{output } q'_o; \text{input } \bar{q}; \overline{op}'$
 (T_c^p-S2R)

$F_b, \text{output } q_o; \text{input } \bar{q}; \overline{op} = \llbracket F_c, P_{cr} \rrbracket_c^p$
 $q'_o = \text{freshId}() \quad v = \text{freshId}()$
 $F'_b = [R2S \mapsto \text{wrapR2S}(F_c(R2S))]F_b$
 $\overline{op}' = \overline{op}, (q'_o, v) \leftarrow R2S(q_o, v);$

$\llbracket F_c, R2S(P_{cr}) \rrbracket_c^p = F'_b, \text{output } q'_o; \text{input } \bar{q}; \overline{op}'$
 (T_c^p-R2S)

$\overline{F_b}, \text{output } q_o; \text{input } \bar{q}; \overline{op} = \llbracket F_c, P_{cr} \rrbracket_c^p$
 $n = |\overline{P_{cr}}| \quad q'_o = \text{freshId}() \quad \bar{q}' = \bar{q}_1, \dots, \bar{q}_n$
 $\forall i \in 1 \dots n : v_i = \text{freshId}() \quad \overline{op}' = \overline{op}_1, \dots, \overline{op}_n$
 $F'_b = [R2R \mapsto \text{wrapR2R}(F_c(R2R))](\cup \overline{F_b})$
 $\overline{op}'' = \overline{op}', (q'_o, \bar{v}) \leftarrow R2R(\bar{q}_o, \bar{v});$

$\llbracket F_c, R2R(\overline{P_{cr}}) \rrbracket_c^p = F'_b, \text{output } q'_o; \text{input } \bar{q}'; \overline{op}''$
 (T_c^p-R2R)

CQL operator wrappers:

$\frac{\sigma, \tau = d_q \quad s = d_v}{s' = s \cup \{\langle e, \tau \rangle : e \in \sigma\} \quad \sigma' = f(s', \tau)}$
 $\text{wrapS2R}(f)(d_q, -, d_v) = \langle \sigma', \tau \rangle, s'$
 (W_c-S2R)

$\frac{\sigma, \tau = d_q \quad \sigma' = d_v \quad \sigma'' = f(\sigma, \sigma')}{\text{wrapR2S}(f)(d_q, -, d_v) = \langle \sigma'', \tau \rangle, \sigma}$
 (W_c-R2S)

$\frac{\sigma, \tau = d_q \quad d'_i = d_i \cup \{\langle \sigma, \tau \rangle\}}{\forall j \neq i \in 1 \dots n : d'_j = d_j}$
 $\exists j \in 1 \dots n : \nexists \sigma : \langle \sigma, \tau \rangle \in d_j$
 $\text{wrapR2R}(f)(d_q, i, \bar{d}) = \bullet, \bar{d}'$
 (W_c-R2R-WAIT)

$\frac{\sigma, \tau = d_q \quad d'_i = d_i \cup \{\langle \sigma, \tau \rangle\}}{\forall j \neq i \in 1 \dots n : d'_j = d_j}$
 $\forall j \in 1 \dots n : \sigma_j = \text{aux}(d_j, \tau)$
 $\text{wrapR2R}(f)(d_q, i, \bar{d}) = \langle f(\bar{\sigma}), \tau \rangle, \bar{d}'$
 (W_c-R2R-READY)

$\frac{\langle \sigma, \tau \rangle \in d}{\text{aux}(d, \tau) = \sigma}$
 (W_c-R2R-AUX)



Operator Fission

$$\begin{array}{l}
 op = (q_{out}) \leftarrow f(q_{in}); \\
 \forall i \in 1 \dots n : q_i = freshId() \quad \forall i \in 1 \dots n : q'_i = freshId() \\
 F'_b, op_s = \llbracket \emptyset, \text{split roundrobin}, \bar{q}, q_{in} \rrbracket_s^p \\
 \forall i \in 1 \dots n : op_i = (q'_i) \leftarrow f(q_i); \\
 F''_b, op_j = \llbracket \emptyset, \text{join roundrobin}, q_{out}, \bar{q}' \rrbracket_s^p \\
 \hline
 \langle F_b, op \rangle \longrightarrow_{split}^N \langle F_b \cup F'_b \cup F''_b, op_s \overline{op} op_j \rangle
 \end{array}$$



Dynamism

