A Domain Theoretic Foundation for ProbNetKAT

Steffen Smolka (Cornell)
Praveen Kumar (Cornell)
Nate Foster (Cornell)
Dexter Kozen (Cornell)
Alexandra Silva (UCL)
ProbNetKAT?
A language for **modeling & reasoning** about networks **probabilistically**.

\[\text{Prob} + \text{Net} + \text{KA} + T\]
A language for **modeling & reasoning** about networks **probabilistically**.

\[ \text{Prob} + \text{Net} + \overline{1951} + \text{KA} + T \]

regular expressions

\[ +, \cdot, \ast \]
A language for **modeling & reasoning** about networks **probabilistically**.

\[
\text{Prob} + \text{Net} + \text{KA} + T
\]

- Prob: regular expressions (+, ·, *)
- Net: boolean tests (f=n)
- KA: 1951
- T: 1996
A language for **modeling & reasoning** about networks **probabilistically**.

\[
\text{Prob} + \text{Net} + \text{KA} + T
\]

- **NetKAT** 2013
  - **KAT** 1996
    - 1951
  - regular expressions 
    - \(+, \cdot, \ast\)
  - boolean tests 
    - \(f=n\)

network primitives
- \(f:=n, \text{dup}\)
ProbNetKAT

Prob + Net + KA + T

Prob: probabilistic primitives $p \oplus_r q$

Net: network primitives $f:=n$, $\text{dup}$

KA: regular expressions $+, \cdot, *$

T: boolean tests $f=n$
ProbNetKAT in Action
Randomized Routing

Expected Congestion?
Randomized Routing

Expected Congestion?

ProbNetKAT model $p$, input distribution $\mu$

$\rightarrow$ output distribution $\nu = \mu \ggg [p] \in \text{Dist}(2^H)$
Randomized Routing

Expected Congestion?

ProbNetKAT model $p$, input distribution $\mu$

$\rightarrow$ output distribution $v = \mu \gg= [p] \in \text{Dist}(2^H)$

Congestion Query: Random Variable $Q : 2^H \rightarrow [0,\infty]$
Randomized Routing

ProbNetKAT model $p$, input distribution $\mu$

→ output distribution $v = \mu \gg p \in \text{Dist}(2^H)$

Congestion Query: Random Variable $Q : 2^H \rightarrow [0,\infty]$ 

Expected Congestion: $E_v[Q]$
Computation???

$$E_{\nu}[Q] = \int Q \, d\nu$$

Lebesgue Integral

continuous distribution
A solution, based on Domain Theory
We can define a partial order $\mu \sqsubseteq \nu$ such that:

1. For any $\mu \in \text{Dist}(2^H)$, there exist finite distributions $\mu_1 \sqsubseteq \mu_2 \sqsubseteq \mu_3 \sqsubseteq \ldots$ such that $\mu = \bigsqcup \mu_i$ [Saheb-Djahromi]
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   [Saheb-Djahromi]

2. Lebesgue Integration is Scott-continous w.r.t. $\mu$:

   $$\int Q \, d\mu = \sup \int Q \, d\mu_i$$

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→ can compute $E_{\mu_1}[Q] \leq E_{\mu_2}[Q] \leq E_{\mu_3}[Q] \leq \ldots$ with
   $$\lim E_{\mu_i}[Q] = E_\mu[Q]!$$
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→ implemented in OCaml in ~300 LOC
Thank you!

More on this soon....