A Domain Theoretic Foundation for ProbNetKAT

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ProbNetKAT?

A language for **modeling & reasoning** about networks **probabilistically.**

Prob + Net + KA + T

A language for **modeling & reasoning** about networks **probabilistically.**

1951

Prob + Net + KA + T

regular expressions

+, ·, *

A language for **modeling & reasoning** about networks **probabilistically.**

KAT 1996

1951

Prob + Net + KA + T

Prob	+	Net	+	KA	+	T
				regular expressions		boolean tests
				+, ·, *		f=n

A language for **modeling & reasoning** about networks **probabilistically.**

NetKAT 2013

KAT 1996

1951

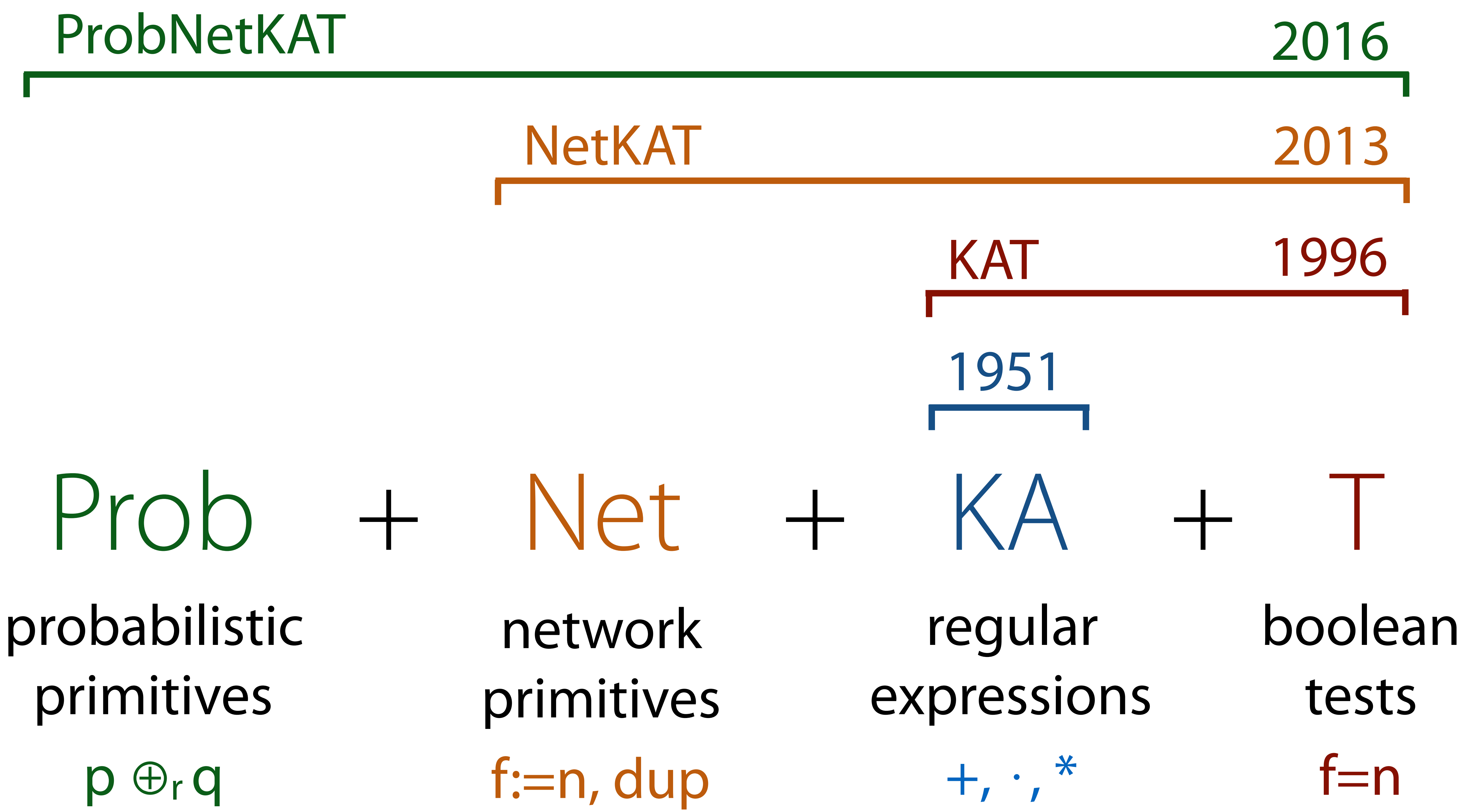
Prob + Net + KA + T

Net — network primitives: f:=n, dup

KA — regular expressions: +, · , *

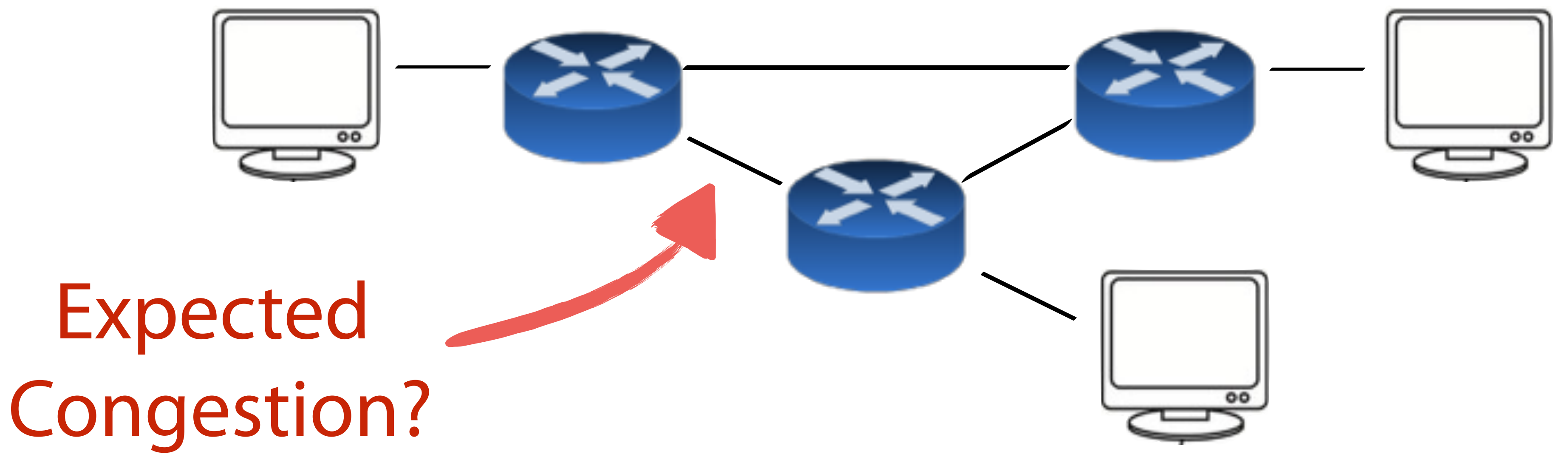
T — boolean tests: f=n

A language for **modeling & reasoning** about networks **probabilistically.**



ProbNetKAT in Action

Randomized Routing



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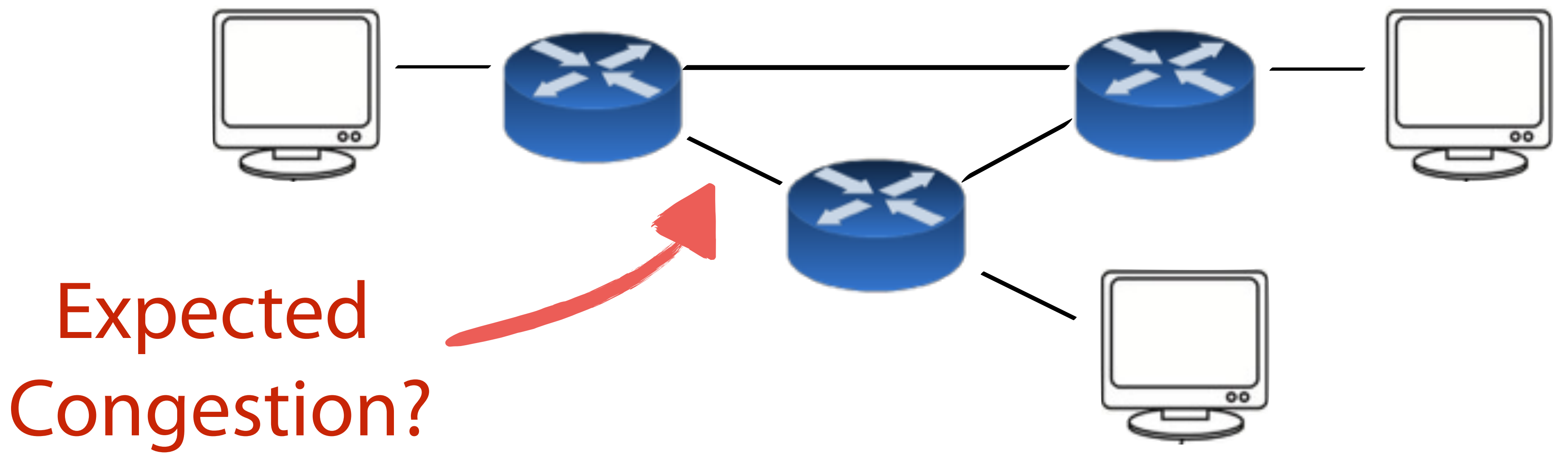
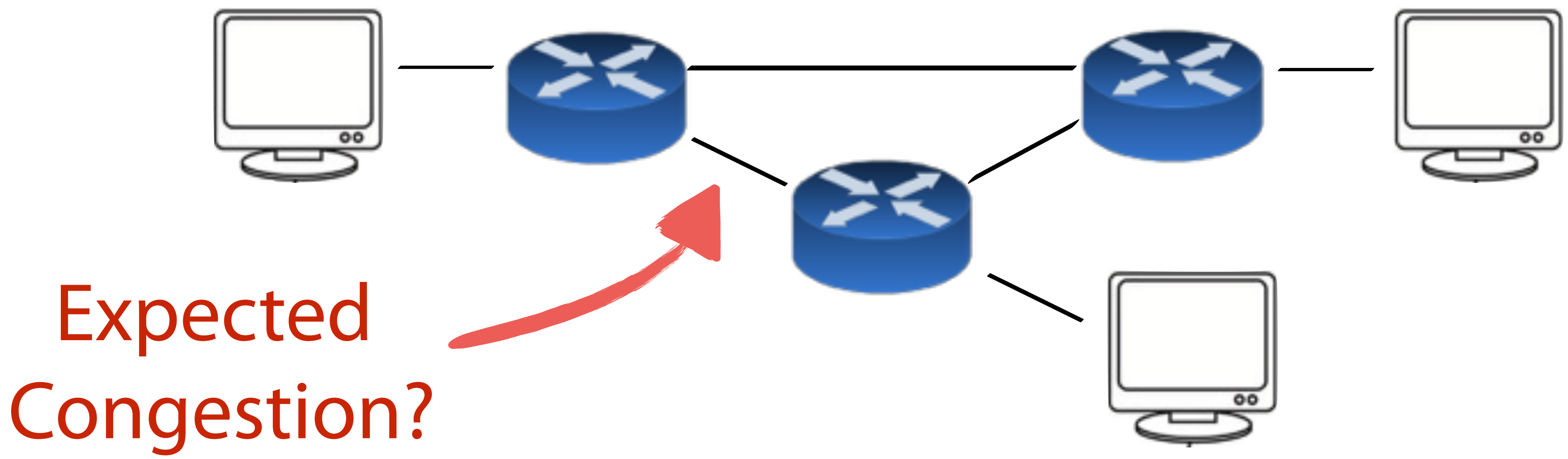


ProbNetKAT model **p**, input distribution **μ**

→ output distribution **ν** $= \mu \gg= [\![p]\!] \in \mathrm{Dist}(2^H)$

Randomized Routing



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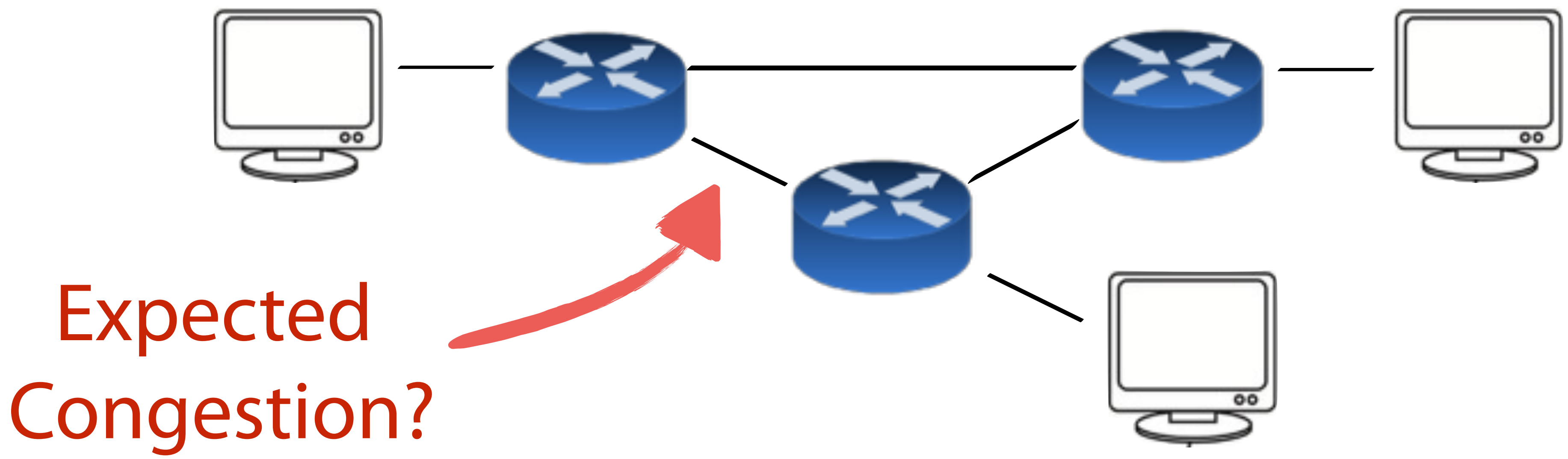
Congestion Query: Random Variable **Q** $: 2^H \rightarrow [0,\infty]$

Randomized Routing



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Congestion Query: Random Variable **Q** : 2^H → [0,∞]

Expected Congestion: E_ν[**Q**]

Computation???

$$E_{\nu}[\mathbf{Q}] = \int Q \, d\nu$$

Lebesgue Integral

continuous distribution

A solution, based on Domain Theory

We can define a partial order $\mu \sqsubseteq \nu$ such that:

1. For any $\mu \in \mathrm{Dist}(2^H)$, there exist **finite distributions** $\mu_1 \sqsubseteq \mu_2 \sqsubseteq \mu_3 \sqsubseteq \ldots$ such that $\mu = \bigsqcup \mu_i$ [Saheb-Djahromi]

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→ implemented in OCaml in ~300 LOC

Thank you!

More on this soon….