Known facts:

- Solution space of random combinatorial problems \textit{fractures into clusters} as constraint density (\& hardness) increases
- The fastest solution technique relies on \textit{marginal probability estimates over clusters}

Our results:

- An expression to count the number of clusters with high precision

\[
Z_{(-1)} = \sum_{y \in \text{DomExt}^a} (-1)^{\#e(y)} \prod_{\alpha} f_{\alpha}(\vec{y}_{\alpha})
\]

- A message-passing scheme similar to BP that approximates \(Z_{(-1)}\) well