From Streamlined Combinatorial Search to Efficient Constructive Procedures

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Motivation

- Significant progress in the area of search, constraint satisfaction and automated reasoning driven by challenge problems in combinatorics.
- One shortcoming is that these methods do not provide further mathematical insights, as they are, in essence, a form of proof by demonstration.
- We propose a framework that combines specialized combinatorial search (streamlining) with human insights, in a complementary, iterative approach.
- Ultimately, the goal is to discover efficient constructive procedures (polynomial-time algorithms that take as input a size parameter, N, and generate a certain combinatorial object of size N).

Example Domains

Spatially Balanced Latin Square (SBLS) Problem:

- Each symbol from 1 to n appears exactly once in each row and column (Latin Square structure).
- The average distance (column-wise) of a pair of symbols is the same for any pair (Balanced structure).
- Computationally challenging combinatorial design problem with applications in the area of crop rotation and drug design.

Weak Schur Number (WS) Problem:

- Schur Numbers are closely related to Ramsey theory, and are a notoriously hard area of combinatorics.
- Schur Numbers are defined as weakly sum free sets.
- The Weak Schur Number of order k, WS(k), is the largest integer n for which there exists a partition of [1,n] into k weakly sum-free sets.

Overview of the proposed strategy:

1) Analyze smaller size solutions, and conjecture potential regularities in the solutions.
2) Validate through streamlining the observed regularities.
3) If the streamlined search does not give a larger size solution, the proposed regularity is quite likely accidental and one looks for a new pattern in the small scale solutions.
4) Otherwise, one proceeds by generating a number of new solutions that all contain the proposed structural regularity and are used to expand the solution set and to reveal new regularities.

Discover-Construction Procedure

Successful Key Streamliners:

- We introduce the first constructive procedure for Spatially Balanced Latin Squares.
- The largest SBLS known to exist was of order 35 and took about 2 weeks of computation. Our algorithm generates a SBLS of order 999 in 0.01 second.
- Our constructive procedure confirms a 2004 conjecture on the existence of arbitrary large SBLSs and of an effective way of constructing them.

Results on the SBLS Problem

Successful Key Streamliners:

- We provide a new lower-bound for the Weak Schur Numbers, proving WS(6) ≥ 581.
- The best known lower-bound was WS(6) ≥ 575, found by (Eliahou et al., 2012) and improving on the ‘WS(6) ≥ 574’ result of (Fonlupt et al., 2011).
- Although not an example of a fully constructive procedure yet, any progress on Schur numbers is quite significant given their long history.

Acknowledgments

The authors gratefully acknowledge the support of the National Science Foundation, award number 0832782.