

Structured Learning of Two-Level Dynamic Rankings

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ABSTRACT

For ambiguous queries, conventional retrieval systems are bound by two conflicting goals. On the one hand, they should diversify and strive to present results for as many query intents as possible. On the other hand, they should provide depth for each intent by displaying more than a single result. Since both diversity and depth cannot be achieved simultaneously in the conventional *static* retrieval model, we propose a new *dynamic* ranking approach. Dynamic ranking models allow users to adapt the ranking through interaction, thus overcoming the constraints of presenting a one-size-fits-all static ranking. In particular, we propose a new *two-level* dynamic ranking model for presenting search results to the user. In this model, a user's interactions with the first-level ranking are used to infer this user's intent, so that second-level rankings can be inserted to provide more results relevant for this intent. Unlike for previous dynamic ranking models, we provide an algorithm to efficiently compute dynamic rankings with provable approximation guarantees for a large family of performance measures. We also propose the first principled algorithm for learning dynamic ranking functions from training data. In addition to the theoretical results, we provide empirical evidence demonstrating the gains in retrieval quality that our method achieves over conventional approaches.

Keywords

Diversified Retrieval, Structured Learning, Submodular Optimization, Web Search & Information Retrieval

1. INTRODUCTION

Search engine users often express different information needs using the same query. This leads to the well-known problem of *ambiguous queries*, where a single query can represent multiple intents. In some cases, the ambiguity in intent can be coarse (*e.g.*, queries such as *apple*, *jaguar* and *SVM*). In other cases, the distinctions can be more fine-

grained (*e.g.*, the query *apple ipod* with the intent of either buying the device or reading reviews).

Conventional retrieval methods do not explicitly model query ambiguity, but simply select a ranking of results by maximizing the probability of relevance independently for each document [17]. A major limitation of this approach is that it favors results for the most prevalent intent. In the extreme, the retrieval system focuses entirely on the prevalent intent, but produces no relevant results for the less popular intents. Diversification-based methods (*e.g.* [4, 23, 5, 22]) try to alleviate this problem by including at least one relevant result for as many intents as possible. However, this necessarily leads to fewer relevant results for each intent. Clearly, there is an inherent trade-off between *depth* (number of results provided for an intent) and *diversity* (number of intents served). In the conventional ranked-retrieval setting, choosing one invariably leads to the lack of the other. A natural question that arises in this context is: how can we obtain diversity while not compromising on depth?

We argue that a key to solving the conflict between depth and diversity lies in the move from conventional *static* retrieval models to *dynamic* retrieval models [3] that can take advantage of user interactions. Instead of presenting a single one-size-fits-all ranking, dynamic retrieval models allow users to adapt the ranking dynamically through interaction. Brandt et al. [3] have already given theoretical and empirical evidence that even limited interactivity can greatly improve retrieval effectiveness. However, Brandt et al. [3] did not provide an efficient algorithm for computing dynamic rankings with provable approximation guarantees, nor did they provide a principled algorithm for learning dynamic ranking functions from training data. In this paper, we resolve these two open questions.

In particular, we propose a new *two-level dynamic ranking model*. The intuition behind the models is that the first level provides a diversified ranking of results. The system then senses the user's interactions with the first-level and interactively provides a second-level rankings conditioned on this feedback. A possible layout for such two-level rankings is given in Figure 1. The left-hand panel shows the first-level ranking initially presented to the user. The user then chooses to expand the second document (*e.g.* by mousing over or clicking) and a second-level ranking is inserted as shown in the center panel. Conceptually, the retrieval system maintains two levels of rankings as illustrated in the right-hand panel, where each second-level ranking is conditioned on the head document in the first-level ranking.

To operationalize the construction and learning of two-

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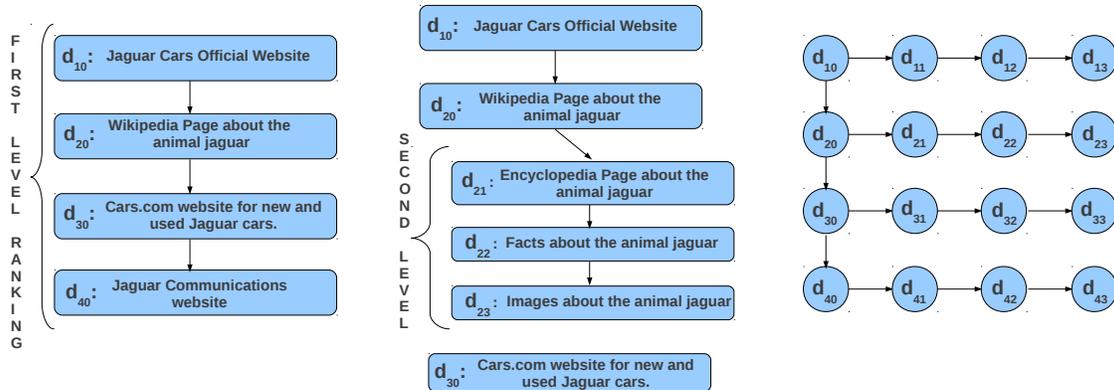


Figure 1: A typical two-level ranking for the query “jaguar”. A user interested in the animal “jaguar” interacts with the first-level ranking (left panel) and can expand results of interest to see additional results (middle panel). A two-level rankings can be thought of as a two-dimensional matrix (right panel).

level rankings in a rigorous way, we define a new family of performance measure for diversified retrieval. Many existing retrieval measures (e.g., Precision@k, DCG, Intent Coverage) are special cases of this family. We then operationalize the problem of computing an optimal two-level ranking as maximizing the given performance measure. While this optimization problem is NP-hard, we provide an algorithm that we show has a $(1 - \frac{1}{e^{1-\frac{1}{e}}})$ approximation guarantee.

Finally, we also propose a new method for learning the (mutually dependent) relevance scores needed for two-level rankings. Following a structural SVM approach, we learn a discriminant model that resembles the desired performance measure in structure, but learns to approximate unknown intents based on query and document features. This method generalizes the learning method from [22] to two-level rankings and a large class of loss functions. In addition to theoretical results, we evaluate the properties of our model, the algorithm for computing two-level rankings, and the learning methods through a detailed empirical analysis.

2. RELATED WORK

Traditional non-diversified methods for retrieval focus on ranking documents based on their probability of relevance to the query [17, 12, 13]. However, these models are problematic in the case of *ambiguous queries*, as they tend to favor the most common user intent while ignoring the others.

Diversified retrieval aims to overcome the challenge of query ambiguity by providing diversity in search results [4, 23, 5, 22, 6]. In the extreme case diversified retrieval methods maximize *intent coverage*, meaning that they aim at covering as many intents in the ranking as possible by providing just a single document per intent. The methods in [19, 16, 22] formulate this problem as a set coverage problem. Most concretely, [16] proposed a multi-armed bandit algorithm, showing that it maximizes the number of users presented with at least one relevant document with provable guarantees. While diversification methods alleviated the problem of ignoring less frequent intents, they all explicitly or implicitly improve diversity at the expense of depth (i.e., they present only a few relevant documents for any given intent).

Recent work by Brandt et al. [3] has focused on addressing the above issue. They propose a *dynamic* ranked-retrieval model that allows user interaction. User can interactively expand results so that a dynamic ranking gets created on the fly. Instead of following a static list of results, users construct their individual ranking as a path through a ranking tree. Since users with different intents take different paths, it is possible to tailor both the distribution and content of each path. Brandt et al. [3] have shown that this small amount of interactivity overcomes the inherent trade-off between diversity and depth of conventional static rankings. We also follow this idea of dynamic ranking, but with the following differences. First, we focus on a simpler and more plausible model of user behavior. Unlike in [3], we do not assume that users are willing to provide feedback that is more than one level deep (see Section 3), and we allow users to backtrack to a higher level. Unlike the algorithms for constructing dynamic rankings presented in [3], we present an algorithm that has provable approximation guarantees. Furthermore, our algorithm and model apply to a large class of submodular performance measures, which include those of [3] and [16, 22] as special cases. And finally, we propose a principled method for learning dynamic ranking functions.

Our learning method follows a long line of research on training retrieval functions [13]. However, with the exception of [22], virtually no other work directly addresses the problem of learning diversified retrieval functions. Following an idea first presented in [19], Yue and Joachims [22] relate diversity in word occurrences to diversity in intents. They propose to learn this relationship using a structural SVM method, where they formulate the discriminant function as a coverage problem and optimize intent coverage as the loss function. We also employ a similar learning technique and also draw a correlation between intents and words. However, our learning method goes beyond single-level static ranking to predict two-level dynamic ranking, and it can optimize a large family of loss functions as defined in Section 4.2.

The dynamic retrieval model we present in this paper is a special case of *interactive retrieval*. An interactive retrieval setting involves multiple interactions between users and a system. Our model is most closely related to relevance feed-

Intent	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9
t_1	1	1	1	0	0	0	0	0	0
t_2	0	0	0	1	1	1	0	0	0
t_3	0	0	0	0	0	0	1	1	0
t_4	0	0	0	0	0	0	1	0	1

Table 1: Utility $U(d_j|t_i)$ of a document d_j given an intent t_i .

back (e.g. [1, 18, 21, 24]), where the system presents a set of results and the user provides either implicit or explicit feedback. The feedback can then be used by the system to update the ranking. Note that the interface sketched in Figure 1 is inspired by SurfCanyon.com [7].

3. TWO-LEVEL DYNAMIC RANKINGS

Current methods for diversified retrieval are *static* in nature. Such *static* rankings stay unchanged through a user session. On the other hand, a *dynamic* model can adapt the ranking based on interactions with the user. The primary motivation for using a dynamic model is the inherent trade-off between depth and diversity in static models. Figure 1 illustrates the two-level dynamic rankings considered in this paper. We now provide a simple quantitative example to show how such two-level dynamic rankings can achieve both diversity and depth.

Consider four user intents $\{t_1, \dots, t_4\}$ and nine documents $\{d_1, \dots, d_9\}$ with relevance judgments $U(d_j|t_i)$ as given in Table 1. In this example, we assume that the user intents are equally likely. On the one hand, a non-diversified static ranking method could present $d_7 \rightarrow d_8 \rightarrow d_9$ as its top three documents. This means that users with intents t_3 and t_4 get two relevant documents, but it fails to provide any relevant documents to users with intents t_1 and t_2 . On the other hand, a diversified static ranking with $d_7 \rightarrow d_1 \rightarrow d_4$ as the top three documents covers all intents, but no user gets more than one relevant document. Therefore, this ranking lacks depth.

As an alternative, now consider a two-level dynamic ranking as follows. The user is presented with $d_7 \rightarrow d_1 \rightarrow d_4$ as the first-level ranking. Users can now expand any of the first-level results to receive a second-level ranking. Assume that users interested in d_7 (and thus having intent t_3 or t_4) will *expand* that result and receive a second-level ranking consisting of d_8 and d_9 . Similarly, users interested in d_1 will get d_2 and d_3 . And users interested in d_4 will get d_5 and d_6 . Note that every intent is covered in the top three results of the first-level ranking. At the same time, users with intents t_3 and t_4 receive two relevant results in the top three positions of their dynamically constructed ranking $d_7 \rightarrow d_8 \rightarrow d_9 \rightarrow d_1 \rightarrow d_4$; users with intent t_1 also receive two relevant results in the top three positions of $d_7 \rightarrow d_1 \rightarrow d_2 \rightarrow d_3 \rightarrow d_4$; and users with intent t_2 still receive one relevant result in the top three of $d_7 \rightarrow d_1 \rightarrow d_4 \rightarrow d_5 \rightarrow d_6$. This example illustrates how a dynamic two-level ranking can provide diversity and increased depth simultaneously.

In the above example, interactive feedback from the user was the key to achieving both depth and diversity. More generally, we assume that users interact with the dynamic ranking according to the following **User Model**, which we

denote as policy π_d . While other policies of user behavior are possible, we focus on π_d for the sake of simplicity. Users expand a first-level document if and only if that document is relevant to their intent. When users skip a document, they continue with the next first-level result. When users expand a first-level result, they go through the second-level rankings before continuing from where they left off in the first-level ranking. It is thus possible for a user to see multiple second-level rankings. Hence we do not allow documents to appear more than once across all two-level rankings.

Note that this user model differs from the one proposed in [3] in several ways. First, it assumes only one level of feedback, while the model in [3] assumes that users are willing and able to provide feedback many levels of rankings deep. Second, we model that users return to the top-level ranking, which is not allowed in the model of [3]. We believe that these differences make the two-level model easier to understand for the user and therefore more plausible for practical use.

We now define some notation used in the rest of this paper. The set of documents shown on the first level are called the **head** documents. The number of documents shown in this level is the **length** of the first-level, and it is denoted by L . The set of documents shown on the second level are called the **tail** documents. A **row** denotes a particular **head** document and all the **tail** documents that follow it in the second level. The length of a row, excluding the head document, is denoted by W and is referred to as the **width**. Static rankings are denoted as θ while two-level rankings are denoted as $\Theta = (\Theta_1, \Theta_2, \dots, \Theta_i, \dots)$. Here $\Theta_i = (d_{i0}, d_{i1}, \dots, d_{ij}, \dots)$ refers to the i^{th} row of a two-level ranking, with d_{i0} representing the head document of the row and d_{ij} denoting the j^{th} tail document of the second-level ranking. We denote the candidate set of documents to rank for a query q by $\mathcal{D}(q)$, the set of possible intents by $\mathcal{T}(q)$ and $\mathbf{P}[t|q] \forall t \in \mathcal{T}(q)$ denotes a distribution over the intents given a query q . Unless otherwise mentioned, dynamic ranking refers to a two-level dynamic rankings in the rest of this paper.

4. PERFORMANCE MEASURES FOR DIVERSIFIED RETRIEVAL

To define what constitutes a good two-level dynamic ranking, we start by defining the measure of retrieval performance we would like to optimize. We then design our retrieval algorithms and learning methods to maximize this measure. In the following, we first consider evaluation measures for one-level rankings, and then generalize them to the two-level case.

4.1 Measures for Static Rankings

Existing performance measures range from those that do not explicitly consider multiple intents (e.g. NDCG, Avg Prec), to measures that reward diversity. Measures that reward diversity give lower marginal utility to a document, if the intents the document is relevant to are already well represented in the ranking. We call this the *diminishing returns* property. The extreme case is the ‘‘intent coverage’’ measure (e.g. [19, 16, 22]). It attributes utility only to the first document relevant for an intent and no additional utility to any additional documents.

We now define a family of measures that includes a whole range of diminishing returns models, and that includes most

existing retrieval measures. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ with $g(0) = 0$ be a concave, non-negative, and non-decreasing function that models the diminishing returns, then we define utility of the ranking $\theta = (d_1, d_2, \dots, d_k)$ conditioned on a given intent t as

$$U_g(\theta|t) = g\left(\sum_{i=1}^{|\theta|} \gamma_i U(d_i|t)\right). \quad (1)$$

The $\gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_k \geq 0$ are discount factors that decrease with position in the ranking, and $U(d|t)$ is the relevance rating of document d for intent t . The above definition of utility is with respect to a single user intent t . For a distribution of user intents $\mathbf{P}[t|q]$ for query q , we take the expectation

$$U_g(\theta|q) = \sum_{t \in \mathcal{T}(q)} \mathbf{P}[t|q] \times U_g(\theta|t). \quad (2)$$

Note that many existing retrieval measures are special cases of the definition above. For example, if one chose g to be the identity function, one recovers the intent-aware measures proposed in [2] and the modular measures defined in [3]. Further restricting $\mathbf{P}[t|q]$ to put all probability mass on a single intent leads to conventional measures like DCG [10] for appropriately chosen γ_i . At the other extreme, choosing $g(x) = \min(x, 1)$ leads to the intent coverage measure [16, 22, 19] that assigns utility only to the first relevant document for an intent. Beyond these special cases, g can be chosen from a large class of functions, implying that this family of performance measures covers a wide range of diminishing returns models.

4.2 Measures for Dynamic Rankings

We are now ready to extend our family of performance measures to dynamic rankings. The key change for dynamic rankings is that users interactively adapt which results they view.

How users expand first-level results was defined in Section 3 as π_d . Under π_d , it is natural to define the utility of a dynamic ranking Θ as follows. For a user intent t and a concave, non-negative, and non-decreasing function g ,

$$U_g(\Theta|t) = g\left(\sum_{i=1}^{|\Theta|} \left(\gamma_i U(d_{i0}|t) + \sum_{j=1}^{|\Theta_i|} \gamma_{ij} U(d_{i0}|t) U(d_{ij}|t)\right)\right). \quad (3)$$

Like for static rankings, $\gamma_1 \geq \gamma_2 \geq \dots$ and $\gamma_{i1} \geq \gamma_{i2} \geq \dots$ are position-dependent discount factors. Furthermore, we again take the expectation as in Equation 2 to average over multiple user intents to obtain $U_g(\Theta|q)$.

Note that the utility of a second-level ranking for a given intent is zero unless the head document in the first-level ranking has non-zero relevance for that intent. This encourages second-level rankings to only contain documents relevant to the same intents as the head document, thus providing depths. The first-level ranking, on the other hand, provides diversity as controlled through the choice of function g . The “steeper” g diminishes returns of additional relevant documents, the more diverse the first-level ranking gets. We will explore this in more detail in Section 7.1.

A key advantage of modeling utility in the above form is that it allows for an efficient algorithm for finding a dynamic ranking which maximizes the utility. We present this in the next section.

5. COMPUTING DYNAMIC RANKINGS

In this section, we provide a greedy algorithm to compute dynamic rankings. These rankings are computed by maximizing the performance measures defined in the previous section. Computing rankings to exactly maximize our performance measure is an NP-hard problem. However, we show that our two-level greedy algorithm admits a $(1 - e^{-1+1/e})$ -approximation guarantee.

Our proposed greedy algorithm is given in Algorithm 1. We are given a query q , a candidate set of documents $\mathcal{D}(q)$, intents $\mathcal{T}(q)$ with their distribution $\mathbf{P}[t|q]$, and a concave, non-negative and non-decreasing function g that defines the utility in (2). Our goal is to construct a two dimensional ranking of length L and width W to maximize the performance measure (2). In the algorithm, the “overloaded operator” \oplus denotes either adding a document to a row, or adding a row to an existing ranking.

The proposed algorithm works as follows. Every document in the remaining collection is considered as the head document of a candidate row. For each candidate row, W documents are greedily added to maximize the utility of the resulting partial dynamic ranking. Once rows of length W are constructed, the row which maximizes the utility is added to the ranking. The above steps are repeated until the ranking has L rows.

Algorithm 1 for computing a two-level dynamic ranking.

Input: $(q, \mathcal{D}(q), \mathcal{T}(q), \mathbf{P}[t|q] : t \in \mathcal{T}(q)), g(\cdot)$, length L and width W .

Output: A dynamic ranking Θ .

$\Theta \leftarrow \text{new_two_level}()$

while $|\Theta| \leq L$ **do**

$bestU \leftarrow -\infty$

for all $d \in \mathcal{D}(q)$ s.t. $d \notin \Theta$ **do**

$row \leftarrow \text{new_row}()$; $row.head \leftarrow d$

for $j = 1$ to W **do**

$bestDoc \leftarrow \text{argmax}_{d' \notin \Theta \cup row} U_g(\Theta \oplus (row \oplus d')|q)$

$row \leftarrow row \oplus bestDoc$

end for

if $U_g(\Theta \oplus row|q) > bestU$ **then**

$bestU \leftarrow U_g(\Theta \oplus row|q)$; $bestRow \leftarrow row$

end if

end for

$\Theta \leftarrow \Theta \oplus bestRow$

end while

The proposed algorithm is extremely simple and efficient. The algorithm requires $O(|\mathcal{T}|)$ space and $O(|\mathcal{T}||\mathcal{D}|^2)$ time, where $|\mathcal{T}|$ is the total number of intents and $|\mathcal{D}|$ is the number of candidate documents. The run time of the algorithm can be further improved using techniques such as lazy evaluation [14].

We now derive an approximation bound for the greedy algorithm by relating it to the well-known problem of optimizing submodular set functions. First, recall the following definition of a submodular function.

DEFINITION 1. Given a set U , a function $f : 2^U \rightarrow \mathbb{R}$ is said to be **submodular** iff for all elements $u \in U$ and all sets X and Y , s.t. $X \subseteq Y \subseteq U$, we have

$$f(X \cup \{u\}) - f(X) \geq f(Y \cup \{u\}) - f(Y). \quad (4)$$

When a submodular function is monotonic (i.e., $f(Y) \leq f(X)$, whenever $Y \subseteq X$) and normalized (i.e., $f(\emptyset) = 0$), greedily constructing a set of size k gives an $(1 - 1/e)$ approximation [15] to the optimal.

Since the definition of our utility in (2) involves a concave function, it is not hard to show that finding the next best row to add (**outer step**) is a submodular maximization problem. Moreover, given the head document, finding the best row (**inner step**) is also a submodular maximization problem. Thus, finding a dynamic ranking to maximize our utility is a *nested* submodular maximization problem. Since submodular function maximization is a hard problem, we can only find an *approximately* good row (rather than the best greedy row) to add in each step. In spite of this, we can show an approximation guarantee for the greedy two-level ranking algorithm. Our result generalizes submodular function maximization from one level to two levels in the same spirit as Hochbaum and Pathria [9] generalize the coverage problem from one level to two levels.

LEMMA 1. *The nested greedy algorithm for the nested submodular optimization problem has a $(1 - \frac{1}{e(1-\frac{1}{2})})$ approximation bound.*

PROOF. The submodular function in question is denoted f . We have f normalized since the utility of the empty ranking is 0. Further, f is monotonic since the score can only increase on adding more rows.

Let S_i be the solution of the method after i iterations of the outer step of the greedy algorithm. Let OPT be the optimal solution to the problem with k elements. First, we define

$$\delta_i = f(S_i) - f(S_{i-1}). \quad (5)$$

By monotonicity of the function f , we have:

$$\forall i, f(OPT) \leq f(S_i \cup OPT). \quad (6)$$

Since at every step we greedily pick the best element, by submodularity we get:

$$f(S_i \cup OPT) \leq f(S_i) + k\delta_{i+1}. \quad (7)$$

The above inequality follows from the fact that adding the elements of OPT to the current solution has no more benefit than k times the benefit achieved by the current best element.

In the problem that we are considering, finding the best element (i.e., the inner step to get S_i from S_{i-1}) is submodular as well. Hence, we are not assured of finding the best element to add to S_{i-1} ; we can only obtain a β -approximate solution (where $\beta = 1 - \frac{1}{e}$). Thus we have $\delta_i = \beta \times \delta_i^{best}$. In this case the inequality (7) becomes:

$$f(S_i \cup OPT) \leq f(S_i) + k\delta_{i+1}/\beta,$$

which along with (6) gives,

$$\delta_{i+1} \geq \beta(f(OPT) - f(S_i))/k.$$

The above inequality in conjunction with (5) implies,

$$\begin{aligned} f(S_{i+1}) &\geq f(S_i) + \beta(f(OPT) - f(S_i))/k \\ &= (1 - \frac{\beta}{k})f(S_i) + \frac{\beta}{k}f(OPT). \end{aligned} \quad (8)$$

Using the above inequality, we can show by induction that $f(S_i) \geq (1 - (1 - \frac{\beta}{k})^i)f(OPT)$. The base case with $i = 1$ can

be easily shown. For the induction step, using the inequality (8) and the induction hypothesis we get:

$$\begin{aligned} f(S_{i+1}) &\geq (1 - \frac{\beta}{k})(1 - (1 - \frac{\beta}{k})^i)f(OPT) + \frac{\beta}{k}f(OPT) \\ &= (1 - (1 - \frac{\beta}{k})^{i+1})f(OPT). \end{aligned}$$

Thus, after k steps, the final solution S satisfies $f(S) \geq (1 - (1 - \frac{\beta}{k})^k)f(OPT)$ which implies $f(S) \geq (1 - e^{-\beta})f(OPT)$. We get the required result by substituting the value of β . \square

6. LEARNING DYNAMIC RANKINGS

In the previous section, we showed that a dynamic ranking can be efficiently computed when all the intents and relevance judgments for a given query are known. Of course, in practice these are not available. In this section, we therefore propose a learning algorithm that can predict dynamic rankings on previously unseen queries. Following the approach of Yue and Joachims [22], our algorithm makes use of word-level features to discriminatively learn a model of the intent distribution for new queries. In particular, given a training set of documents with known intents, our algorithm learns the weight vector of a linear discriminant function. This discriminant function can then be used in Algorithm 1 as a substitute for intents and relevance judgments in order to predict dynamic rankings on unseen queries.

We now describe our learning approach, which is based on structural SVMs [20]. Our goal here is to learn a mapping from a query q to a dynamic ranking Θ . We pose this as the problem of learning a weight vector $\mathbf{w} \in \mathbb{R}^N$ from which we can make a prediction as follows:

$$h_{\mathbf{w}}(q) = \underset{\Theta}{\operatorname{argmax}} \mathbf{w}^\top \Psi(q, \Theta). \quad (9)$$

In the above equation $\Psi(q, \Theta) \in \mathbb{R}^N$ is a joint feature-map between the candidate set of documents $\mathcal{D}(q)$ and Θ given a query q .¹

In the structural SVM framework, given a set of training examples, $(q^i, \Theta^i)_{i=1}^n$, a discriminant function is obtained by minimizing the empirical risk $\frac{1}{n} \sum_{i=1}^n \Delta(\Theta^i, h_{\mathbf{w}}(q^i))$ where Δ is a loss function.

The above equation requires the knowledge of Θ^i in order to compute empirical risk. However, in practice, we are not given Θ^i with the training documents. Assuming that we are given $(q^i, \mathcal{D}(q^i), \mathcal{T}(q^i), \mathbf{P}[t|q] : t \in \mathcal{T}(q^i))_{i=1}^n$, we first compute a dynamic ranking Θ^i that maximizes the utility U_g (approximately) from Algorithm 1. These Θ^i 's will be treated as the training examples in the rest of this section.

A key aspect of structural SVMs is to appropriately define the joint-feature map for the problem at hand. For our problem, the joint-feature map in (9) is defined such that:

$$\mathbf{w}^\top \Psi(q, \Theta) := \sum_{v \in V_{\mathcal{D}(q)}} \mathbf{w}_v^\top \phi_v U_g(\Theta|v) + \sum_{s \in V_{\mathcal{D}(q)} \times \mathcal{D}(q)} \mathbf{w}_s^\top \phi_s(\Theta), \quad (10)$$

where $V_{\mathcal{D}(q)}$ denotes an index set over the words in the candidate set $\mathcal{D}(q)$. The vector ϕ_v denotes word-level features (for example, how often a word occurs in a document) for the word corresponding to index v . The utility $U_g(\Theta|v)$ is analogous to (3) but is now over the words in the vocabulary (rather than over intents). In particular, a document

¹Strictly, the joint feature-map should be $\Psi(\mathcal{D}(q), \Theta|q)$ for a given query q . For brevity, we simply denote this as $\Psi(q, \Theta)$.

provides utility $U(d|v)$ for a word v , if that word occurs in the document. The word-level features are reminiscent of the features used in diverse subset prediction [22]. The key assumption is that the words in a document are correlated with the intent. This seems natural since documents relevant to the same intent are likely to share more words than documents that are relevant to different intents.

The second term in Eq. 10 captures the similarity between head and the tail documents. In this case, $V_{\mathcal{D}(q) \times \mathcal{D}(q)}$ denotes an index set over all document pairs in $\mathcal{D}(q)$. Consider a particular index s that corresponds to documents d_1 and d_2 in $\mathcal{D}(q)$. $\phi_s(\Theta)$ is a feature vector describing the similarity between d_1 and d_2 in Θ when d_1 is a head document in Θ and d_2 occurs in the same row as d_1 . If either d_1 is not a head document in Θ or when d_2 is not in the same row as d_1 , $\phi_s(\Theta)$ is simply a vector of zeros. An example of a feature in $\phi_s(\Theta)$ that captures the similarity between two documents is their TFIDF cosine. In the second term, the diminishing returns property does not hold strictly. However, it is easy to see that this term is modular (*i.e.*, Equation (4) holds with equality) and hence our greedy algorithm and its guarantee still hold even with these similarity features.

From the above definition of the feature-map (10), it is clear that $\mathbf{w}^\top \Psi(q, \Theta)$ models the utility of a given dynamic ranking Θ . Thus, a good discriminant function must give a higher value to rankings with higher utility. This is achieved by solving the following structural SVM optimization problem for \mathbf{w} [20]:

$$\begin{aligned} \min_{\mathbf{w}, \xi \geq 0} \quad & \frac{1}{2} \|\mathbf{w}\|^2 + \frac{C}{n} \sum_{i=1}^n \xi_i & (11) \\ \text{s.t.} \quad & \mathbf{w}^\top \Psi(q^i, \Theta^i) - \mathbf{w}^\top \Psi(q^i, \Theta) \geq \Delta(\Theta^i, \Theta|q^i) - \xi_i, \\ & \forall \Theta \neq \Theta^i, \quad \forall 1 \leq i \leq n. \end{aligned}$$

The constraints in the above formulation ensure that the predicted utility for the dynamic ranking Θ^i is higher than the predicted utility for any other Θ . The objective function in (11) minimizes the empirical risk while maximizing margin. The risk and the margin are traded off by the scalar parameter $C > 0$. The loss between Θ^i and Θ is given by:

$$\Delta(\Theta^i, \Theta|q^i) := 1 - \frac{U_g(\Theta|q^i)}{U_g(\Theta^i|q^i)}.$$

The above definition ensures that the loss is zero when $\Theta = \Theta^i$. It is easy to see that a dynamic ranking Θ has a large loss when its utility is low compared to the utility of Θ^i .

The quadratic program in Eq. 11 is convex and it can be solved efficiently using a cutting-plane algorithm [11, 20]. Even though Eq. (11) has an exponential number of constraints, the cutting-plane algorithm can be shown to always terminate in polynomial time [11, 20]. In each iteration of the cutting-plane algorithm, the most violated constraints in (11) are added to a working set and the resulting quadratic program is solved. Given a current \mathbf{w} , the most violated constraints are obtained by solving:

$$\arg \max_{\Theta} \mathbf{w}^\top \Psi(q^i, \Theta) + \Delta(\Theta^i, \Theta|q^i) \quad (12)$$

It is easy to see that Algorithm 1 can be used to solve this problem, even though the formal approximation guarantee does not hold in this case. While the original structural SVM was proposed for exact inference in Eq. (12), the approach has been shown effective [22, 8] even when only approximate

Statistic	TREC	WEB
No. of queries	17	28
Avg. # of documents per query	46.3	76.1
Avg. # of intents per query	20.8	4.5
Avg. # of docs with > 1 intent per query	9.6	25.6
Frac. of docs with > 1 intent per query	0.21	0.34
Avg. # of intents per document	1.33	1.41
Frac. of docs on prevalent intent	0.376	0.734

Table 2: Key statistics of the two datasets.

inference is possible. Once a weight vector \mathbf{w} is obtained, the dynamic ranking for a test query can be obtained from Eq. (9).

7. EXPERIMENTS

This section explores the properties of our two-level ranking method empirically. In particular, we first investigate how the choice of concave function g impacts diversity and depth. We also compared against several static and dynamic baselines, and finally evaluate how accurately two-level dynamic rankings can be learned using the Structural SVM method.

All experiments were conducted on two datasets, namely, the TREC 6-8 Interactive Track (TREC) and the Diversity Track of TREC 18 using the ClueWeb collection (WEB). Each query in TREC contains between 7 to 56 different manually judged intents. In the case of WEB, we used 28 queries with 4 or more intents. Unless noted otherwise, we consider the probability $\mathbf{P}[t]$ of an intent proportional to the number of documents relevant to that intent. Key statistics describing the two datasets are provided in Table 2. Note that the two datasets differ vastly in terms of some criteria, therefore spanning a wide range of application scenarios. In particular, the most prevalent intent covers 73.4% of all queries in the WEB dataset, while the most prevalent intent in TREC is far less dominating with 37.6%.

Unless noted otherwise, the number of documents in the first-level ranking is set to 5. The width of the second-level rankings is set to 2 (*i.e.* one head document plus 2 second-level results). For simplicity, we chose all factors γ_i and γ_{ij} in Equations (1) and (3) to be 1. Further, we chose $U(d|t) = 1$ if document d was relevant to intent t and set $U(d|t) = 0$ otherwise.

7.1 Controlling Diversity and Depth

The key design choice of our family of utility measures is the concave function g . Since Algorithm 1 directly optimizes utility, we first explore how different choices of g affect various properties of the two-level rankings produced by our method.

We experiment with four different concave functions g , each providing a different diminishing-returns model. At one extreme, we have the identity function $g(x) = x$ which corresponds to modular returns (*i.e.* Eq. (4) holds with equality). Using this function in Eq. (1) leads to the intent-aware Precision measure proposed in [2], and it is the only function considered in [3]. We therefore refer to this function as PREC. It is not hard to show that Algorithm 1 actually computes the optimal two-level ranking for this choice of g . On the other end of the spectrum, we study $g(x) = \min(x, 2)$.

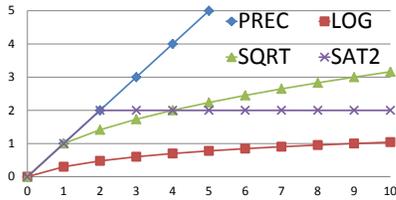


Figure 2: Illustrating the diminishing-returns properties of four concave functions.

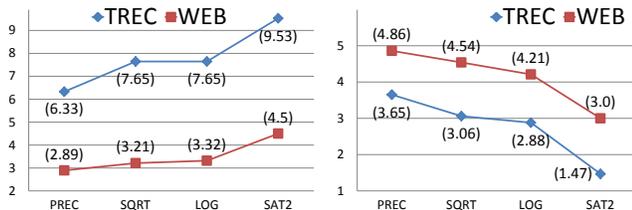


Figure 3: Average number of intents covered (left) and average number of documents for prevalent intent (right) in the first-level ranking.

By remaining constant after two, this function discourages presenting more than two relevant documents for any intent. The measure obtained using this function in Eq. (3) will be referred to as SAT2 (short for “satisfied after two”). In between these two extremes, we study the square root function (SQRT) $g(x) = \sqrt{x}$ and the log function (LOG) $g(x) = \log(1 + x)$. A plot of all four functions is shown in Figure 2.

To explore how dynamic rankings differ for different choices of g , we used Algorithm 1 to compute the two-level rankings (approximately) maximizing the respective measure for known relevance judgments $U(d|t)$ and $\mathbf{P}[t|q]$. Figure 3 shows how g influences diversity. The left-hand plot shows how many different intents are represented in the top 5 results of the first-level ranking on average. The graph shows that the stronger the diminishing-returns model, the more different intents are covered in the first-level ranking. In particular, the number of intents almost doubles on both dataset when moving from PREC to SAT2. In return, the number of documents on the most prevalent intent in the first-level ranking decreases, as shown in the right-hand plot. This illustrates how the choice of g can be used to elegantly control the desired amount of diversity in the first-level ranking.

Tables 3 (TREC) and 4 (WEB) provide further insight into the impact of g , now also including the contributions of the second-level rankings. The rows correspond to different choices for g when evaluating expected utility according to Eq. (3), while the columns show which g the two-level ranking was optimized for. Not surprisingly, the diagonal entries of Tables 3 and 4 show that the best performance for each measure is obtained when optimizing for it. The off-diagonal entries show that different g used during optimization lead to substantially different rankings. This is particularly apparent when optimizing the two extreme performance measures PREC and SAT2; optimizing one invariably leads to rankings that have a low value of the other. In contrast,

Eval.	Optim.	PREC	SQRT	LOG	SAT2
	PREC	0.315	0.302	0.294	0.164
SQRT	1.612	1.664	1.659	1.333	
LOG	1.216	1.267	1.27	1.046	
SAT2	1.18	1.335	1.349	1.487	

Table 3: Performance when optimizing and evaluating using different performance measures for TREC.

Eval.	Optim.	PREC	SQRT	LOG	SAT2
	PREC	0.746	0.731	0.714	0.443
SQRT	3.083	3.132	3.118	2.472	
LOG	2.236	2.297	2.303	1.908	
SAT2	1.773	1.882	1.892	1.984	

Table 4: Same as Table 3 for WEB.

optimizing LOG or SQRT results in much smoother behavior across all measures, and both seem to provide a good compromise between depths (for the prevalent intent) and diversity.

7.2 Static vs. Dynamic Ranking

The ability to simultaneously provide depth and diversity was a key motivation for our dynamic ranking approach over conventional static rankings. We now evaluate whether this goal is indeed achieved. We compare the two-level rankings produced by Algorithm 1 (denoted *Dyn*) with several static baselines.

First, we compare against a diversity-only static ranking, namely the static rankings obtained by maximizing intent coverage using the set-coverage algorithm proposed in [22] (denoted *Stat-Div*). Second, we compare against a depth-only static ranking, namely the static ranking that optimizes utility with g chosen to be the identity function (denoted *Stat-Depth*). Note that Algorithm 1 can be used for this purpose by setting the width of the second-level rankings to 0. And, third, we similarly use Algorithm 1 to produce static rankings that optimize SQRT, LOG, and SAT2 (denoted *Stat-Util*). Note that both *Dyn* and *Stat-Util* optimize the same measure that is used for evaluation.

To make a fair comparison between static and dynamic rankings, we measure performance in the following way. For static rankings, we compute performance using the expectation of Eq. (1) at a depth cutoff of 5. In particular, we measure PREC@5, SQRT@5, LOG@5 and SAT2@5. For two-level rankings, the number of results viewed by a user depends on how many results he or she expands. So, we truncate any user’s path through the two-level ranking after visiting 5 results and compute PREC@5, SQRT@5, LOG@5 and SAT2@5 for the truncated path.

Results of these comparisons are shown in Figures 4 and 5. First, we see that both *Dyn* and *Stat-Util* outperform *Stat-Div*, illustrating that optimizing rankings for the desired evaluation measure leads to much better performance than using a proxy measure as in *Stat-Div*. Note that *Stat-Div* never tries to present more than one result for each intent, which explains the extremely low “depth” performance in terms of PREC@5. But *Stat-Div* is not competitive even for

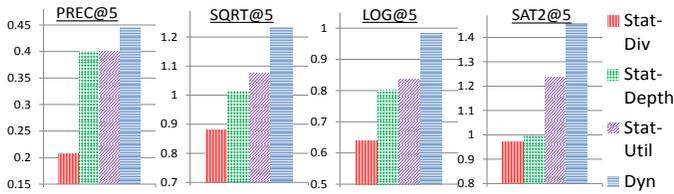


Figure 4: Comparing the retrieval quality of Static vs. Dynamic Rankings for TREC.

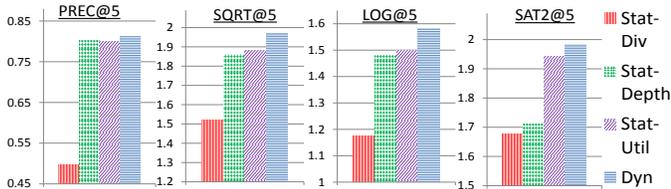


Figure 5: Same as Figure 4 for WEB.

SAT2, since it never tries to provide a second result for the more prevalent intents. Second, at first glance it may be surprising that Dyn outperforms Stat-Depth even on PREC@5, despite the fact that Stat-Depth explicitly (and globally optimally) optimizes depth. As an explanation, consider the following situation where A is the prevalent intent, and there are three documents relevant to A and B and three relevant to A and C. Putting those sets of three documents into the first two rows of the dynamic ranking provides better PREC@5 than sequentially listing them in the optimal static ranking.

Overall, Figures 4 and 5 show that the dynamic ranking method outperform all static ranking schemes on all the metrics – in many cases with a substantial margin. This gain is more pronounced for TREC than for WEB. This can be explained by the fact that WEB queries are less ambiguous, since the single most prevalent intent accounts for more than 70% of all queries on average.

7.3 Width of Second-Level Rankings

In the previous experiments the width of the second-level rankings was limited to 2. To study the effect of width, we varied it from 0 (i.e. single-level, static) to 4. In each case we obtained a dynamic ranking optimized for the respective measure from Algorithm 1. We again use the truncated metrics as defined in Section 7.2 for evaluation. The results are shown in Figure 6. Performance generally increases with increasing width on TREC. However, note that increasing width for SAT2 does not help much beyond width 1, which is to be expected. The improvements from increased width are less strong on WEB, where not much gain is provided beyond width 1. Again, this can be explained by the lower amount of query ambiguity.

7.4 Learning Two-level Ranking Functions

So far we have evaluated how far Algorithm 1 can construct effective two-level rankings if the relevance ratings are known. We now explore how far our learning algorithm can predict two-level rankings for previously unseen queries.

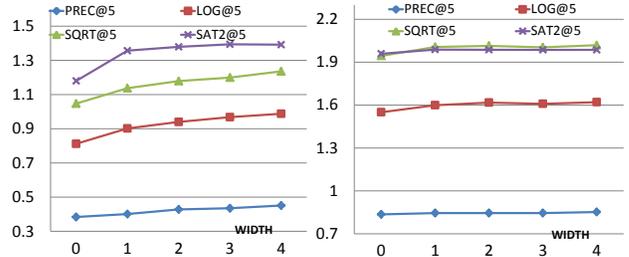


Figure 6: Retrieval performance when the width of the second-level ranking is varied for TREC (left) and WEB (right).

For all experiments in this section, we learn and predict using SQRT as the choice for g , since it provides a good tradeoff between diversity and depth as shown above. We performed cross-validation as follows and report test-set performance averaged over all splits.

For TREC, each test set consisted of a single held-out query. For each remaining set of 16 queries, 4 further splits were made such that 12 were used for training and 4 were used for validation. For WEB, we divided the data into 28 splits of 16 training, 8 validation and 4 testing. Queries were split such that all the queries were equally often in the training, test and validation sets respectively. The C parameter of the structural SVM was varied from 10^{-5} to 10^{-1} . The C value corresponding to the best performance on the validation set was picked for each split.

To compute features, we performed standard preprocessing such as tokenization, stopword removal and Porter stemming. Since the focus of our work is about diversity and not about relevance, we rank only those documents that are relevant to at least one intent of a query. This simulates a candidate set that may have been provided by a conventional retrieval method. This setup is similar to that used by Yue and Joachims [22].

Many of our features in ϕ_v follow those used in [22]. These features provide information about the importance of a word in terms of two different aspects. A first type of feature describes the overall importance of a word. These features capture, for example, the intuition that a word appearing in 10 documents in the candidate set is more important than a word appearing in only one. Examples include:

- Word appears in at least $x\%$ of the documents?
- Word appears in the title of at least $x\%$ of the documents?

A second type of feature describes the importance of a word in a particular document. This captures the intuition that a word that appears 10 times in a document is more important than a word that appears only once. Examples include:

- Word appears with frequency of at least $y\%$ within the document?
- Word that appears in $x\%$ of the documents, appears with frequency of at least $y\%$ within the document?

Finally, we also use features ϕ_s that model the relationship between the documents in the second-level ranking and the corresponding head document of that row. Examples of this type of feature include:

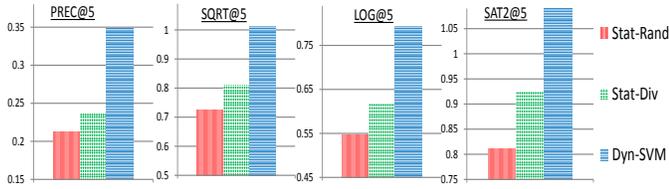


Figure 7: Performance of learned retrieval functions, comparing static vs. dynamic rankings for TREC.

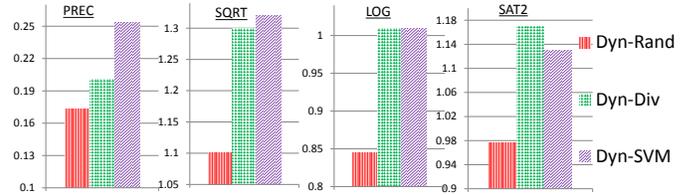


Figure 9: Comparing learned dynamic rankings with heuristic baselines for TREC.

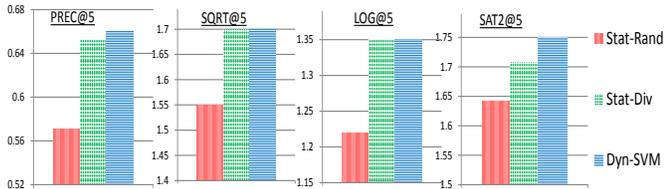


Figure 8: Same as Figure 7 for WEB.

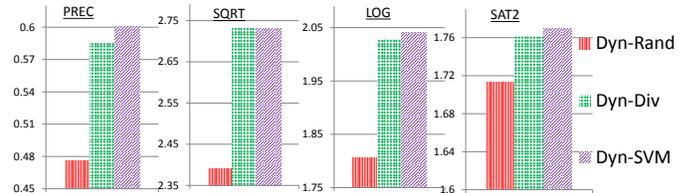


Figure 10: Same as Figure 9 for WEB.

- TFIDF similarity of both documents, binned into multiple binary features,
- Number of common words that appear in both documents with frequency at least $x\%$.

Dynamic vs. Static.

In the first set of experiments, we compare our learning method (*Dyn-SVM*) for two-level rankings with two static baselines. The first static baseline is the learning method from [22] which optimizes diversity, and is henceforth referred to as *Stat-Div*. It is one of the very few learning methods for learning diversified retrieval functions, and it was shown to outperform non-learning methods like Essential Pages [19]. We also consider a *random* static baseline (referred to as *Stat-Rand*), which randomly orders the candidate documents. This is a competent baseline, since all our candidate documents are relevant for at least one intent.

Figure 7 shows the comparison between static and dynamic rankings for TREC. *Dyn-SVM* substantially outperforms both static baselines across all performance metrics, mirroring the results we obtained in Section 7.2 where the relevance judgments were known. This shows that our learning method can effectively generalize the multi-intent relevance judgments to new queries. On the less ambiguous WEB dataset, Figure 8 shows again that the differences between static and dynamic rankings are smaller. While *Dyn-SVM* substantially outperforms *Stat-Rand*, *Stat-Div* is quite competitive on WEB.

Learning vs. Heuristic Baselines.

We also compare against alternative methods for constructing two-level rankings. In particular, we extend the static baselines *Stat-Rand* and *Stat-Div* using the following heuristic. For each result in the static ranking, we add a second-level ranking using the documents with the highest TFIDF similarity from the candidate set. This results in two dynamic baselines, which we call *Dyn-Rand* and *Dyn-Div*.

The results are shown in Figure 9 for TREC and in Figure 10 for WEB. Since we are now comparing two-level

rankings of equal size, we measure performance in terms of expected utility. On both datasets *Dyn-SVM* performs substantially better than *Dyn-Rand*. This implies that our method can effectively learn which documents to place at the top of the first-level ranking. Surprisingly, simply extending the diversified ranking of *Dyn-Div* using the TFIDF heuristic produces dynamic rankings that are competitive with *Dyn-SVM*. In retrospect, this is not too surprising for two reasons. First, our experiments with *Dyn-SVM* use rather simple features to describe the relationship between the head document and the documents in the second-level ranking – most of which are derived from their TFIDF similarity. Stronger features exploiting document ontologies or browsing patterns could easily be incorporated into the feature vector. Second, the learning method of *Dyn-Div* is actually a special case of *Dyn-SVM* when using the SAT1 loss (i.e. users are satisfied after a single relevant document) and second-level rankings of width 0. However, we argue that it is still highly preferable to directly optimize the desired loss function and two-level ranking using *Dyn-SVM*, since the reliance on heuristics may fail on other datasets.

8. CONCLUSIONS AND FUTURE WORK

We proposed a two-level dynamic ranking approach that provides both diversity and depth for ambiguous queries by exploiting user interactivity. In particular, we showed that the approach has the following desirable properties. First, it covers a large family of performance measures, making it easy to select a diminishing returns model for the application setting at hand. Second, we presented an efficient algorithm for constructing two-level rankings that maximizes the given performance measure with provable approximation guarantees. Finally, we provided a structural SVM algorithm for learning two-level ranking functions, showing that it can effectively generalize to new queries.

The idea of dynamic ranking models that allow and actively anticipate user interactions, as well as the proof that such models can be learned and implemented with provable

approximation guarantees, opens a wide range of further questions. First, we need user studies that investigate what types of user-interaction policies π_d are most accurate in practice. While the two-level model used in this paper appears to be more plausible than the infinite-level model of [3], a detailed user study needs to investigate this question. Second, the learning approach presented in this paper requires relevance judgments for each intent of a query. While manually collecting such judgments is feasible in commercial settings like Web search engines, it would be desirable to have algorithms that can learn such models from implicit feedback in settings with resource constraints. Finally, there is a whole range of additional information that could be incorporated into the model. For example, a taxonomy (of words or documents) is likely to provide valuable features for modeling the dependencies between head document and the results in the second-level ranking.

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