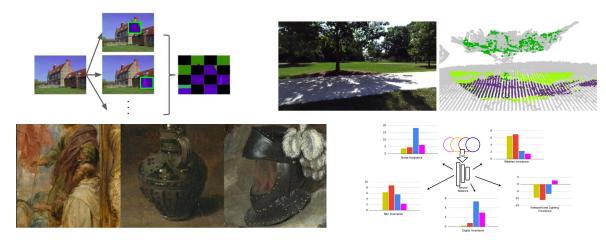
# Towards Robust Perception Systems in Real World Environments

Feb 10, 2022 B Exam

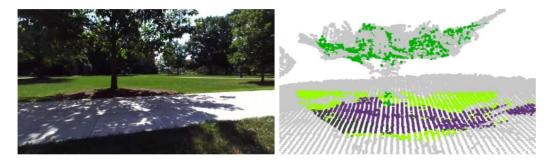
Hubert Lin



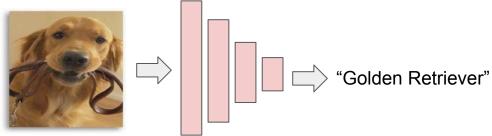
# Introduction

Visual perception systems useful for many applications:

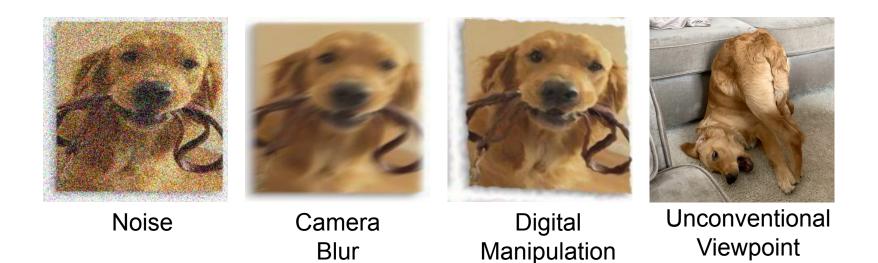
- Robotics
- Self-driving
- Visual discovery
- Medical diagnostics

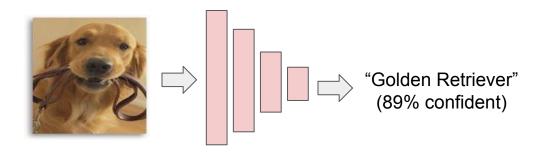


Many modern systems are based on neural networks.

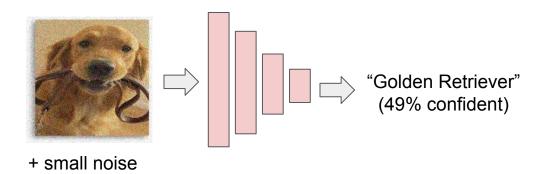


#### Many imperfections in real world images...

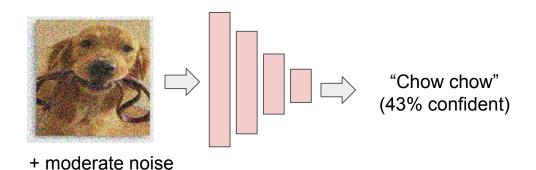




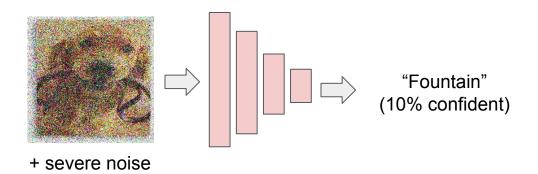
- Humans can recognize objects<sup>1</sup> and materials<sup>2</sup> in non-ideal images.
- Neural networks may struggle to perform well.



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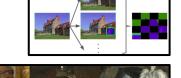
- Humans can recognize objects<sup>1</sup> and materials<sup>2</sup> in non-ideal images.
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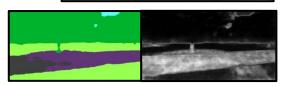
# Talk Outline

Many challenges in improving perception systems in real world.

- 1. Better annotation tools. [ICCV 2019]
- 2. Learning robust visual invariances. [PLOS One 2021, FAPER ICPR 2020, CVPR 2021]
- **3. Reasoning about perception uncertainties.** [ICRA 2020]
- 4. Summary.

This talk will primarily focus on (2).

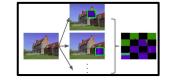




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Many challenges in improving perception systems in real world.

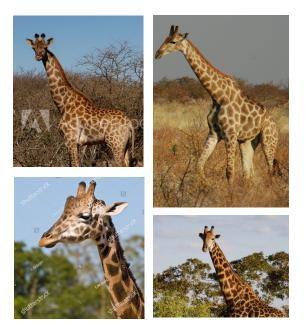
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#### Illustrative example

Giraffes face left in training set.

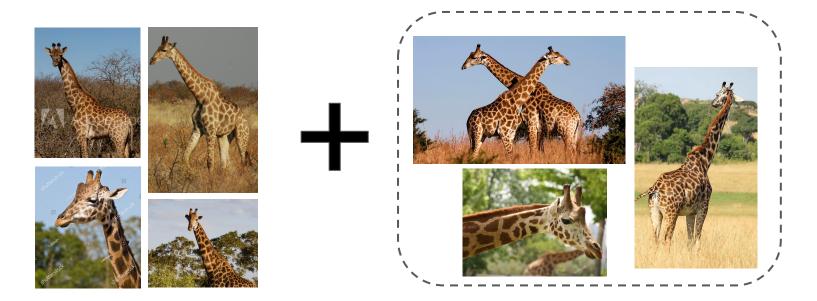


#### Unseen image: Giraffe?



#### Need more data

One possible solution: train with more images of giraffes in different poses.



**Original Dataset** 

New Labeled Data

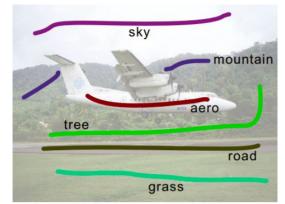
#### What kind of labels are useful?



#### Cheap but coarse alternatives



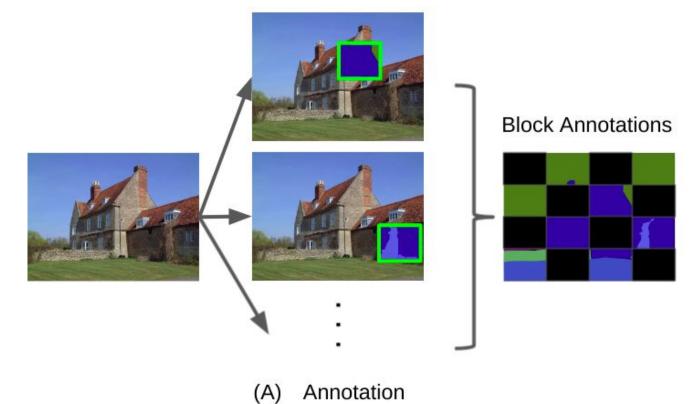






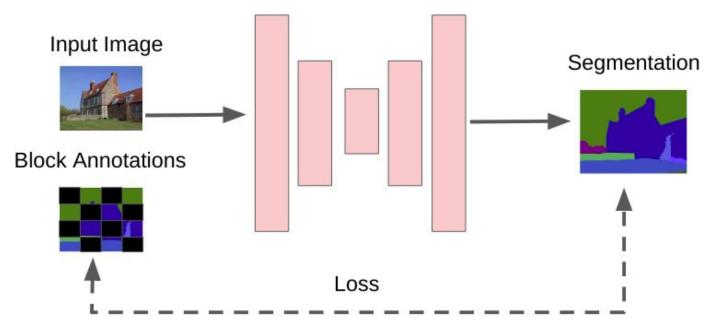


# **Block Annotation**



Lin, Upchurch, Bala, "Block Annotation: Better Image Annotation with Sub-Image Decomposition", ICCV 2019

# **Block Annotation**

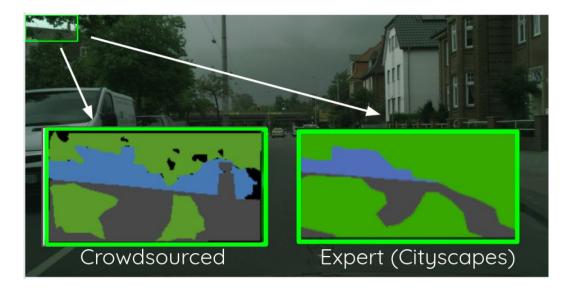


(B) Segmentation

Lin, Upchurch, Bala, "Block Annotation: Better Image Annotation with Sub-Image Decomposition", ICCV 2019

# Key Findings: Block Annotation

Crowdworkers produce high quality annotations, and more cheaply than conventional methods.



# Key Findings: Block Annotation

High performing semantic segmentation models learned – up to 97% of full supervision performance with 1/10th annotation time.

Cityscapes mIOU (%)	Ours: Block (7 min) 72.1	Coarse (7 min [14]) 68.8	Full Supervision (90 min [14]) 77.7	
Pascal mIOU (%)	Ours: Block (25 sec) 67.2	Scribbles (25 sec [36]) 63.1 [36]	Full Supervision (4 min [41]) 69.6	

# Talk Outline

Many challenges in improving perception systems in real world.

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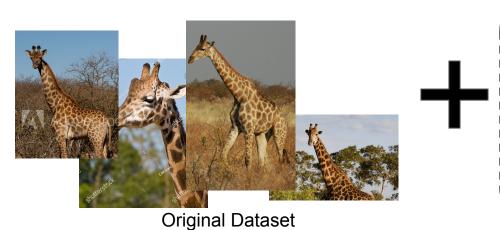


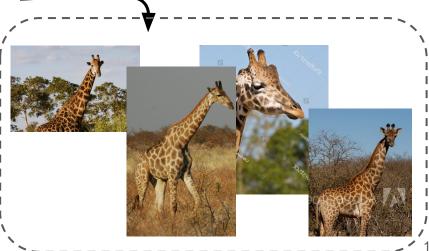


#### What about synthetic data?

Problem: Giraffes always face towards left in original dataset.

- Label more data expensive.
- Alternative? Synthetically create images+labels by applying a left-right reflection to the existing set of images.





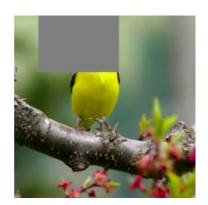
# **Data Augmentation**

Image transformations encourage the network to ignore some signals in the data.

• Reflected image pairs: Network will not rely on left-right orientation when classifying an animal.



#### **Data Augmentation**



CutOut



AutoAugment



RandAugment



AugMix



DeepAugment

Common goals: (a) Preserve semantics. (b) Manipulate non-robust features.



Artwork: Fra Carnevale "The Birth of the Virgin" 1467





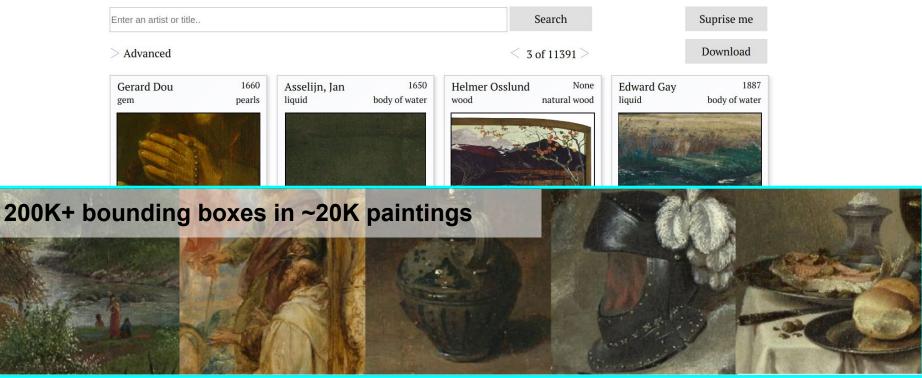
Cavanagh 2005, "The Artist as Neuroscientist"

# Paintings as Implicit Data Augmentation

Artworks implicitly encode human visual invariances by omitting or altering unimportant details for perception.



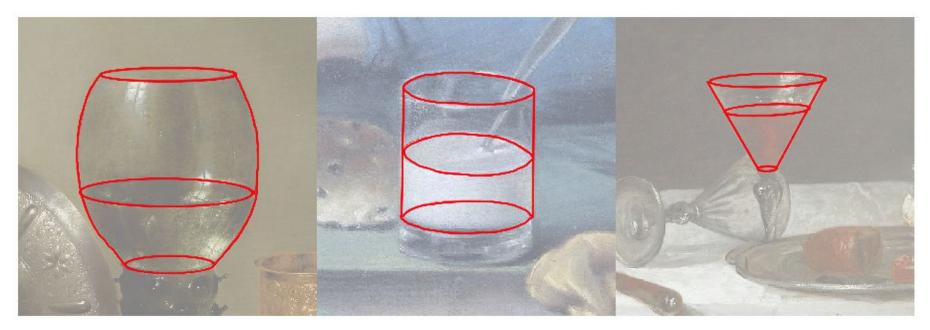
# Materials In Paintings <a href="https://materialsinpaintings.tudelft.nl/">https://materialsinpaintings.tudelft.nl/</a>



Van Zuijlen, <u>Lin</u>, Bala, Pont, Wijntjes, *"Materials In Paintings (MIP): An interdisciplinary dataset for perception, art history, and computer vision"*, *PLOS One 2021* 

# **Painterly Biases**

Highlights on glass cups:

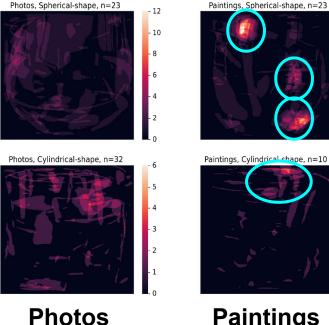


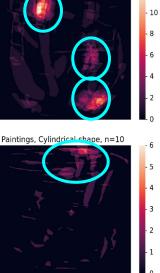
# How are glass highlights depicted?

1. Paintings have more localized highlights.

- 2. Painting highlights are less ambiguous.
  - 50% higher agreement (recall) • between participants.

#### **Highlight Heatmaps**





- 12

#### **Paintings**

#### Learning From Painterly Biases

Local cues like highlights on the satin or silk fabrics are emphasized by artists.

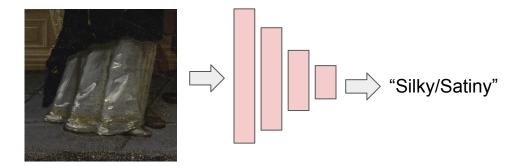


Lin, Van Zuijlen, Wijntjes, Pont, Bala, "Insights from a Large-Scale Database of Material Depictions in Paintings", FAPER ICPR 2020

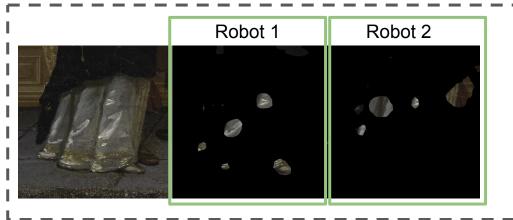
#### Learning From Painterly Biases

Compare models trained on paintings or photos to distinguish satin from cotton.

• Assess human preference for cues used by models.



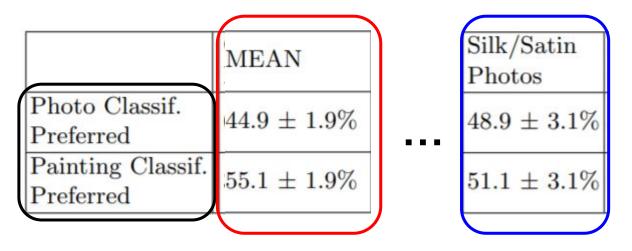
## Human Preferences for Cues



"Two different robots think these regions I in the image look like silk/satin. Which robot do you agree with more?"

Humans are shown cues used by each classifier and prompted to select which set of cues they prefer.

#### Which cues do humans prefer?



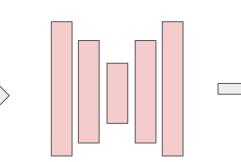
- Overall, cues learned from paintings preferred over cues learned from photos.
- For photos of silk/satin, cues learned from paintings equally preferred to cues learned from photos despite domain shifts.

# "Fake" Paintings via Style Transfer

Style transfer: methods for creating painting-like images from photos



Giraffe photo





Giraffe in the style of a Monet painting

## "Fake" Paintings via Style Transfer

Style transfer: methods for creating painting-like images from photos

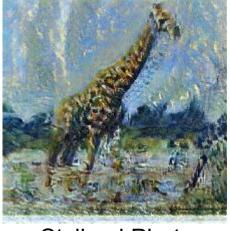


# Learning from Paintings vs Stylized Images

Do models learn similar behaviors from paintings and stylized images?

• Does style transfer allow us to replace paintings altogether?





Stylized Photo

Lin, Van Zuijlen, Wijntjes, Pont, Bala, **"What Can Style Transfer and** *Paintings Do For Model Robustness?"*, CVPR 2021

# Datasets

Materials:

- Photographs of materials from existing datasets (MINC<sup>1</sup>, COCO<sup>2</sup>)
- Paintings of materials from Materials in Paintings (MIP<sup>3</sup>)



Objects:

• Existing dataset of photos, paintings, cartoons, and sketches (PACS<sup>4</sup>).



# **Evaluating Model Behavior**

Interested in model behavior in real-world settings with imperfect images.

• High accuracy on these images = model is more "robust"



Noise

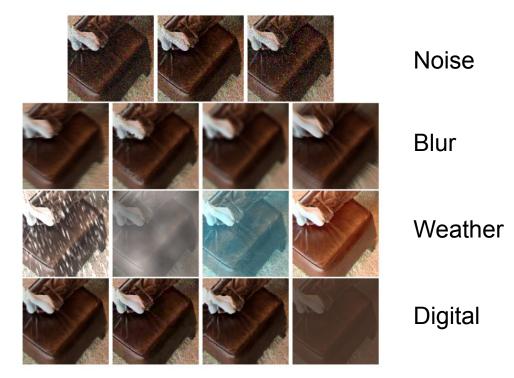
Camera Blur

Digital Manipulation

Unconventional Viewpoint

#### **Evaluating Model Behavior**

Accuracy with respect to common image corruptions:



## **Evaluating Model Behavior**

Accuracy with respect to out-of-distribution photos (different viewpoint, lighting):

 $Materials \rightarrow FMD^1$ 



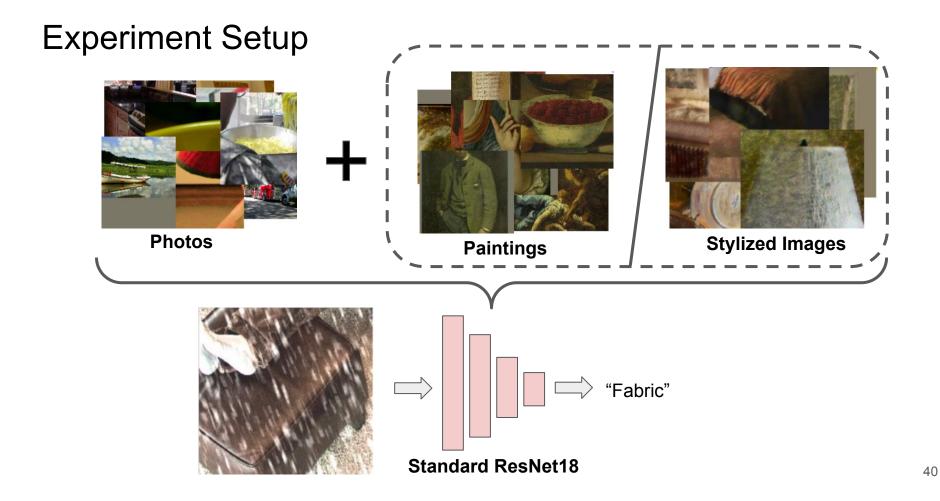
 $PACS \rightarrow Subset \ of \ YFCC100 M^2$ 



1. Does learning from paintings improve model robustness?

2. Does learning from stylized images improve model robustness?

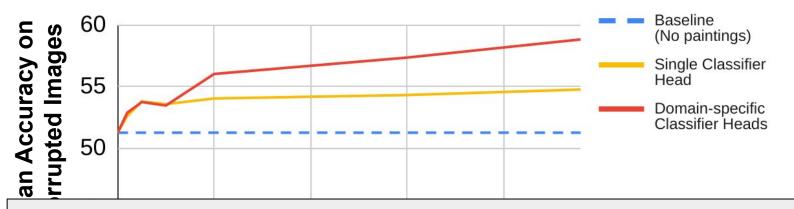
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1. Does learning from paintings improve model robustness?

# Does learning from paintings improve robustness?

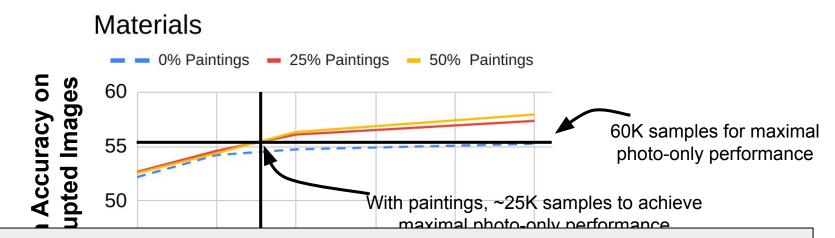
Materials



Learning from paintings improves model robustness to image corruptions, even without accounting for domain shift.

(plus 10000 Photos)

Is learning from paintings data-efficient?



Learning from paintings is sample-efficient, so it is good to use a mix of photos and paintings even with a fixed budget.

10000 20000 30000 40000 50000 60000 Total Data Samples

#### Can sketches and cartoons work too?

Other artwork like sketches and cartoons are also perceptually meaningful.



#### Can sketches and cartoons work too?

Training Data (# Samples)	Mean Corruption Acc (%)	
Materia	als	
Photo (30K)	54.73±0.25	
Photo + Painting (15K + 15K)	<b>56.31</b> ±0.27 (+)	
PACS	5	
Photo (1500)	76.16±0.34	
Photo + <b>Painting</b> (750 + 750)	<b>79.41</b> ±0.55 (+)	
Photo + Cartoon (750 + 750)	75.38±0.36 (-)	
Photo + Sketch (750 + 750)	73.85±0.39 (-)	
DomainNe	t [28]	
Photo (120K)	36.59±0.12	
Photo + Painting (90K + 30K)	<b>39.00</b> ±0.14 (+)	
Photo + Sketch $(90K + 30K)$	$37.57 \pm 0.22 (+)$	

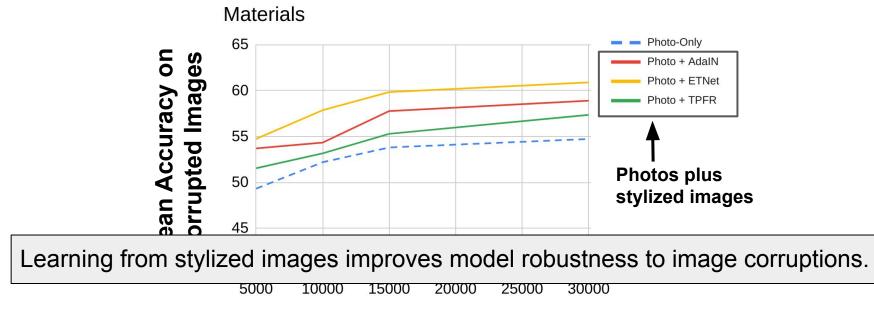
Paintings are uniquely useful due to their balance of realism and abstraction.

Photo + Intograph (90K + 50K)	34.00±0.18 (-)
VisDA [2	9]
Photo (30K)	<b>65.97</b> ±0.33
Photo + Rendering (15K + 15K)	$63.90 \pm 0.21 (-)$

- 1. Does learning from paintings improve model robustness?
  - YES improves robustness to image corruptions.
  - Cost-effective compared to only photos.
  - More abstract art forms do not enable such improvements.

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- 2. Does learning from stylized images improve model robustness?

# Does learning from stylized images improve robustness?



**Data Samples** 

AdalN: Huang and Belongie 2017; ETNet: Song et al 2019; TPFR: Svoboda et al 2020

# Do stylized images need painting styles?

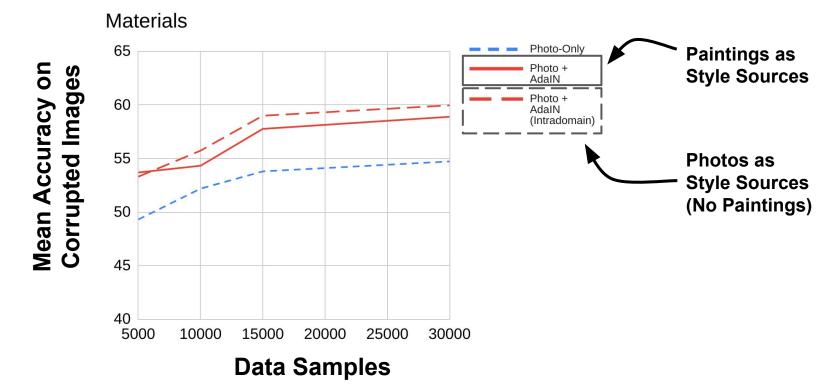
Arbitrary style transfer applies style from a source image to a target image.

• Do style source images need to be paintings?

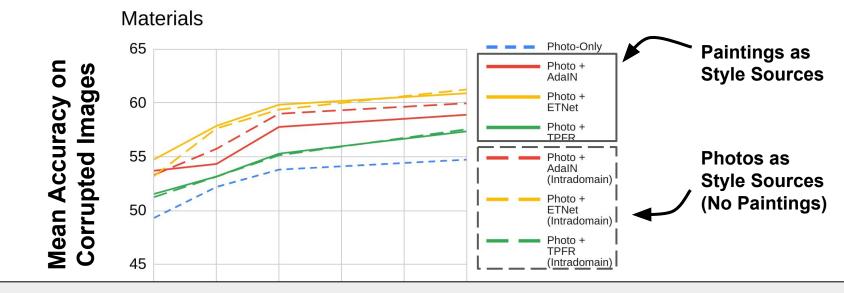


TOP: Photos stylized by paintings. BOTTOM: Photos stylized by photos.

#### Do stylized images need painting styles?



# Do stylized images need painting styles?



Stylization does not necessarily rely on painting style sources to improve robustness.

2000 10000 12000 20000 22000 30000

**Data Samples** 

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# How do paintings and stylized images differ?

Method	Accuracy (Image Corruptions)	Accuracy (OOD Photos)
	Materials (30K samples / do	omain)
Photos-only	54.73±0.25	41.33±0.62
Photos + Stylization		
Photos + Paintings		
PACS (1.5K samples / domain)		
Photos-only	76.16±0.34	82.57±0.00
Photos + Stylization		
Photos + Paintings		

# How do paintings and stylized images differ?

Method	Accuracy (Image Corruptions)	Accuracy (OOD Photos)
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Photos-only	54.73±0.25	41.33±0.62
Photos + Stylization	62.67±0.03	34.54±0.91 🕂
Photos + Paintings		
	PACS (1.5K samples / domain)	
tylization improves rob	ustness to image corruption	ns, but hurts view generalizat

Photos + Stylization	87.27±0.10	77.43±0.84 📕
Photos + Paintings		

# Stylized images have diverse textures, but same background contexts and views



**Diverse textures:** helps against image corruptions **Same background and views:** hurts against new views

# How do paintings and stylized images differ?

Ρ

Method	Accuracy (Image Corruptions)	Accuracy (OOD Photos)		
Materials (30K samples / domain)				
Photos-only	54.73±0.25	41.33±0.62		
Photos + Stylization	<b>62.67</b> ±0.03	34.54±0.91		
Photos + Paintings	57.92±0.09 🕇	<b>43.92</b> ±0.47		
	PACS (1.5K samples / dor	nain)		
aintings improve robustness to both image corruptions and novel views.				
Photos + Stylization	<b>87.27</b> ±0.10	77.43±0.84		
Photos + Paintings	79.65±0.49	<b>85.43</b> ±0.70		

# Paintings have diverse textures, and have ambiguous backgrounds



Diverse textures: helps against image corruptions. Ambiguous background, focused on foreground: helps against new views.

# Why stylization > paintings against image noise?

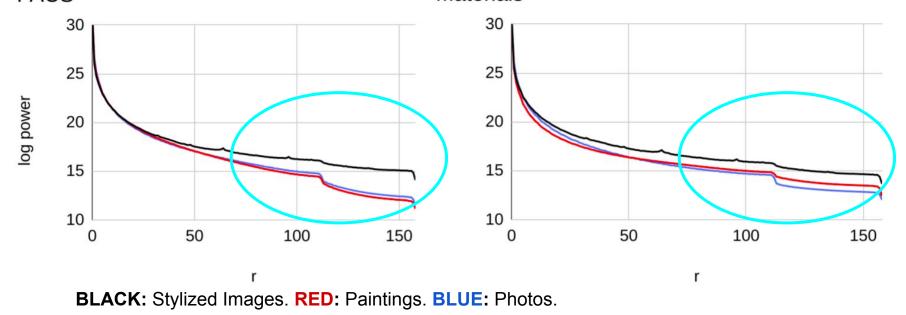
Similar textures, but stylization much better.

Reasons?

- 1. Corrupted images are share similar background and views to training; model uses these features.
- 2. Invisible high frequency textures?

Method	Accuracy (Image Corruptions)	
Materials (3	OK samples / domain)	
Photos-only	54.73±0.25	
Photos + Stylization	<b>62.67</b> ±0.03	
Photos + Paintings	57.92±0.09	
PACS (1.5K samples / domain)		
Photos-only	76.16±0.34	
Photos + Stylization	<b>87.27</b> ±0.10	
Photos + Paintings	79.65±0.49 59	

#### Why stylization > paintings against image noise? Image Power Spectrum Materials



Stylized images contain larger magnitude high frequency components.

## Why stylization > paintings against image noise?

**Original Image** 

Low Frequency Only



# Why stylization > paintings against image noise?

	Method	Accuracy (Images with Noise)	
		Materials (30K samples / domain)	PACS (1.5K samples / domain)
Γ	Photos + Stylization	61.87±0.16	85.98±0.56
	Photos + Stylization (Low Freq. Images)	45.82±1.36 🖊 🦊	77.55±2.60 📕 📕
$\left[ \right]$	Photos + Paintings	49.82±0.56	68.83±0.83
	Photos + Paintings (Low Freq. Images)	44.95±0.66 🖊	71.16±1.31

Stylized images contain imperceptible high-frequency signals that greatly improve noise robustness.

# Are paintings and stylized images complementary?

Method	Accuracy (Image Corruptions + OOD Photos)	
	Materials (30K samples / domain)	PACS (1.5K samples / domain)
Photos-only	48.03±0.21	79.37±0.17
Photos + Stylization		
Photos + Paintings		
Photos + Stylization + Paintings		

# Are paintings and stylized images complementary?

Method	Accuracy (Image Corruptions + OOD Photos)	
	Materials (30K samples / domain)	PACS (1.5K samples / domain)
Photos-only	48.03±0.21	79.37±0.17
Photos + Stylization	48.56±0.45 🔶	82.35±0.37
Photos + Paintings	50.92±0.22 🔶	82.54±0.59
Photos + Stylization + Paintings		

# Are paintings and stylized images complementary?

Method	Accuracy (Image Corruptions + OOD Photos)	
	Materials (30K samples / domain)	PACS (1.5K samples / domain)
Photos-only	48.03±0.21	79.37±0.17
Photos + Stylization	48.56±0.45 🔶	82.35±0.37 🔶
Photos + Paintings	50.92±0.22 🔶	82.54±0.59 🛖
Photos + Stylization + Paintings	<b>51.49</b> ±0.69	85.42±0.18

Models learn complementary invariances from paintings and stylization.

- 1. Does learning from stylized images improve model robustness?
  - YES improves robustness to image corruptions.
  - Does not necessarily require painting styles.
- 2. Does learning from paintings improve model robustness?
  - YES improves robustness to image corruptions.
  - Cost-effective compared to only photos.
  - More abstract art forms do not enable such improvements.
- 3. How do models trained on paintings differ from models trained on stylized images?
  - Stylized images greatly improve robustness to corruptions, but hurts generalization to new views. Paintings improve robustness to both.
  - Stylized images contain imperceptible noise that improve robustness.

# Key Findings: Learning Robust Invariances from Paintings

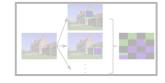
How can paintings help our perception models?

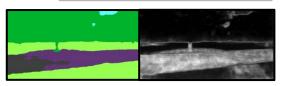
- 1. Artists emphasize cues like highlights to help viewers understand scenes.
- 2. Models trained on paintings may learn to use more interpretable cues.
- 3. Models trained on paintings are robust to image corruptions and novel views.
- 4. "Fake" paintings produced by style transfer greatly strengthen model robustness to noise while harming generalization to novel views.
- 5. Learning from both paintings and stylized images allow models to learn useful complementary invariances that boost robustness overall.

## Talk Outline

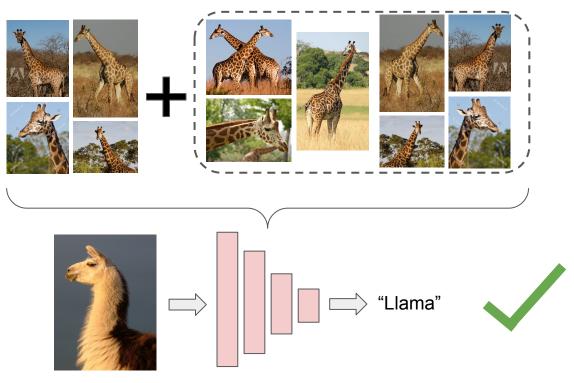
Many challenges in improving perception systems in real world.

- 1. Better annotation tools. [ICCV 2019]
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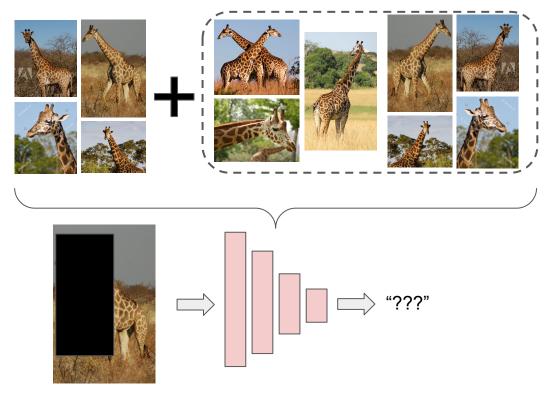




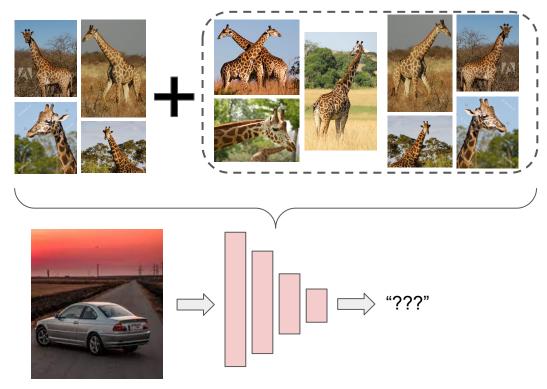
## Toy problem: Solved?



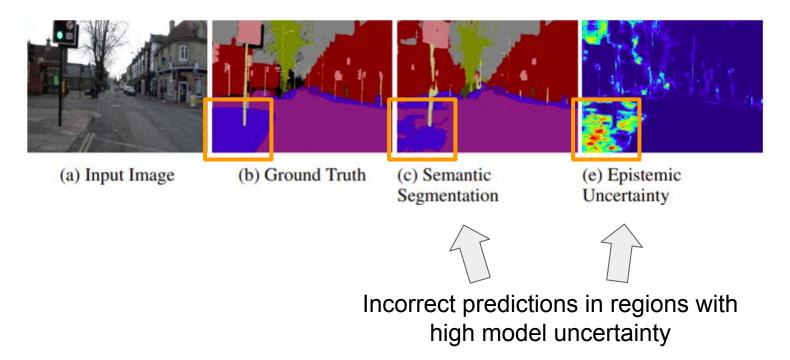
## Toy problem: Solved?



# Toy problem: Solved?



# Modeling Uncertainty

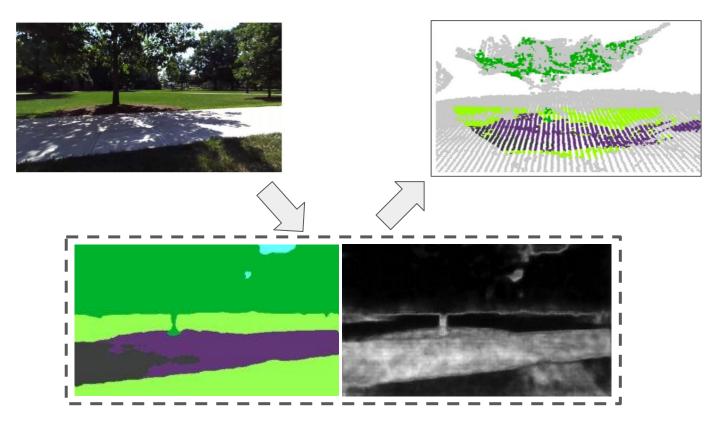


### **Unstructured Real World Navigation**

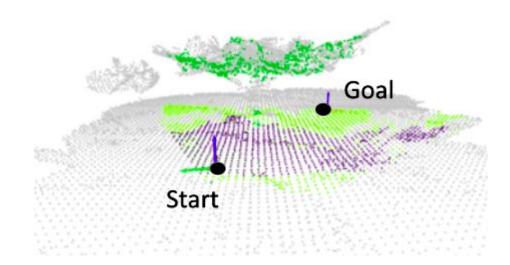


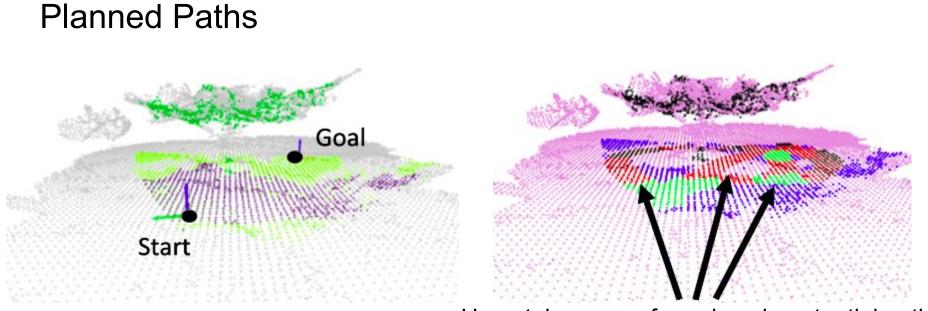
Han\*, <u>Lin\*</u>, Banfi\*, Bala, Campbell, **"DeepSemanticHPPC:** *Hypothesis-based Planning over Uncertain Semantic Point Clouds"*, *ICRA 2020* 

### **Environment Semantics and Uncertainty**



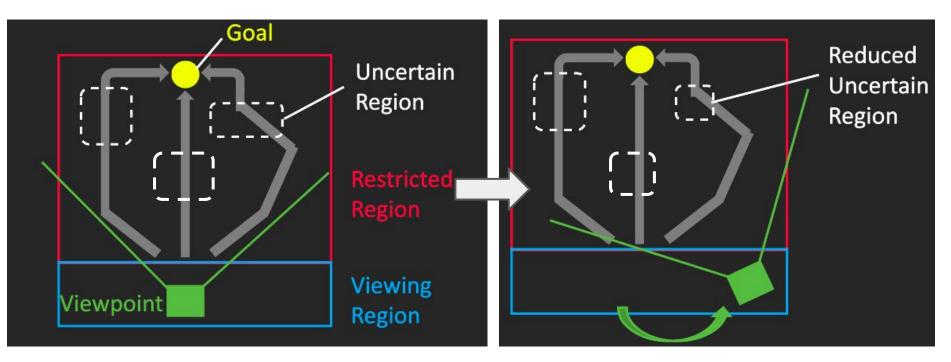
# Navigation Task





#### Uncertain or unsafe regions in potential paths

# **Uncertainty Reduction**



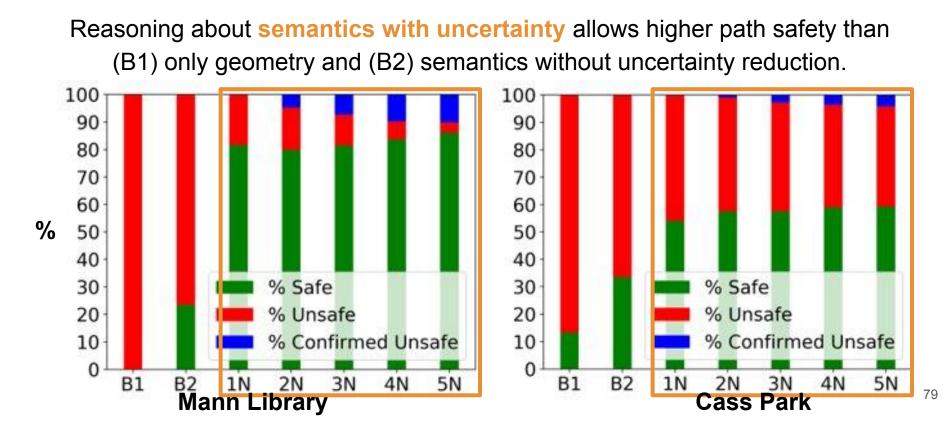
#### **Current Measurement**

Updated Measurement

### **Real World Test Environments**



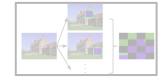
# Key Findings: Unstructured Real World Navigation



# Talk Outline

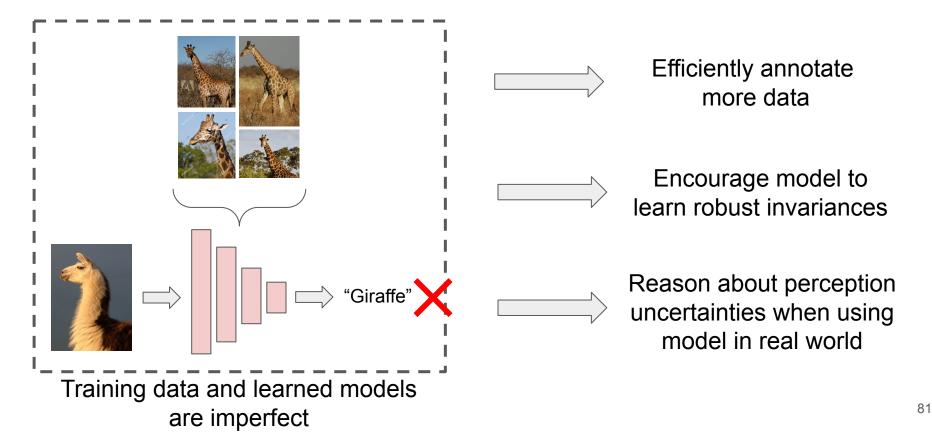
Many challenges in improving perception systems in real world.

- 1. Better annotation tools. [ICCV 2019]
- 2. Learning robust visual invariances. [PLOS One 2021, FAPER ICPR 2020, CVPR 2021]
- **3. Reasoning about perception uncertainties.** [ICRA 2020]
- 4. Summary.





# Summary



# **Future Directions**

- 1. Better annotation tools.
  - a. Which images or image regions to label?
- 2. Learning robust invariances from paintings.
  - a. Improved style transfer algorithms.
  - b. Implications for synthetic data in computer vision physical realism goal?
  - c. Better methods for learning from paintings domain generalization methods fail.
- 3. Reasoning about perception failures.
  - a. Combine with online adaptation and continual learning.

### Acknowledgements

#### Research group and collaborators from 2016 to 2022





Wijntjes

Mitchell Van Zuijlen Sylvia Pont



Yutao Jacopo Ma Han Banfi Cam

Mark Campbell

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Thank you!

#### References

Lin, Upchurch, Bala, "Block Annotation: Better Image Annotation with Sub-Image Decomposition", ICCV 2019

Van Zuijlen, <u>Lin</u>, Bala, Pont, Wijntjes, *"Materials In Paintings (MIP): An interdisciplinary dataset for perception, art history, and computer vision"*, *PLOS One 2021* 

Lin, Van Zuijlen, Wijntjes, Pont, Bala, "Insights from a Large-Scale Database of Material Depictions in Paintings", FAPER ICPR 2020

Lin, Van Zuijlen, Wijntjes, Pont, Bala, "What Can Style Transfer and Paintings Do For Model Robustness?", CVPR 2021

Han\*, Lin\*, Banfi\*, Bala, Campbell, *"DeepSemanticHPPC: Hypothesis-based Planning over Uncertain Semantic Point Clouds"*, ICRA 2020

# Removed + Backup Slides...

# Painting + Style Robustness Backup Slides

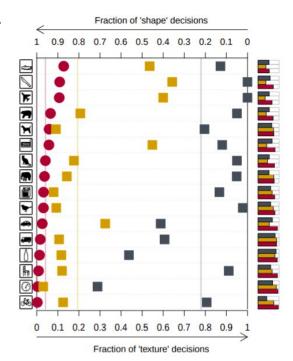
# Learning from Stylized Images

What do models learn from stylized images ("fake" paintings)?

• More shape-based decisions, similar to humans.

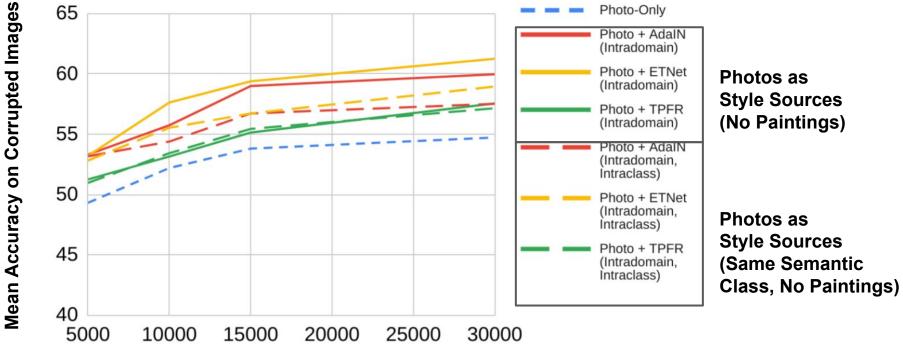


Cat shape with elephant texture



# Style Semantic Diversity vs Robustness

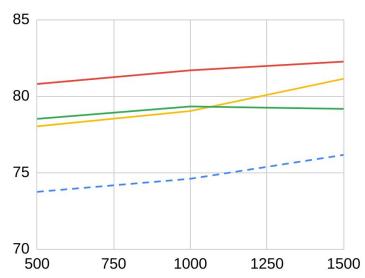
Materials



Data Samples

### Style Strength vs Robustness

PACS



Method	Painting	Intradomain	Intradomain (Intraclass)
AdaIN	$1.58 \pm 0.93$	$1.28 \pm 0.79$	$1.16 \pm 0.85$
ETNet	$2.33 \pm 1.09$	$2.13 \pm 1.04$	$1.81 \pm 1.03$
TPFR	$1.52 \pm 0.90$	$1.38 {\pm} 0.87$	$1.27 \pm 0.91$

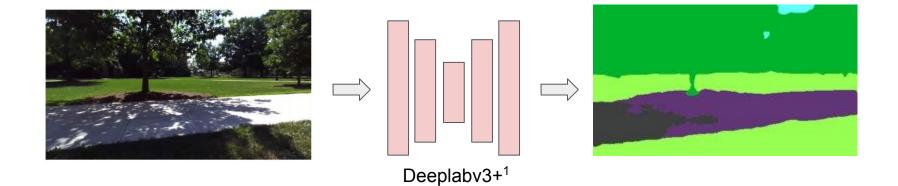
Table 6: Style (Gram Matrix) Distance. Gram matrices computed from ImageNet pretrained ResNet18 features on PACS. Mean distance between (image, stylized image) pairs is reported.  $\uparrow$  distance implies  $\uparrow$  style difference.  $\pm$  denotes standard deviation across 1.5K pairs.

# Stylization vs Paintings: Per-Corruption Accuracy

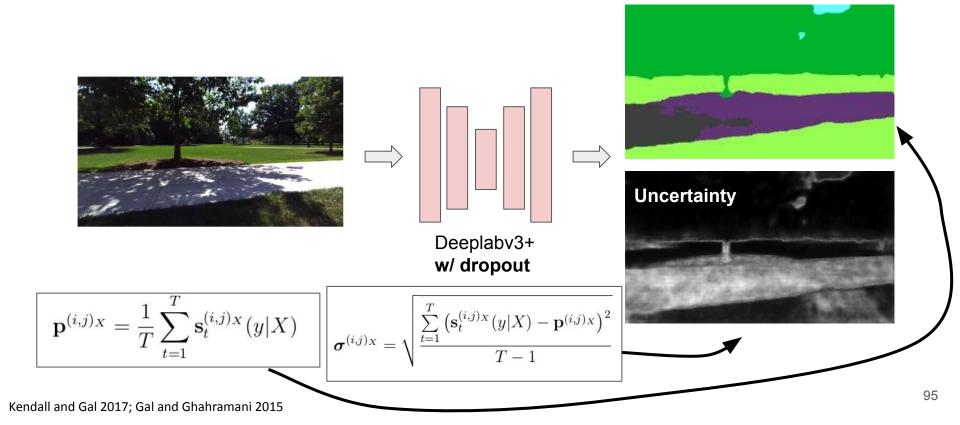
Method	Noise	Blur	Weather	Digital			
Materials (30K Samples/Domain)							
Photo-Only	43.70±0.65	58.76±0.14	$55.25 \pm 0.33$	61.20±0.69			
Photo + SACL	61.87±0.16	64.36±0.20	57.49±0.24	66.55±0.17			
Photo + Painting	<b>49.82</b> ±0.56	$61.03 \pm 0.13$	$56.69 \pm 0.10$	64.15±0.14			
Photo+SACL (LF)	45.82±1.36	64.24±0.39	57.06±0.13	66.37±0.29			
Photo+Painting (LF)	44.95±0.66	60.87±0.29	$56.82 \pm 0.23$	63.69±0.46			
PACS (1.5K Samples/Domain)							
Photo-Only	62.64±1.48	$72.75 \pm 0.04$	83.24±0.22	86.33±0.14			
Photo + SACL	85.98±0.56	84.61±0.15	89.73±0.33	88.74±0.48			
Photo + Painting	68.83±0.83	$75.80 {\pm} 0.95$	$86.88 \pm 0.66$	87.07±0.14			
Photo+SACL (LF)	$77.55 \pm 2.60$	85.4±0.11	88.93±0.22	88.53±0.15			
Photo+Painting (LF)	71.16±1.31	$75.97 \pm 0.71$	$86.82 \pm 0.37$	87.35±0.36			

# Path Planning Backup Slides

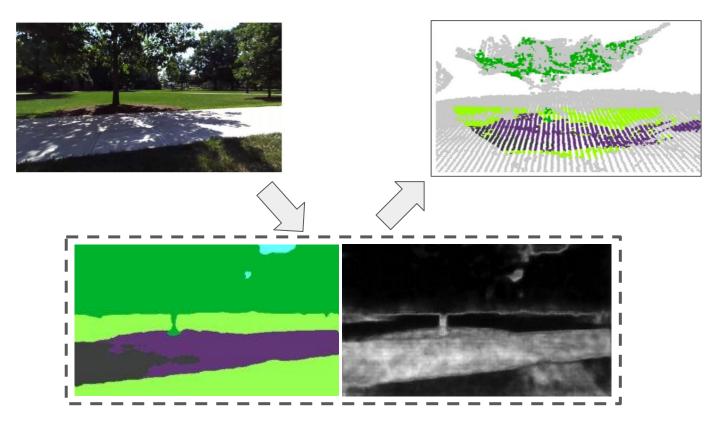
### Semantic Segmentation with Uncertainty



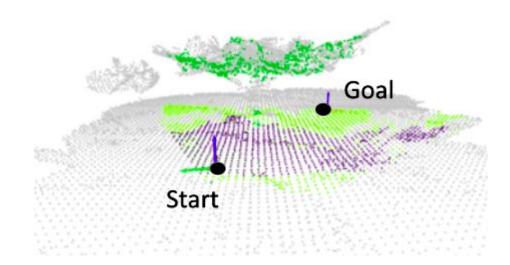
#### Semantic Segmentation with Uncertainty



### **Environment Semantics and Uncertainty**



# Navigation Task



# Multihypothesis Path Planner

#### Planner:

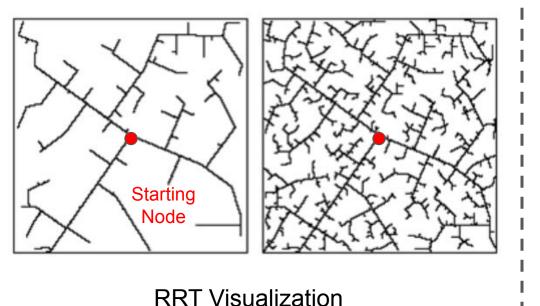
- RRT with sampling biased towards goal.
- Multiple paths: remove large regions after planning, and re-plan a new path.

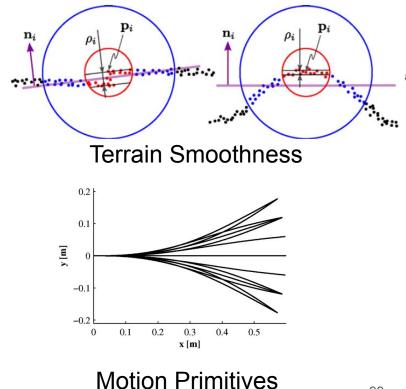
#### Feasibility constraints: [Krusi et al 2017]

- Contact with the terrain surface.
- Static traversability (bounded roll and pitch angles).
- Kinematic constraints (motion primitives + bounded continuous curvature).

### **Multihypothesis Path Planner**

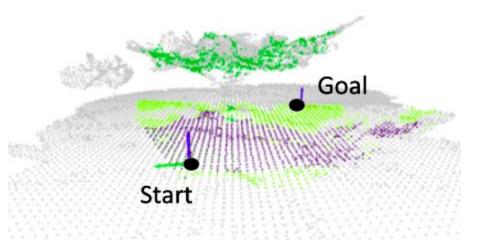
Basic RRT:

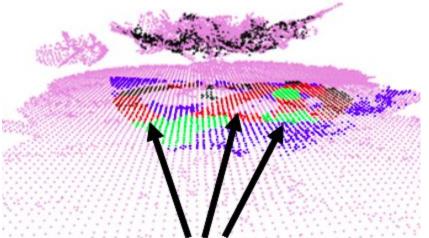




LaValle 1998 (Figure: Wikipedia); Krusi et al 2017

### **Planned Paths**





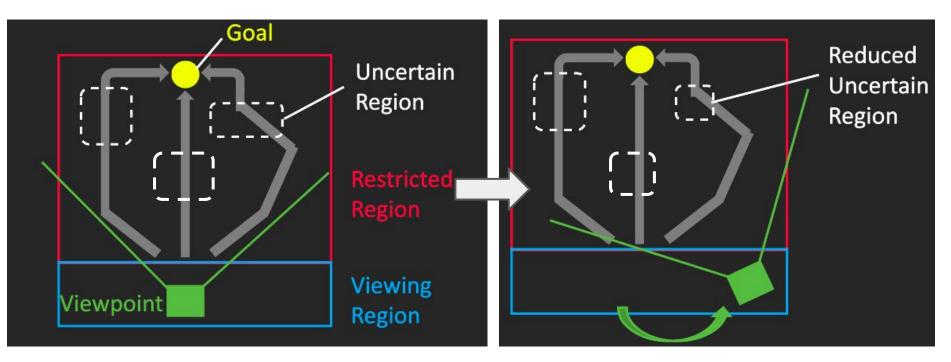
Uncertain or unsafe regions in potential paths

safe if 
$$p_S^i - w_\sigma \sigma^i \ge \theta_s$$
;

$${unsafe if } p_U^i - w_\sigma \sigma^i \ge \theta_u;$$

#### unclear otherwise.

# **Uncertainty Reduction**



#### **Current Measurement**

Updated Measurement

# **Uncertainty Reduction**

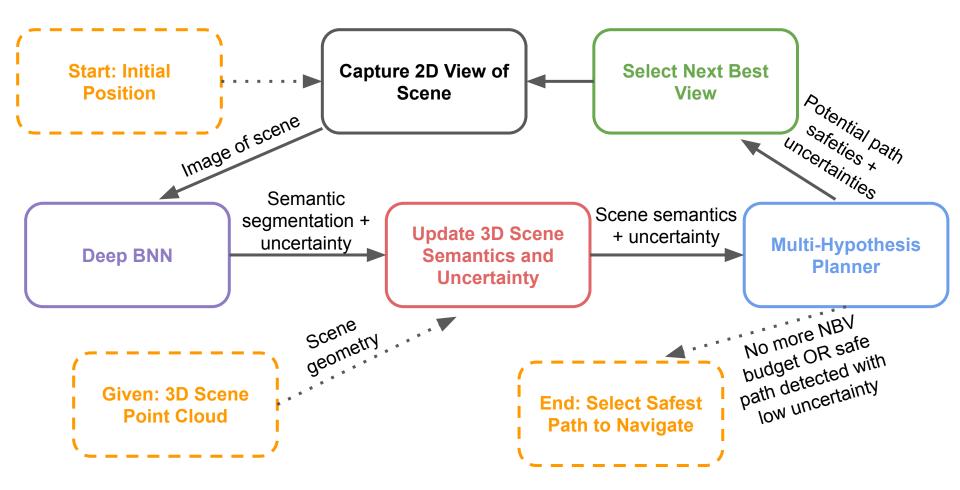
How to select new viewpoint?

#### Camera pose heuristics:

- - Distance to visible path nodes.
- <u>Viewing angle</u> (vs initial viewing orientation).
- <u>Number of path nodes</u> seen from view.

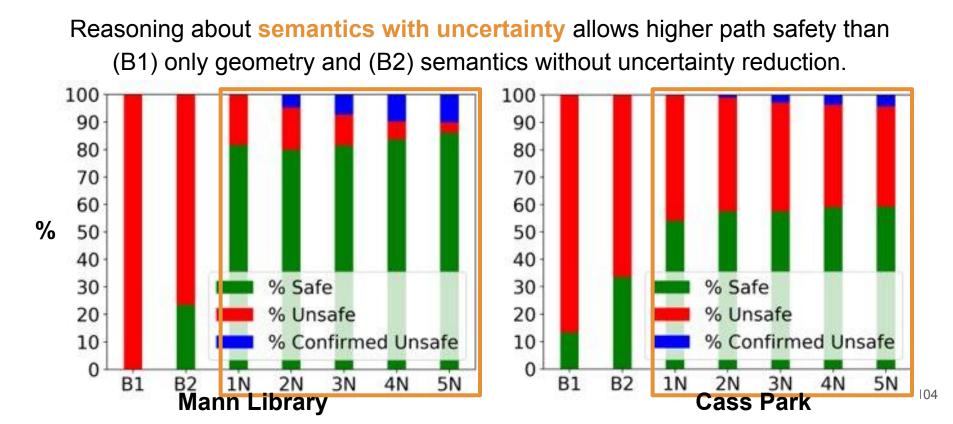
#### **Uncertainty reduction heuristics:**

- <u>Pixel coverage</u> visible path nodes projected onto view.



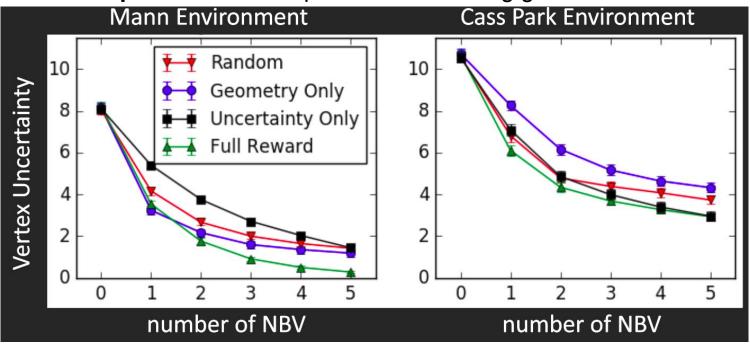
Han\*, <u>Lin\*</u>, Banfi\*, Bala, Campbell, **"DeepSemanticHPPC:** *Hypothesis-based Planning over Uncertain Semantic Point Clouds"*, *ICRA 2020* 

## Key Findings: Unstructured Real World Navigation



# Key Findings: Unstructured Real World Navigation

Accounting for viewing angle+distance (geometry) and uncertainty of viewable path nodes is important for selecting good measurements.



#### **Uncertainty Reduction**



Projected / Estimated View (given point cloud) True Captured View of Environment

$$p_S^i = \sum_{j \in S} p_j^i, \, p_U^i = 1 - p_S^i = \sum_{j \in U} p_j^i$$
$$\sigma^i = \min(\sqrt{\sum_{j \in S} \sigma_j^{i2}}, \sqrt{\sum_{j \in U} \sigma_j^{i2}})$$