Network Games

Éva Tardos [*]
Department of Computer Science
Cornell University
Ithaca, NY 14853
eva@cs.cornell.edu

ABSTRACT

Network games approach some of the traditional algorithmic questions in networks from the perspective of game theory, which gives rise of a wide range of interesting issues. In this talk we will give an overview of recent progress in many of these areas, and show strong ties to certain algorithmic techniques.

Network games model the interaction between parts of information and computer systems controlled by different parties. This perspective suggests approaching some of the traditional algorithmic questions in networks as games: each participant in an algorithm is viewed as a player in a noncooperative game, where each player acts to selfishly optimize his or her own objective function. Information and computer systems involve the interaction of multiple participants, such as servers, routers, etc., each controlled by different parties. The future of much of the complex technology we are developing today, depends on our ability to ensure that participants cooperate despite their diverse goals and interests. In this talk we will survey a few recent developments and a few open problems.

The question that has maybe received the most attention in recent years is quantifying the loss of efficiency due to selfishness. Here we will mostly focus on Nash equilibria, solutions in a network game that are stable, in the sense that no user wants to "defect". Selfishness often leads to bad outcomes, as is well known by the classical Prisoner's Dilemma. Papadimitriou [27] introduced the term *price of anarchy* for the ratio of worst Nash equilibrium and the best optimal design. There has been considerable progress on questions arising from this perspective for many problems including routing [29, 31], load balancing [4, 6, 25, 30, 32], and facility location [33, 8], network design [10]. In [2] we suggest the term *price of stability* [2] for the ratio of best equilibrium and the best optimal design. The best Nash equilibrium is an interesting middle ground between total anarchy and centralized design. The best Nash is the best solution that is stable despite the selfish interest of the participants. Work in price of stability has focused on network design [1, 2] and routing [5, 2]. Quantifying efficiency loss is the game theory analog of approximation algorithms, and the best response process of finding a Nash equilibrium is analogous to local search. Techniques from these areas of algorithms have played key roles in the above developments.

There are many important issues in networks like the Internet, where the effect of selfishness is much less well understood. For an example, consider the routing between network providers on the Internet [21], or more generally consider the process by which independent network providers build and use the Internet. For many simple and natural models the price of anarchy/stability is very high, but it may be possible to realistic or assumptions that allow an improved bound. For example, the price of anarchy in routing game of [31] for general delay functions can be very bad. However, such bad behaviors does not seem to happen impractical environments [28]. There a few ways to prove better bounds: [31] shows that a good bicriteria bound is possible, while [14] provides an improved analysis for most flow rates.

Studying Nash equilibria appears to assume implicitly that natural game pay, such as letting each player greedily update his solution (best response), reaches such an equilibrium. While there are some nice positive results, there is also strong evidence that in network games simple learning strategies do not converge [15, 16]. However, we can also think of equilibria as stable design, rather than outcomes of natural play dynamics. This leads us to the question of price of stability, and also whether equilibria can be efficiently found algorithmically. Papadimitriou, in his influential STOC 2001 survey of algorithmic game theory [27] called this one of the most fundamental computational problems whose complexity is wide open. There has been progress in the area, mostly focused on market equilibria [7, 9, 22], but also including other problems, such as routing [11]. While equilibria are not directly optimization problems, the techniques used to find them in polynomial time are often borrowed from optimization. For example, primal-dual algorithms play a key role.

The most developed area of algorithmic game theory is

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mechanism design, mostly developed in the context of auctions, and cost-sharing games. In the talk we will focus on the network games such as routing [26, 3, 13, 17], and cost-sharing games for multicast [12, 18], and network design [19, 24]. Generally speaking, the goals of mechanism design is to design simple games (mechanisms or protocols) such that selfish users interacting with this game will result in a good quality solution. Traditional mechanism design focuses on auction type problems and uses social welfare as the measure of solution quality. However, from the above broader perspective, we also consider the network design game of [2], the bandwidth allocation of [23, 20], and the cost-sharing of [12, 19, 24] also as mechanisms. From this perspective, mechanism design ties back to the price of anarchy/stability. When the price of anarchy is high, one should consider the question if there is a simple variant of the game, a simple change in the mechanism, that allows an improved analysis.

1. REFERENCES

[1] E. Anshelevich, A. Dasgupta, E. Tardos, T. Wexler. Near-Optimal Network Design with Selfish Agents. Proc. 35th ACM Symposium on Theory of Computing, 2003.

[2] E. Anshelevich, A. Dasgupta, J. Kleinberg, E. Tardos, T. Wexler, and T. Roughgarden. Price of Stability for Network Design with Fair Allocation of Costs. Unpublished manuscript, 2004.

[3] A. Archer and É. Tardos. Frugal Path mechanisms. in *SODA* 2002.

[4] A. Czumaj, P. Krysta, and B. Vöcking. Selfish Traffic Allocation for Server Farms. In *STOC*, 287–296, 2002.

[5] J. R. Correa, A. S. Schulz, and N. E. Stier Moses. Selfish Routing in Capacitated Networks. To appear in *Mathematics of Operations Research*, 2004.

[6] A. Czumaj and B. Vöcking. Tight bounds for worst-case equilibria. In *SODA*, 413–420, 2002.

[7] X. Deng, C. Papadimitriou and S. Safra. On the Complexity of Equilibria. In *STOC* 2002.

[8] N. Devanur, N. Garg, R. Khandekar, V. Pandit, A. Saberi, and V. Vazirani. Price of anarchy, locality gap, and a network service provider game. Unpublished manuscript, 2004.

[9] N. Devanur, C. H. Papadimitriou, A. Saberi and V. V. Vazirani. Market equilibrium via a primal-dual-type algorithm. In *FOCS* 2002.

[10] A. Fabrikant, A. Luthra, E. Maneva, S. Papadimitriou, and S. Shenker. On a Network Creation Game. In *PODC*, 2003.

[11] A. Fabrikant, C. Papadimitriou, K. Talwar. The complexity of pure Nash equilibria. To appear in *STOC*, 2004.

[12] J. Feigenbaum, C. Papadimitriou, S. Shenker. Sharing the Cost of Multicast Transmissions. Journal of Computer and System Sciences 63 (2001), pp. 21-41.

[13] J. Feigenbaum, C. Papadimitriou, R. Sami, S. Shenker. A BCG-based Mechanism for Lowest-Cost Routing. In *PODC* 2002.

[14] E. Friedman. Genericity and Congestion Control in Selfish Routing. Unpublished manuscript, 2003.

[15] E. Friedman, A. Greenwald and S. Shenker. Learning in Network Context: Experimental Results from Simulations. *Games and Economic Behavior*, 35:80-123, 2001.

[16] E. Friedman, M. Shor, S. Shenker, and B. Sopher. Experiment on Learning with Limited Information: Non-convergence, Experimentational Cascades, and the Advantage of Being Slow. To appear in *Games and Economic Behavior*.

[17] J. Hershberger, and S. Suri. Vickerey Prices and Shortest paths: What is an edge worth? In *FOCS*, 2001.

[18] Shai Herzog, Scott Shenker, Deborah Estrin. Sharing the "Cost" of Multicast Trees: An Axiomatic Analysis. IEEE/ACM Transactions on Networking, Dec. 1997.

[19] K. Jain and V. Vazirani. Applications of Approximation Algorithms to Cooperative Games. In *STOC*, 2001.

[20] R. Johari and J. Tsitsiklis. Efficiency loss in a network resource allocation game. Mathematics of Operations Research, to appear.

[21] R. Johari and J. Tsitsiklis. Routing and Peering in a Competitive Internet. Unpublished manuscript, 2004.

[22] S. Kapoor and R. Garg. Auction Algorithms for Market Equilibrium. To appear in *STOC*, 2004.

[23] F. Kelly. Charging and rate control for elastic traffic. European Transactions on Telecommunications, volume 8 (1997) pages 33-37.

[24] K. Kent and D. Skorin-Kapov. Population monoton cost allocation on mst's. In Operations Research Proceedings KOI, pages 43-48, 1996.

[25] E. Koutsoupias and C. Papadimitriou. Worst-case equilibria. In *STACS*, 404–413, 1999.

[26] N. Nisan and A. Ronen. Algorithmic Mechanism Design In *STOC* 1999.

[27] C. Papadimitriou. Algorithms, Games, and the Internet. In *STOC*, 749–753, 2001.

[28] L. Qui, Y. R. Yang, Y. Zhang, S. Shenker. On Selfish Routing in Interner-Like Environments. In *SIGCOMM*, 2003.

[29] T. Roughgarden. The price of anarchy is independent of the network topology. To appear in *Journal of Computer and System Sciences*, 2004.

[30] T. Roughgarden. Stackelberg scheduling strategies. In *STOC*, pages 104–113, 2001.

[31] T. Roughgarden and É. Tardos. How bad is selfish routing? *Journal of the ACM* 2002.

[32] S. Suri, C. Toth, Y. Zhou. Selfish Load Balancing and Atomic Congestion Games. To appear in *SPAA*, 2004.

[33] A. Vetta. Nash equilibria in competitive societies with applications to facility location, traffic routing and auctions. In *FOCS*, 2002.