Recursive Serial and Parallel QR Factorization

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> ABSTRACT

The QR Factorization is a computational routine widely used to solve everything from least square to eigenvalue problems. It is provided in the ubiquitous Linear Algebra Package (LAPACK) via an iterative algorithm. Prior work indicates recursive algorithms lead to better performance. We present both serial and parallel implementations of the Recursive GEneral QR Factorization (rgeqrf).

> METHODOLOGY

Our code is written in C with calls to LAPACK Fortran Routines. The standard QR Factorization (dgeqrf) divides a matrix into blocks. The left-most block is factored, and then the rest of blocks are updated with a matrix multiply (gemm). This is repeated by shifting the leftmost block right and continuing the same process.

On the other hand, the recursive rgeqrf performs recursive block merges instead of iterative block updates. Recursive block merges recursively halve a block and combine halves when both are factored. Practically speaking, the recursive method uses more BLAS-3 operations than the iterative one, leading to quicker runtimes.

Parallelization is accomplished through synchronous openmp tasking. A matrix is divided into an rgeqrf (of size ib) portion to factor and gemm portions (of size nb) to update, similar to the original dgeqrf. Load balancing threads can be done either with heuristics or flop balancing.

> CONCLUSIONS AND FUTURE WORK

On tall skinny matrices, rgeqrf performs up to 30% faster than dgeqrf. On square matrices, rgeqrf has roughly comparable performance to dgeqrf. We also benchmarked plasma_dgeqrf, another of algorithm using block-tiling, which rgeqrf beats as well. We perform a rgeqrf scaling test – scaling is quite good; this is due to the fact that parallelization of rgeqrf is straightforward. To improve scaling, future work includes writing an asynchronous version to remove sync time between threads.

> WORK DIVISION AMONG THREADS

Serial Comparison

Parallel Comparison

Scaling Test

Parallel rgeqrf Execution Trace