Background Information
The Largest Machines in the World
Some Terms

- **Bus**: A node in the network, often correlated with some sort of substation or distribution center.
- **Loads on buses**: Power Consumption/Demand.
- **Branch/Line**: A physical line connecting two buses (transmission level).
- **Generator**: Generates power to meet demand.
- **Contingency**: Failure element e.g. Downed Line.
AC Power

- Voltage: \( V(t) = |V| \cos(\omega t) \)
- Current: \( I(t) = |I| \cos(\omega t + \phi) \)
- Power: \( V(t)I(t) \)
AC Power

Waveforms

Phase relations

Vector representations

(of A' waveform with reference to B' waveform)

Phase shift = 0 degrees
A and B waveforms are in perfect step with each other

Phase shift = 90 degrees
A is ahead of B (A 'leads' B)

Phase shift = 90 degrees
B is ahead of A (B 'leads' A)

Phase shift = 180 degrees
A and B waveforms are mirror-images of each other
Phasor Measurement Unit (PMU)

- Delivers GPS-Synchronized Sensor Readings
- Measures Voltage Magnitudes and Angles 10-30 readings/second of bus and buses neighbors
- Placed only at certain buses, so one cannot see the entire network
A power system is given by a system of differential algebraic equations (DAE)

$$\Omega = \begin{cases} 
    x' = f(x, y) \\
    0 = g(x, y)
\end{cases}$$

With initial conditions $x(t_0) = x_0$ and $y(t_0) = y_0$
General Differential Equation Theory says near steady state, linearization is accurate

\[ E z' = J z \]

Where \( E, J, \) and \( z \) are defined as

\[
\begin{align*}
  z &= \begin{bmatrix} x \\ y \end{bmatrix} \\
  J &= \begin{bmatrix} F_x & F_y \\ G_x & G_y \end{bmatrix} \\
  E &= \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}
\end{align*}
\]
Recall the solution to the linear ODE $x' = Ax$ is given by

$$x(t) = \sum_{i=1}^{k} c_i e^{\lambda_i t} u_i$$

Where $\langle \lambda_i, u_i \rangle$ are the eigenpairs of the eigenvalue problem $Au_i = \lambda_i u_i$
ODE theory somewhat generalizes to DAE theory; the solution to the linear DAE $Ez' = \mathcal{J}z$ is given by

$$z(t) = \sum_{i=1}^{k} c_i e^{\mu_i t} v_i$$

Where $\langle \mu_i, v_i \rangle$ are the non-infinite eigenpairs of the generalized eigenvalue problem $\mathcal{J}v_i = \mu_i Ev_i$
So we have a closed form for the solution of the DAE. The PMUs ”pick out” a subset of $z(t)$ associated with the voltages and angles of the buses with PMUs on them. So the signal we see is

$$f(t) = Hz(t) = \sum_{i=1}^{k} c_i e^{\mu_i t} Hv_i$$

Where $I$ is the identity matrix and $H \subset I$
What does sensor data look like?

![Graph showing sensor data]
Euler’s Formula

Why does the sensor data look like waves? Don’t confuse the sensor readings with $V(t)$ and $I(t)$ — the sensors don’t measure these precise quantities!

\[ e^{it} = \cos(t) + i\sin(t) \]

\[ e^{\mu t} = e^{\text{real}(\mu)t} [\cos(\text{imag}(\mu)t) + i\sin(\text{imag}(\mu)t)] \]
We know the exact solution to our power grids model and therefore what exactly our sensors should be outputting. So how can we use this information to actually diagnose a contingency?
Assuming we have an existing system model $\Omega$, a contingency will represent a shift in system models $\Omega \rightarrow \Omega'$ to a new system model $\Omega'$. This change in systems will lead to what is known as a ”transient response” as the system moves from one steady state ($\Omega$) to another ($\Omega'$).
Then given the mathematical formulation in the previous section, we may boil down the problem of contingency identification into the following: given a PMU signal $f(t)$ and a dictionary of contingency models

$$D = \{\Omega_1, \Omega_2, \ldots, \Omega_m\}$$

determine which contingency model $\Omega_c$ “best” explains $f(t)$. 
Solution: A Model Driven Approach

- First, fit damped sinusoids to signal $f(t)$ to get form that we want. Since $f(t)$ gives us a subset of eigenvectors, we need to fit the rest.
- Fit eigenvector against each contingency model; we expect the correct contingency to give the best fit.
Fitting Method

\[
\min_{\alpha, q_j} \left\| \left( E \lambda_j - J_i \right) \left( \begin{array}{c} \alpha p_j \\ q_j \end{array} \right) \right\|^2_2
\]

s.t.

\[
\left\| \left( \begin{array}{c} \alpha p_j \\ q_j \end{array} \right) \right\|^2_2 = 1
\]

\[
\min_{\alpha, q_j} \left\| \left( E \lambda_j - J_i \right) \left( \begin{array}{cc} p_j & 0 \\ 0 & l \end{array} \right) \left( \begin{array}{c} \alpha \\ q_j \end{array} \right) \right\|^2_2
\]

s.t.

\[
\left\| \left( \begin{array}{c} \alpha \\ q_j \end{array} \right) \right\|^2_2 = 1
\]

\[
G_{ij} = \left( E \lambda_j - J_i \right) \left( \begin{array}{cc} p_j & 0 \\ 0 & l \end{array} \right) \quad z_j = \left( \begin{array}{c} \alpha \\ q_j \end{array} \right)
\]
Fitting Method

Point is, we end up solving, for each Contingency and each eigenvector,

\[
\min_b \|Mb\|_2^2 \quad \text{s.t.} \quad \|b\|_2^2 = 1
\]

Standard Linear Algebra results tell us that the solution given by \( \lambda^s \), the smallest eigenvalue of \( MTM \). The contingency in our dictionary \( D \) generating the smallest minima is the correct contingency.
Fitting Method

Identifying Contingencies in Power Grids

- $Hv_i$
- Fit w.r.t. Contig 1
- Fit w.r.t. Contig 2
- Fit w.r.t. Contig 3
- Min 1
- Min 2
- Min 3
Results
The power systems community isn’t very exhaustive when it comes to testing. An earlier paper by D. Bindel and C. Ponce reaches about 75 percent accuracy w/ limited sensing
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Our Results

**Figure**: Testing on the IEEE 39 Bus system, we first placed a single PMU on Bus 16. We consider 40+ line failures and reach almost 100 percent accuracy, with one misdiagnosis.
What Went Wrong?

Figure: Left: Misdiagnosis, Right: True Line Failure
Figure: Contingency Scores, sorted by increasing order with the correct diagnosis highlighted in red
Conclusions
Our method is not very resilient to noise in the signal; fitting damped exponentials to a signal is an inherently ill-conditioned problem.

Scalability: we only consider one possible failure at a time. What about K simultaneous failures? Scales exponentially in K

Model dependent; modeling inaccuracies will lead to contingency misidentifications
Conclusions: Ongoing/Future Work

- Moving up to larger systems; 39 bus system is quite small.
- Filtering Method to decrease cost (roughly a logarithmic number of eigenvector fittings as compared to the original) with provably identical results
- Noise. Noise. Noise. Making this algorithm robust to noise is incredibly important (can we actually do this?)
Thank You!