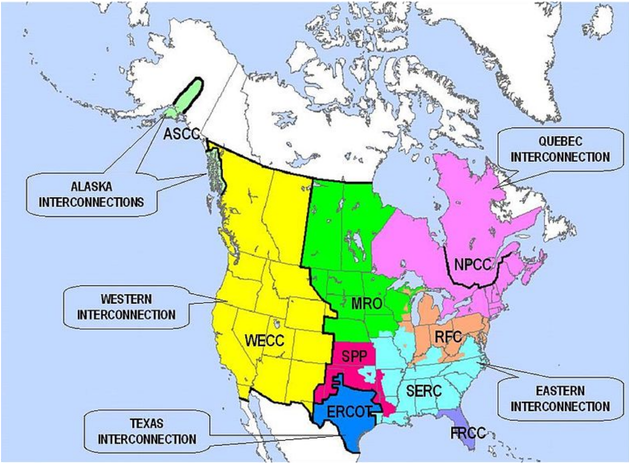
Identifying Contingencies in Power Grids

Student Brown Bag 2/22/2017

Background Information

The Largest Machines in the World



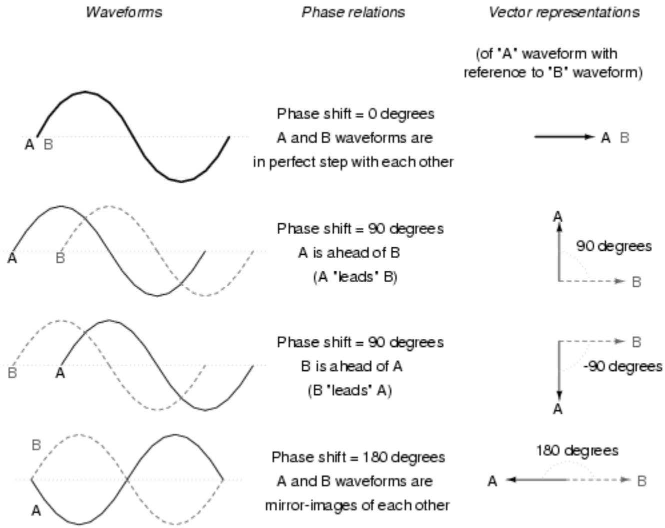
Some Terms

- Bus: A node in the network, often correlated with some sort of substation or distribution center
- Loads on buses   Power Consumption/Demand
- Branch/Line: A physical line connecting two buses (transmission level)
- Generator: Generates power to meet demand
- Contingency: Failure element e.g. Downed Line

AC Power

- Voltage: $V(t) = |V|cos(\omega t)$
- Current: $I(t) = |I|cos(\omega t + \phi)$
- Power: $V(t)I(t)$

AC Power


Waveforms
Phase relations
Vector representations
(of "A" waveform with reference to "B" waveform)
A B
Phase shift = 0 degrees
A and B waveforms are
in perfect step with each other
A B
A B
Phase shift = 90 degrees
A is ahead of B
(A "leads" B)
A
90 degrees
B
B A
Phase shift = 90 degrees
B is ahead of A
(B "leads" A)
B
-90 degrees
A
B
A
Phase shift = 180 degrees
A and B waveforms are
mirror-images of each other
180 degrees
A
B

Phasor Measurement Unit (PMU)

- Delivers GPS-Synchronized Sensor Readings
- Measures Voltage Magnitudes and Angles 10-30 readings/second of bus and buses neighbors
- Placed only at certain buses, so one cannot see the entire network

Problem Formulation

Modeling the Grid

A power system is given by a system of differential algebraic equations (DAE)

$$\Omega = \begin{cases} x' = f(x,y) \\ 0 = g(x,y) \end{cases}$$

With initial conditions $x(t_0) = x_0$ and $y(t_0) = y_0$

Modeling the Grid

General Differential Equation Theory says near steady state, linearization is accurate

$$Ez' = \mathcal{J} z$$

Where $E, \mathcal{J}$, and z are defined as

$$z = \begin{bmatrix} x \\ y \end{bmatrix} \quad \mathcal{J} = \begin{bmatrix} F_x & F_y \\ G_x & G_y \end{bmatrix} \quad E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

ODE Review

Recall the solution to the linear ODE $x' = \mathcal{A}x$ is given by

$$x(t) = \sum_{i=1}^{k} c_i e^{\lambda_i t} u_i$$

Where $\langle \lambda_i, u_i \rangle$ are the *eigenpairs* of the eigenvalue problem $\mathcal{A}u_i = \lambda_i u_i$

Modeling the Grid

ODE theory somewhat generalizes to DAE theory; the solution to the linear DAE $Ez' = \mathcal{J}z$ is given by

$$z(t) = \sum_{i=1}^{k} c_i e^{\mu_i t} v_i$$

Where $\langle \mu_i, v_i \rangle$ are the non-infinite *eigenpairs* of the generalized eigenvalue problem $\mathcal{J}v_i = \mu_i E v_i$

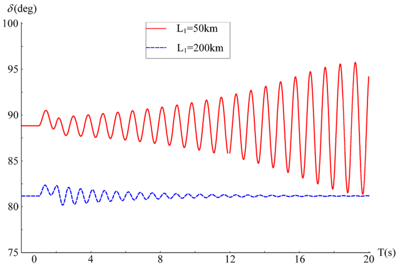
Modeling the Sensors

So we have a closed form for the solution of the DAE. The PMUs "pick out" a subset of $z(t)$ associated with the voltages and angles of the buses with PMUs on them. So the signal we see is

$$f(t) = Hz(t) = \sum_{i=1}^{k} c_i e^{\mu_i t} H v_i$$

Where I is the identity matrix and $H \subset I$

What does sensor data look like?



Euler's Formula

Why does the sensor data look like waves? Don't confuse the sensor readings with $V(t)$ and $I(t)$ —the sensors don't measure these precise quantities!

$$e^{it} = cos(t) + isin(t)$$

$$e^{\mu t} = e^{\mathbf{real}(\mu)t}[cos(\mathbf{imag}(\mu)t) + isin(\mathbf{imag}(\mu)t)]$$

Summary So Far

We know the exact solution to our power grids model and therefore what exactly our sensors should be outputting. So how can we use this information to actually diagnose a contingency?

Modeling a Contingency

Assuming we have an existing system model Ω, a contingency will represent a shift in system models $\Omega \rightarrow \Omega'$ to a new system model Ω'. This change in systems will lead to what is known as a "transient response" as the system moves from one steady state (Ω) to another (Ω')

Solution: A Model Driven Approach

Then given the mathematical formulation in the previous section, we may boil down the problem of contingency identification into the following: given a PMU signal $f(t)$ and a dictionary of contingency models

$$\mathcal{D} = \{\Omega_1, \Omega_2, \ldots, \Omega_m\}$$

determine which contingency model Ω_c "best" explains $f(t)$.

Solution: A Model Driven Approach

- First, fit damped sinusoids to signal $f(t)$ to get form that we want. Since $f(t)$ gives us a subset of eigenvectors, we need to fit the rest.
- Fit eigenvector against each contingency model; we expect the correct contingency to give the best fit

Fitting Method

$$\min_{\alpha, q_j} \left\| \left(E\lambda_j - \mathcal{J}_i \right) \begin{pmatrix} \alpha p_j \\ q_j \end{pmatrix} \right\|_2^2$$

s.t.

$$\left\| \begin{pmatrix} \alpha p_j \\ q_j \end{pmatrix} \right\|_2^2 = 1$$

$$\min_{\alpha, q_j} \left\| \left[\left(E\lambda_j - \mathcal{J}_i \right) \begin{pmatrix} p_j & 0 \\ 0 & I \end{pmatrix} \right] \begin{pmatrix} \alpha \\ q_j \end{pmatrix} \right\|_2^2$$

s.t.

$$\left\| \begin{pmatrix} \alpha \\ q_j \end{pmatrix} \right\|_2^2 = 1$$

$$G_{ij} = \left[\left(E\lambda_j - \mathcal{J}_i \right) \begin{pmatrix} p_j & 0 \\ 0 & I \end{pmatrix} \right] \quad z_j = \begin{pmatrix} \alpha \\ q_j \end{pmatrix}$$

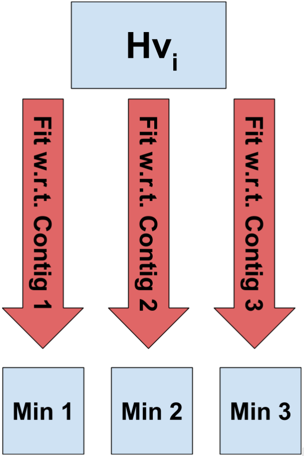
Fitting Method

Point is, we end up solving, for each Contingency and each eigenvector,

$$\min_{b} \|Mb\|_2^2 \quad s.t. \quad \|b\|_2^2 = 1$$

Standard Linear Algebra results tell us that the solution given by λ^s, the smallest eigenvalue of $M^T M$. The contingency in our dictionary $\mathcal{D}$ generating the smallest minima is the correct contingency.

Fitting Method



Results

Lit Review

The power systems community isn't very exhaustive when it comes to testing. An earlier paper by D. Bindel and C. Ponce reaches about 75 percent accuracy w/ limited sensing

	IEEE 14	IEEE 39	IEEE 145
single	100	97.5	TODO
sparse	100	100	TODO
full	100	100	TODO

Our Results

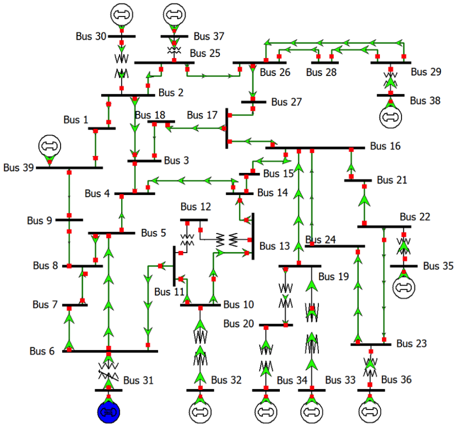


Figure: Testing on the IEEE 39 Bus system, we first placed a single PMU on Bus 16. We consider 40+ line failures and reach almost 100 percent accuracy, with one misdiagnosis

What Went Wrong?

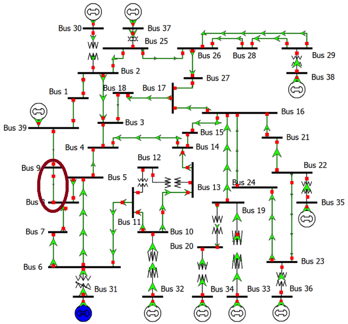


Figure: Left: Misdiagnosis, Right: True Line Failure

What Went Wrong

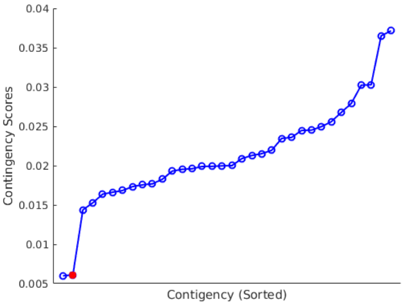


Figure: Contingency Scores, sorted by increasing order with the correct diagnosis highlighted in red

Conclusions

Conclusions: Flaws

- Our method is not very resilient to noise in the signal; fitting damped exponentials to a signal is an inherently ill-conditioned problem.
- Scalability: we only consider one possible failure at a time. What about K simultaneous failures? Scales exponentially in K
- Model dependent; modeling inaccuracies will lead to contingency misidentifications

Conclusions: Ongoing/Future Work

- Moving up to larger systems; 39 bus system is quite small.
- Filtering Method to decrease cost (roughly a logarithmic number of eigenvector fittings as compared to the original) with provably identical results
- Noise. Noise. Noise. Making this algorithm robust to noise is incredibly important (can we actually do this?)

Conclusions

Thank You!