Computer Aided Design of Micro-Electro-Mechanical Systems

From Eigenvalues to Devices

Text

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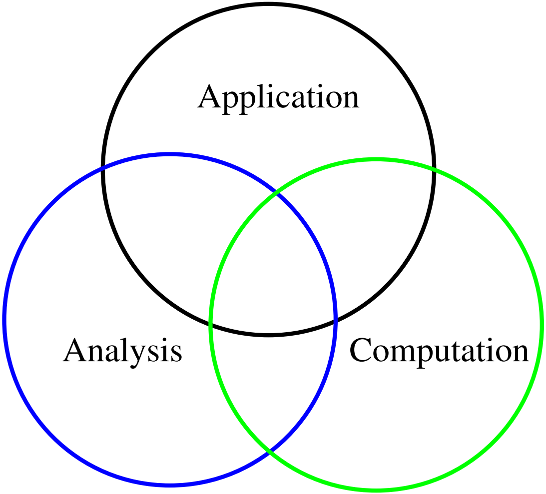
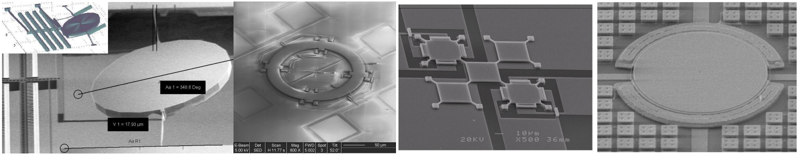
Fudan University, 13 Dec 2012

The Computational Science & Engineering Picture



A Favorite Application: MEMS



I've worked on this for a while:

- SUGAR (early 2000s) – SPICE for MEMS
- HiQLab (2006) – high-Q mechanical resonator device modeling
- AxFEM (2012) – solid-wave gyro device modeling

Goal today: two illustrative snapshots.

Outline

1 Resonant MEMS

2 Anchor losses and disk resonators

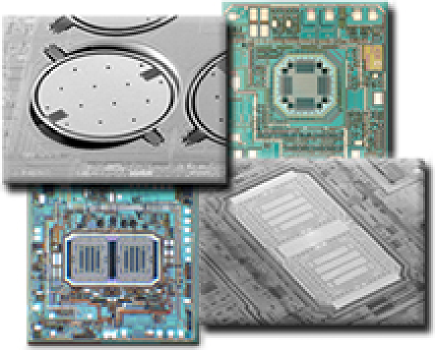
3 Elastic wave gyros

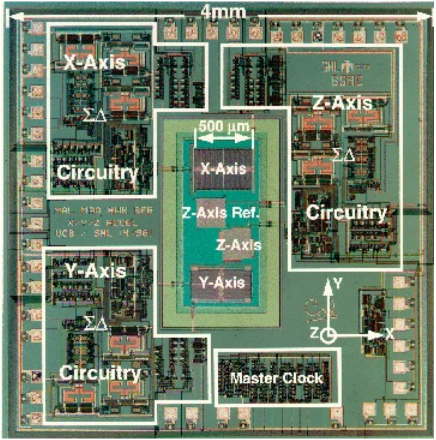
4 Conclusion

MEMS Basics

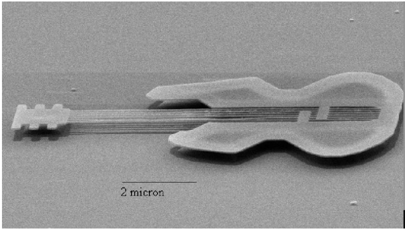
- Micro-Electro-Mechanical Systems
  - Chemical, fluid, thermal, optical (MECFTOMS?)
- Applications:
  - Sensors (inertial, chemical, pressure)
  - Ink jet printers, biolab chips
  - Radio devices: cell phones, inventory tags, pico radio
- Use integrated circuit (IC) fabrication technology
- Tiny, but still classical physics

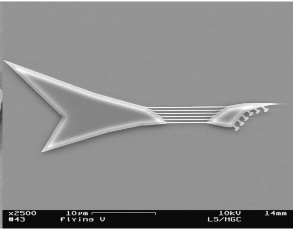
Where are MEMS used?

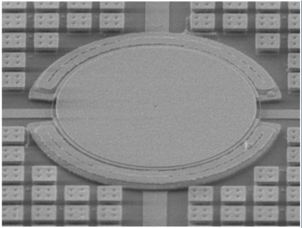


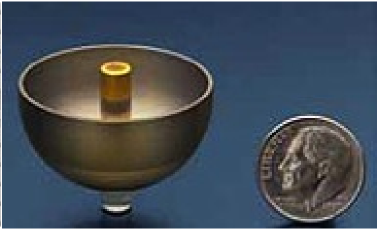


My favorite applications



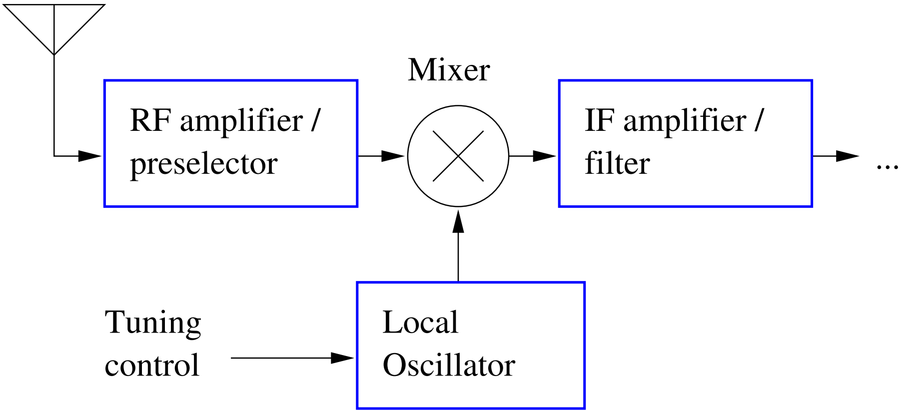






Why you should care, too!

The Mechanical Cell Phone



- ...and lots of mechanical sensors, too!

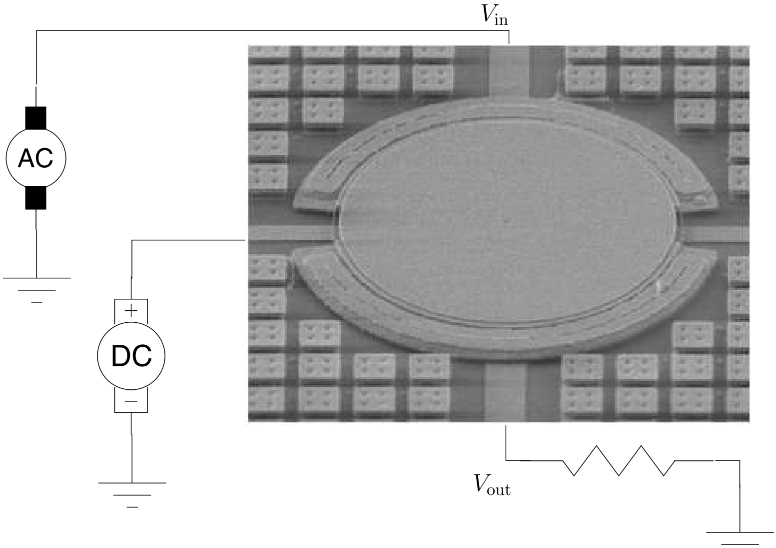
Ultimate Success

"Calling Dick Tracy!"

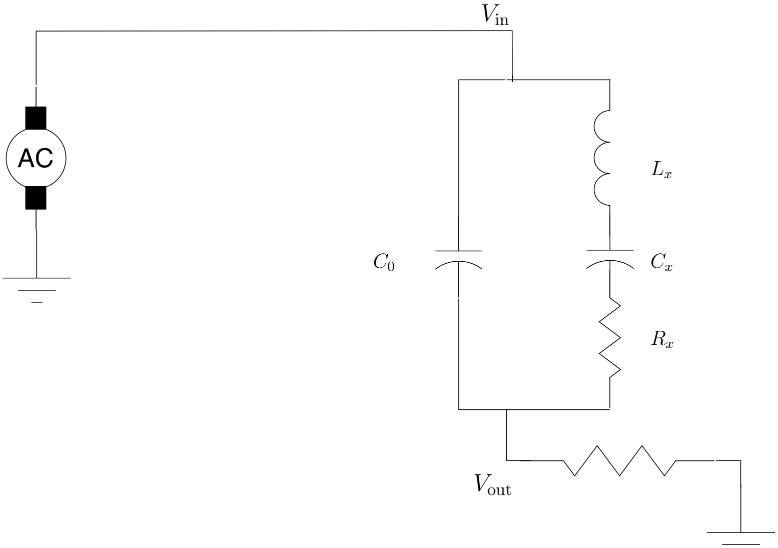
Computational Challenges

Devices are fun – but I'm not a device designer.
Why am I in this?

Model System



The Circuit Designer View



Electromechanical Model

Balance laws (KCL and BLM):

$$\frac{d}{dt}\left(C(u)V\right) + GV = I_{\text{external}}$$

$$Mu_{tt} + Ku - \nabla_u \left(\frac{1}{2} V^* C(u) V\right) = F_{\text{external}}$$

Linearize about static equilibium (V_0, u_0):

$$C(u_0)\,\delta V_t + G\,\delta V + \left(\nabla_u C(u_0) \cdot \delta u_t\right) V_0 = \delta I_{\text{external}}$$

$$M\,\delta u_{tt} + \tilde{K}\,\delta u + \nabla_u \left(V_0^* C(u_0)\,\delta V\right) = \delta F_{\text{external}}$$

where

$$\tilde{K} = K - \frac{1}{2} \frac{\partial^2}{\partial u^2} \left(V_0^* C(u_0) V_0\right)$$

Electromechanical Model

Assume time-harmonic steady state, no external forces:

$$\begin{bmatrix} i\omega C + G & i\omega B \\ -B^T & \tilde{K} - \omega^2 M \end{bmatrix} \begin{bmatrix} \delta\hat{V} \\ \delta\hat{u} \end{bmatrix} = \begin{bmatrix} \delta\hat{I}_{\text{external}} \\ 0 \end{bmatrix}$$

Eliminate the mechanical terms:

$$\begin{aligned} Y(\omega)\,\delta\hat{V} &= \delta\hat{I}_{\text{external}} \\ Y(\omega) &= i\omega C + G + i\omega H(\omega) \\ H(\omega) &= B^T(\tilde{K} - \omega^2 M)^{-1} B \end{aligned}$$

Goal: Understand electromechanical piece ($i\omega H(\omega)$).

- As a function of geometry and operating point
- Preferably as a simple circuit

Damping and Q

Designers want high *quality of resonance* (Q)

- Dimensionless damping in a one-dof system

$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

- For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{|\omega|}{2\,\mathrm{Im}(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

To understand Q, we need damping models!

The Designer's Dream

Reality is messy:

- Coupled physics
- ... some poorly understood (damping)
- ... subject to fabrication errors

Ideally, would like:

- Simple models for behavioral simulation
- Parameterized for design optimization
- Including all relevant physics
- With reasonably fast and accurate set-up

We aren't there yet.

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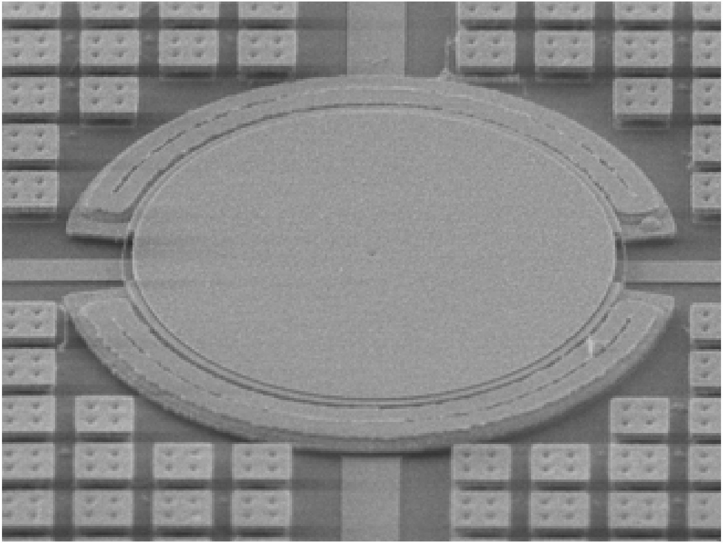
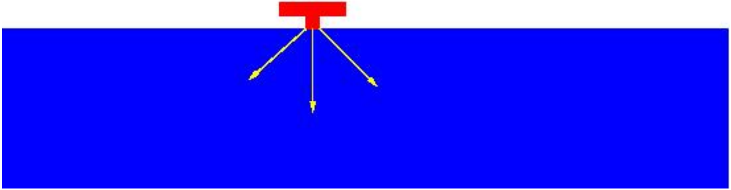
Disk Resonator Simulations



Damping Mechanisms

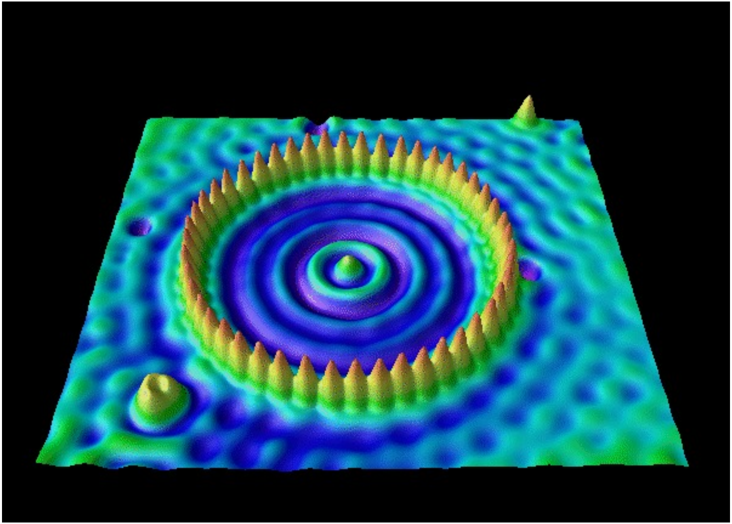


Possible loss mechanisms:

- Fluid damping
- Material losses
- Thermoelastic damping
- Anchor loss

Model substrate as semi-infinite $\Longrightarrow$ resonances!

Resonances in Physics



Resonances and Literature

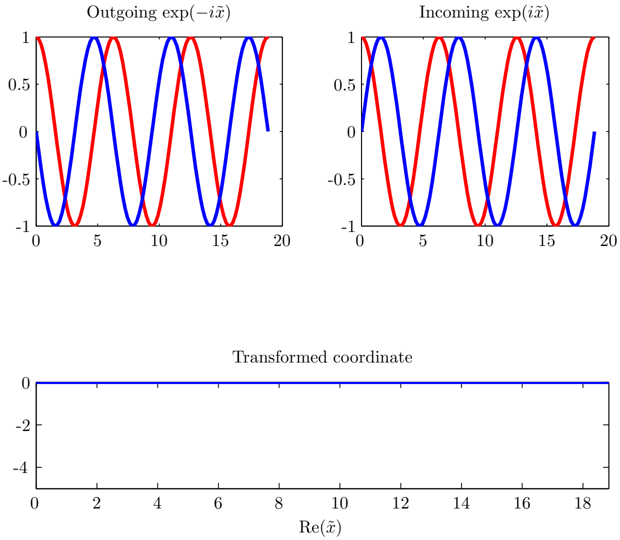
In bells of frost I heard the resonance die.

– Li Bai (translated by Vikram Seth)

Perfectly Matched Layers

- Complex coordinate transformation
- Generates a "perfectly matched" absorbing layer
- Idea works with general linear wave equations
    - Electromagnetics (Berengér, 1994)
    - Quantum mechanics – *exterior complex scaling* (Simon, 1979)
    - Elasticity in standard finite element framework (Basu and Chopra, 2003)
    - Works great for MEMS, too! (Bindel and Govindjee, 2005)

Model Problem Illustrated



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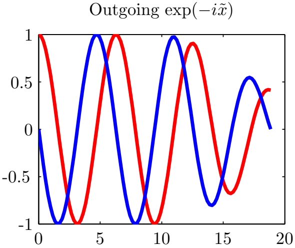

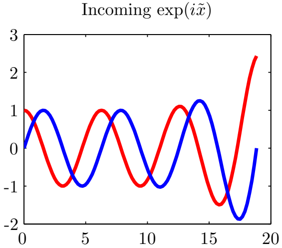

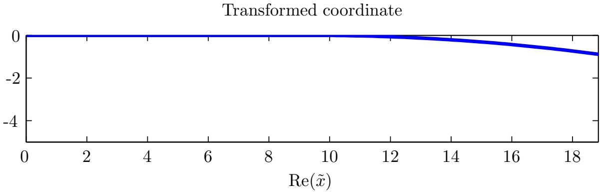

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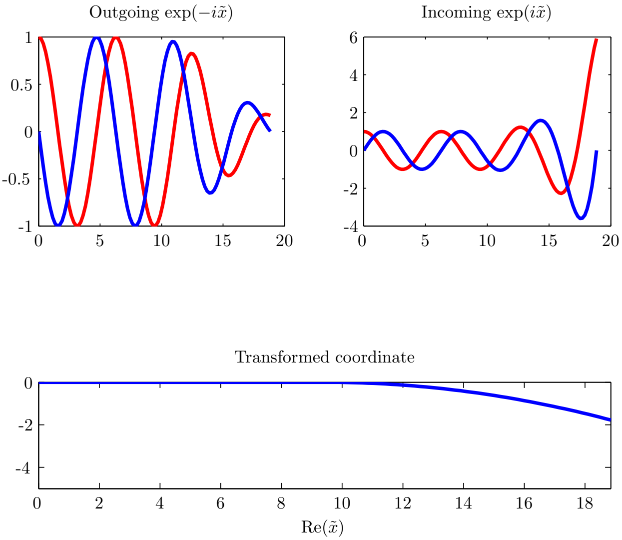

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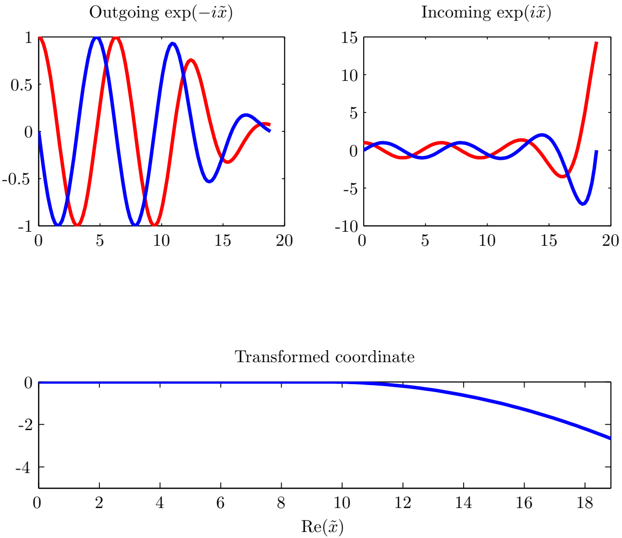

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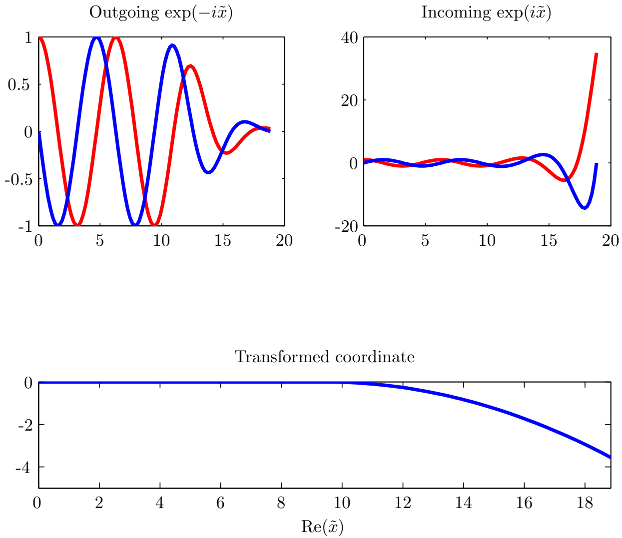

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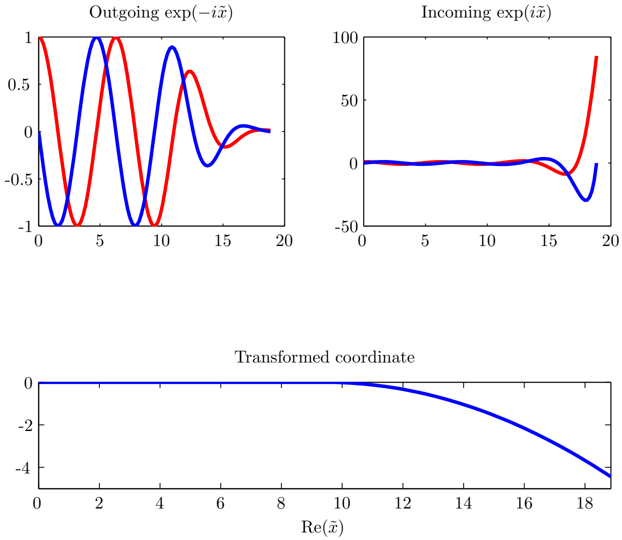
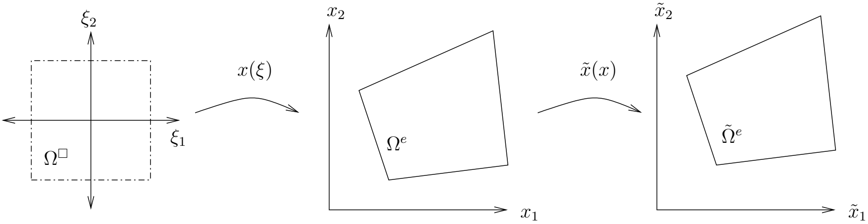
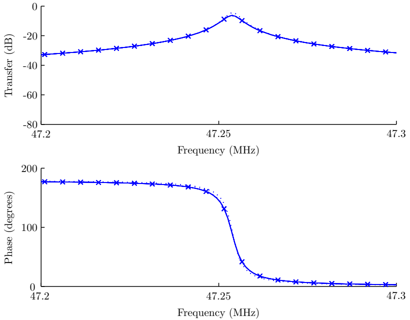


Finite Element Implementation



Matrices are *complex symmetric*

Eigenvalues and Model Reduction



Goal: understand $H(\omega)$:

$$H(\omega) = B^T(K - \omega^2 M)^{-1}B$$

Look at

- Poles of H (eigenvalues)
- Bode plots of H

Model reduction: Replace $H(\omega)$ by cheaper $\hat{H}(\omega)$.

Approximation from Subspaces

A general recipe for large-scale numerical approximation:

1. A subspace $\mathcal{V}$ containing good approximations.
2. A criterion for "optimal" approximations in $\mathcal{V}$.

Basic building block for eigensolvers and model reduction!

Better subspaces, better criteria, better answers.

Variational Principles

- Variational form for complex symmetric eigenproblems:
  - Hermitian (Rayleigh quotient):

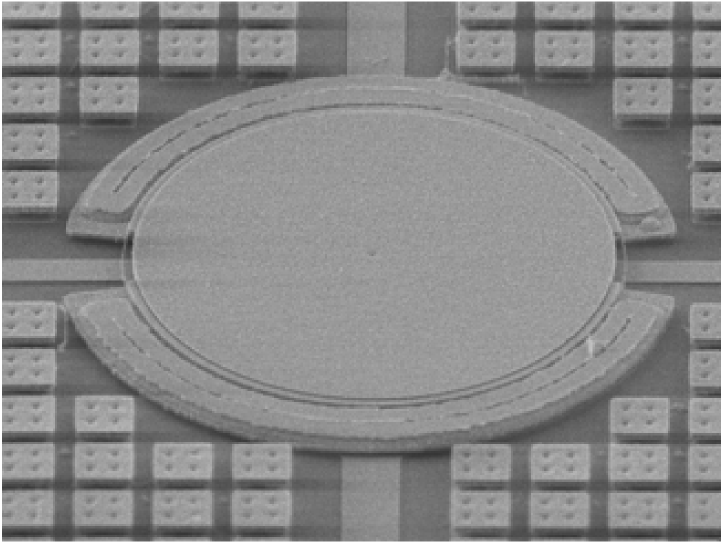
$$\rho(v) = \frac{v^* K v}{v^* M v}$$

  - Complex symmetric (modified Rayleigh quotient):

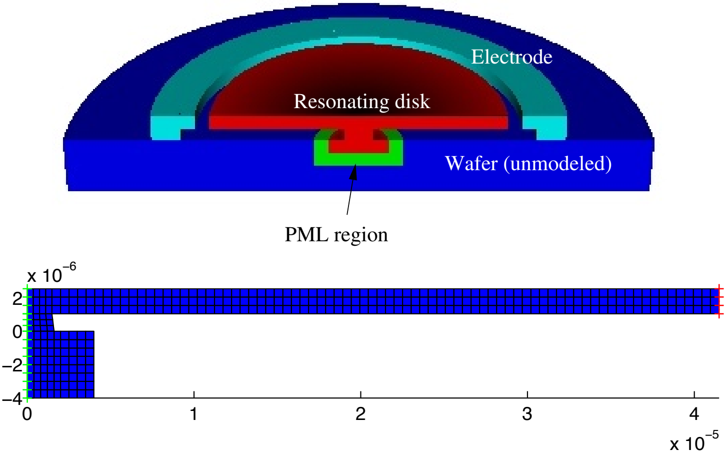
$$\theta(v) = \frac{v^T K v}{v^T M v}$$

- First-order accurate eigenvectors $\Longrightarrow$ Second-order accurate eigenvalues.
- Good for model reduction, too!

Disk Resonator Simulations

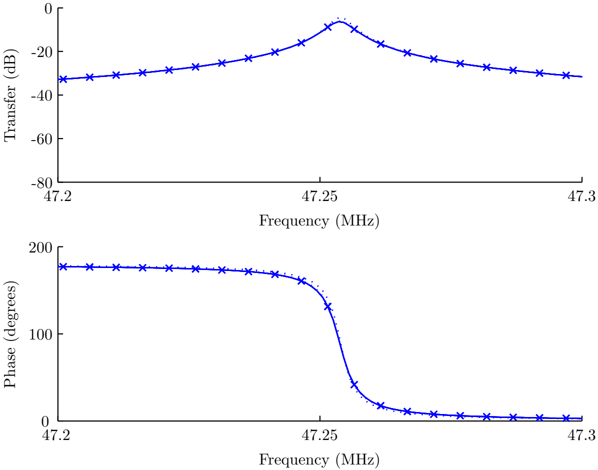


Disk Resonator Mesh



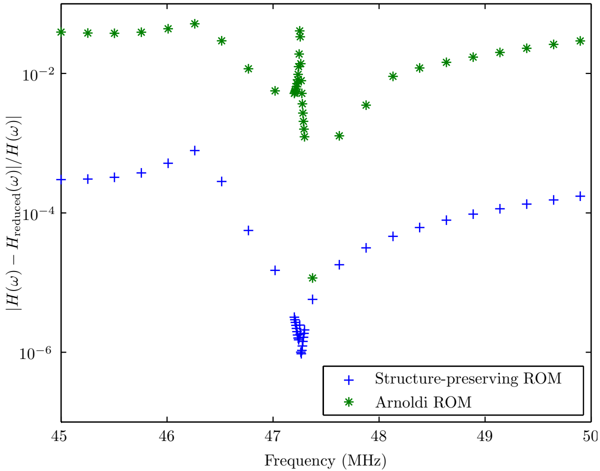
Axisymmetric model, bicubic, $\approx 10^4$ nodal points at convergence

Model Reduction Accuracy



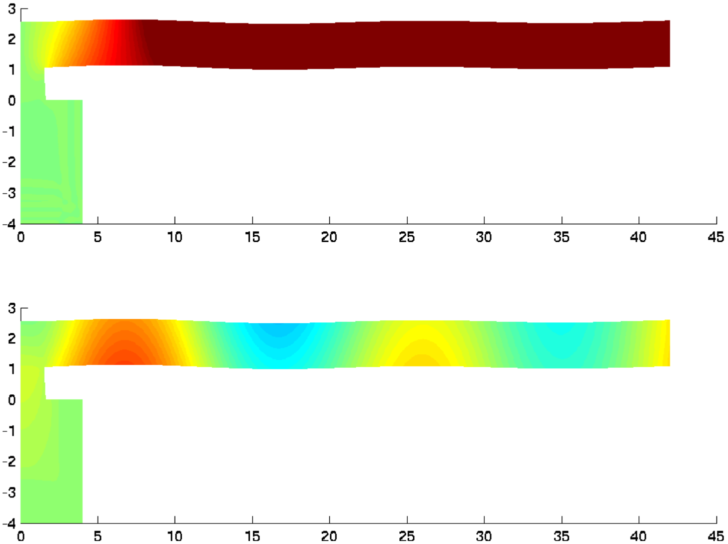
Results from ROM (solid and dotted lines) nearly indistinguishable from full model (crosses)

Model Reduction Accuracy

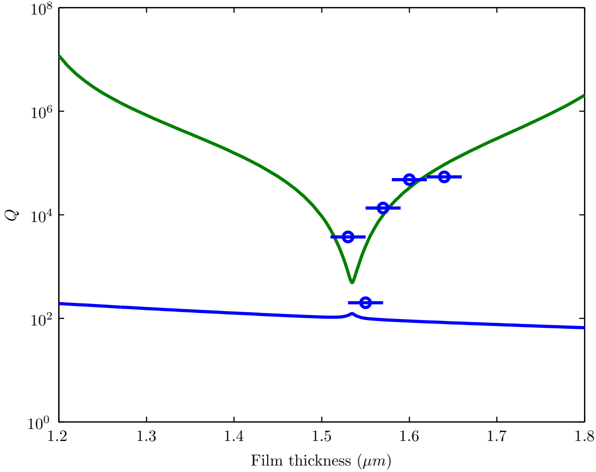


Preserve structure $\implies$
get twice the correct digits

Response of the Disk Resonator

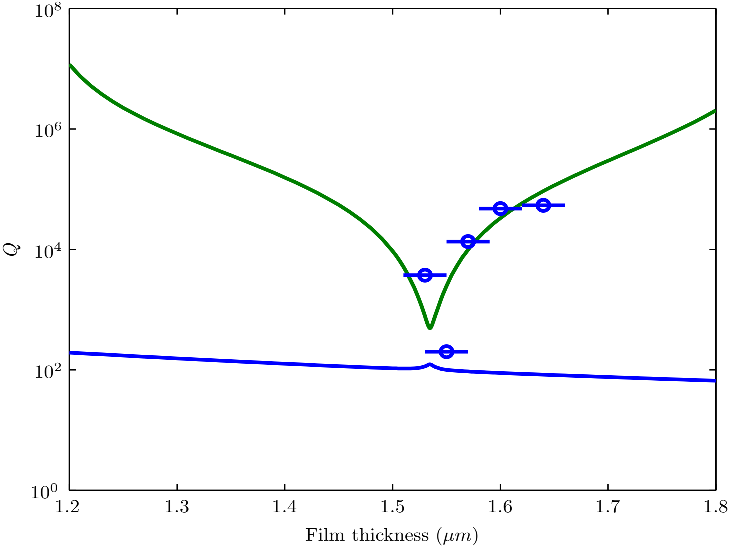


Variation in Quality of Resonance

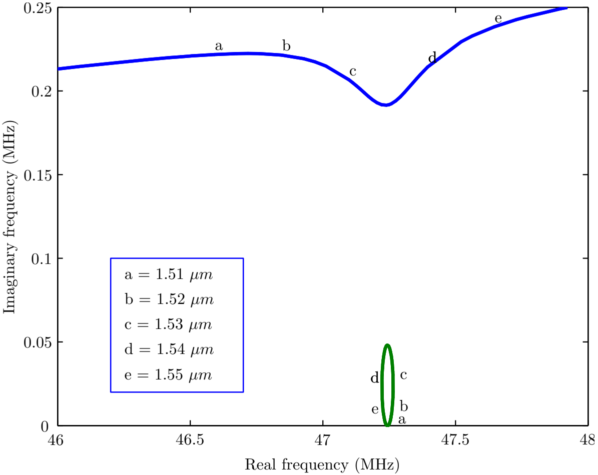


Simulation and lab measurements vs. disk thickness

Explanation of Q Variation



Explanation of Q Variation



Interaction of two nearby eigenmodes

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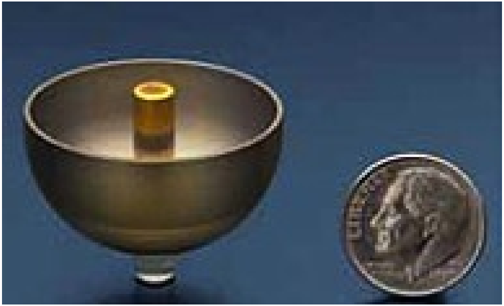
Bryan's Experiment

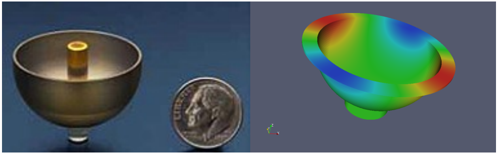
"On the beats in the vibrations of a revolving cylinder or bell"
by G. H. Bryan, 1890

A Small Application



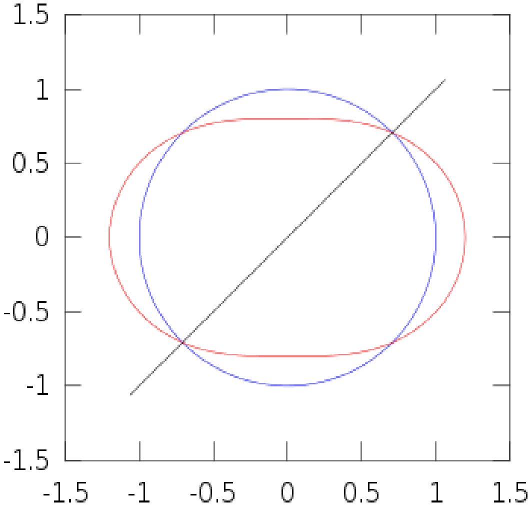
Northrup-Grummond HRG

Current example: Micro-HRG / GOBLiT / OMG

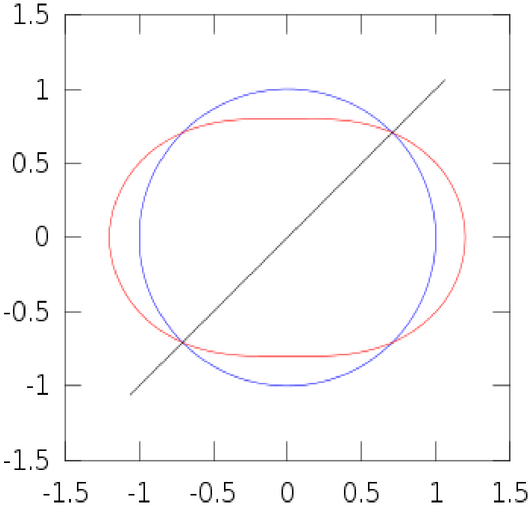


- Goal: Cheap, small (1mm) HRG
- Collaborator roles:
    - Basic design
    - Fabrication
    - Measurement
- Our part:
    - Detailed physics
    - Fast software
    - Sensitivity
    - Design optimization

How It Works



How It Works



Goal state

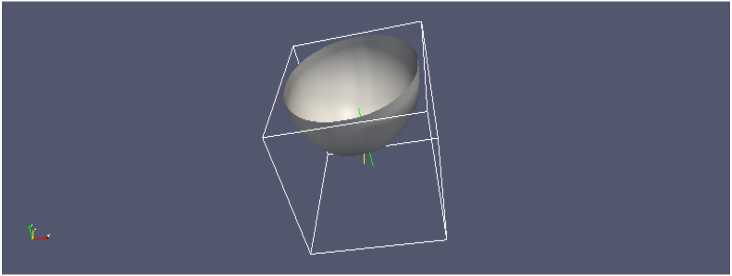
We want to compute:

- Geometry
- Fundamental frequencies
- Angular gain (Bryan's factor)
- Damping (thermoelastic, radiation, material)
- Sensitivities of everything
- Effects of symmetry breaking

For speed and accuracy: use structure!

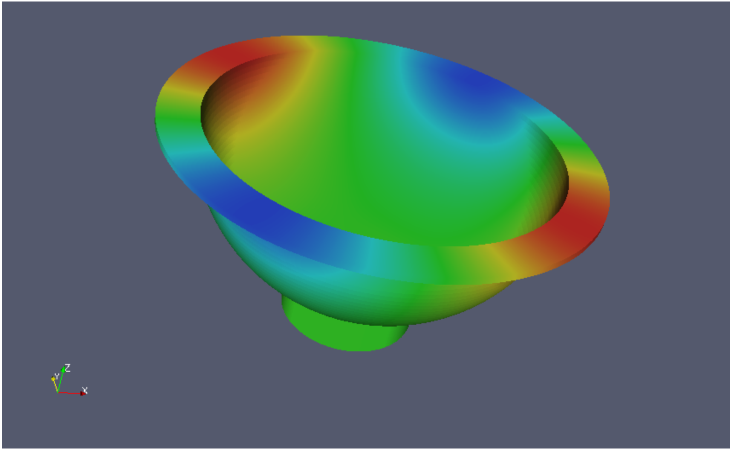
- Axisymmetric geometry $\implies$ 3D to 2D via Fourier
- Perturbed geometry $\implies$ interactions for different wave numbers

Getting the Geometry

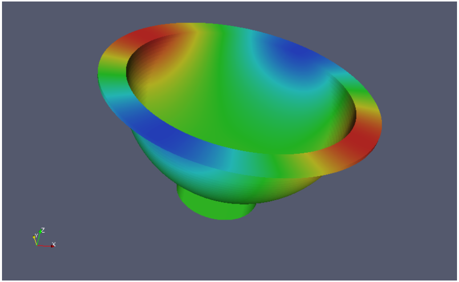


- Simple isotropic etch modeling fails – 1mm is *huge*!
- Working on better simulator (reaction-diffusion).
- For now, take idealized geometries on faith...

Full Dynamics



Essential Dynamics



Dynamics in 2D subspace of degenerate modes:

$$\left(-\omega^2 mI + 2i\omega\Omega gJ + kI\right) c = 0$$

Scaled gain g is *Bryan's factor*

$$\mathrm{BF} = \frac{\text{Angular rate of pattern relative to body}}{\text{Angular rate of vibrating body}}$$

If no parameters in the world were very large or very small, science would reduce to an exhaustive list of everything.

– Nick Trefethen

Fourier Picture

Write displacement fields as Fourier series:

$$\mathbf{u} = \sum_{m=0}^{\infty} \left(\begin{bmatrix} u_{mr}(r,z)\cos(m\theta) \\ u_{m\theta}(r,z)\sin(m\theta) \\ u_{mz}(r,z)\cos(m\theta) \end{bmatrix} + \begin{bmatrix} -u'_{mr}(r,z)\sin(m\theta) \\ u'_{m\theta}(r,z)\cos(m\theta) \\ -u'_{mz}(r,z)\sin(m\theta) \end{bmatrix} \right)$$

- Works whenever *geometry* is axisymmetric
- Treat non-axisymmetric geometries as mapped axisymmetric
  - Now *coefficients* in PDEs are non-axisymmetric
- Problems with different m decouple if *everything* axisymmetric

Fourier Picture

Perfect axisymmetry:

$$\begin{bmatrix} K_{11} & & & \\ & K_{22} & & \\ & & K_{33} & \\ & & & \ddots \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11} & & & \\ & M_{22} & & \\ & & M_{33} & \\ & & & \ddots \end{bmatrix}$$

Fourier Picture

Broken symmetry (via coefficients):

$$\begin{bmatrix} K_{11} & \epsilon & \epsilon & \epsilon \\ \epsilon & K_{22} & \epsilon & \epsilon \\ \epsilon & \epsilon & K_{33} & \epsilon \\ \epsilon & \epsilon & \epsilon & \ddots \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11} & \epsilon & \epsilon & \epsilon \\ \epsilon & M_{22} & \epsilon & \epsilon \\ \epsilon & \epsilon & M_{33} & \epsilon \\ \epsilon & \epsilon & \epsilon & \ddots \end{bmatrix}$$

Perturbing Fourier

Modes "near" azimuthal number m = nonlinear eigenvalues

$$\left(K_{mm} - \omega^2 M_{mm} + E_{mm}(\omega) \right) u = 0.$$

Need:

- Control on E_{mm}
  - Depends on frequency spacing
  - Depends on Fourier analysis of perturbation
- Perturbation theory for nonlinearly perturbed eigenproblems
  - For self-adjoint case, results similar to Lehmann intervals

First-order estimate: $(K_{mm} - \omega_0^2 M_{mm}) u_0 = 0$; then

$$\delta(\omega^2) = \frac{u_0^T E_{mm}(\omega_0) u_0}{u_0^T M_{mm} u_0}.$$

Perturbation and Radiation

Incorporating numerical radiation BCs gives:

$$\left(K - \omega^2 M + G(\omega) \right) u = 0.$$

Perturbation approach: ignore G to get (ω_0, u_0). Then

$$\delta(\omega^2) = \frac{u_0^T \, G(\omega_0) \, u_0}{u_0^T M_{mm} u_0}.$$

Works when BC has small *influence* (coefficients aren't small).

Also an approach to understanding sensitivity to BC!
... explains why PML works okay despite being inappropriate?

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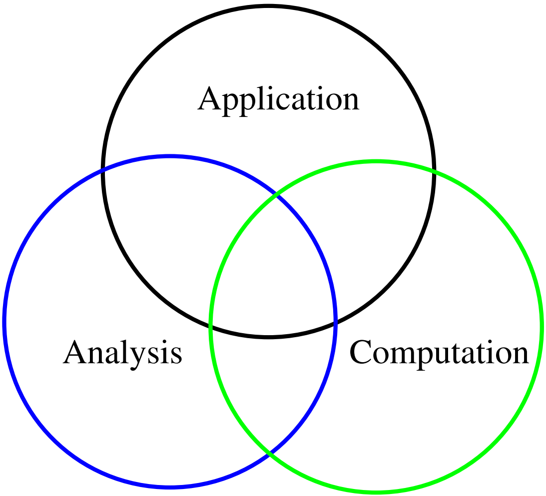

Conclusions

The difference between art and science is that science is what we understand well enough to explain to a computer. Art is everything else.

Donald Knuth

The purpose of computing is insight, not numbers.

Richard Hamming

- Collaborators:
  - Disk: S. Govindjee, T. Koyama, S. Bhave, E. Quevy
  - HRG: S. Bhave, L. Fegely, E. Yilmaz
- Funding: DARPA MTO, Sloan Foundation