Micro-gyro simulation and modeling

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Basic goals

1. Simulate mechanics of axisymmetric micro-gyro designs:
    - Resonant frequencies and mode shapes
    - Angular gain / Bryan's factor
    - Loss mechanisms (anchor, TED)
2. Efficient computational formulations
    - 2.5-dimensional finite element formulation
    - Fast solvers for Bryan's factor, loss mechanisms
3. Sensitivity analysis for design and optimization

deal2lab

- Uses open-source `deal.ii` FE framework.
- Programmatic meshes for geometry parameterization:

```
function hrglip(r_hemisphere,thickness,
                r_lip,
                r_anchor,
                h_anchor)
```

- Fast solvers for angular gain, loss mechanisms.
- Fast sensitivity with respect to parameters.

Kinematic assumptions

Use 2.5D formulation for basic modal computations:

$$\mathbf{u}_1(r,z) = \begin{bmatrix} u_r(r,z)\cos(m\theta) \\ u_\theta(r,z)\sin(m\theta) \\ u_z(r,z)\cos(m\theta) \end{bmatrix}, \quad \mathbf{u}_2(r,z) = \begin{bmatrix} -u_r(r,z)\sin(m\theta) \\ u_\theta(r,z)\cos(m\theta) \\ -u_z(r,z)\sin(m\theta) \end{bmatrix}.$$

- Only mesh cross-section, 3 DOF per node.
- 2D connectivity $\implies$ fast direct solvers.
- Geometric degeneracy preserved by the discretization.

Computational pattern

- Solve for mode with no damping or rotation:

$$(-\omega_0^2 M_{uu} + K_{uu})u_0 = 0.$$

- First-order perturbation theory for damping and rotation.

Thermoelastic damping

Compute mechanical mode + induced temperature fluctuation:

$$\begin{aligned}(-\omega_0^2 M_{uu} + K_{uu})u_0 &= 0 \\ (i\omega_0 C_{\theta\theta} + K_{\theta\theta})\theta_0 &= -i\omega_0 C_{\theta u} u_0.\end{aligned}$$

First-order correction to eigenvalue (generalized Zener):

$$\delta(\omega^2) = -\frac{u_0^T K_{u\theta}\theta_0}{u_0^T M_{uu} u_0}.$$

Anchor loss

Incorporating numerical radiation BCs gives:

$$\left(-\omega^2 M_{uu} + K_{uu} + G(\omega)\right) u = 0$$

where $G(\omega)$ approximates a DtN map (e.g. via PML).

Perturbation approach: ignore G to get (ω_0, u_0). Then

$$\delta(\omega^2) = \frac{u_0^T G(\omega_0) u_0}{u_0^T M_{uu} u_0}.$$

Bryan's factor

Angular gain for a given mode is

$$\mathrm{BF} = \frac{\text{Angular rate of pattern relative to body}}{\text{Angular rate of vibrating body}} = \frac{1}{m}\left(\frac{u^T B u}{u^T M u}\right),$$

where M is the standard FE mass matrix and B is

$$B_{IJ} = \int_\Omega N_I(r,\theta) N_J(r,\theta) \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Geometric sensitivities

Basic strategy is standard[1]:

- Differentiate node positions w.r.t. geometric parameters
- Differentiate FE matrices w.r.t. node positions
- Differentiate ω, Q, BF w.r.t. FE matrices
- Apply chain rule

[1] Haslinger and Mäkinen. 2003. *Introduction to shape optimization theory, approximation, and computation.*

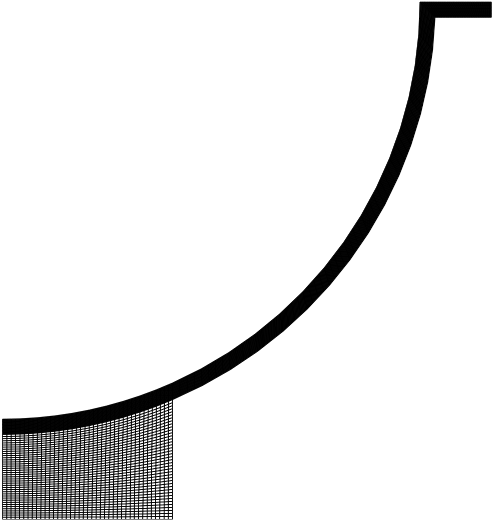
Testing strategy

- Unit tests for basic functionality (run automatically on build)
- Convergence tests
- Validation tests compare against results in the literature
- Finite difference checks for sensitivity computations

Validation testing

1. F. I. Niordson, *Free Vibrations of Thin Elastic Spherical Shells*, International Journal of Solids and Structures, 20 (7), 1984, pp. 667–687.
2. J.J.Hwang C.S.Chou C.O.Chang, *Precession of Vibrational Modes of a Rotating Hemispherical Shell*, Transactions of the ASME, 119, 1997
3. S. Y. Choi, Y. H. Na, and J. H. Kim *Thermoelastic Damping of Inextensional Hemispherical Shell*, World Academy of Science, Engineering and Technology, 56, 2009.
4. S. J. Wong, C.H. Fox, S. McWilliam, C.P. Fell, R. Eley *A preliminary investigation of thermo-elastic damping in silicon rings*. J. Micromech. Microeng. 14, 2004, S108–S113

HRG simulations



Basic parameters

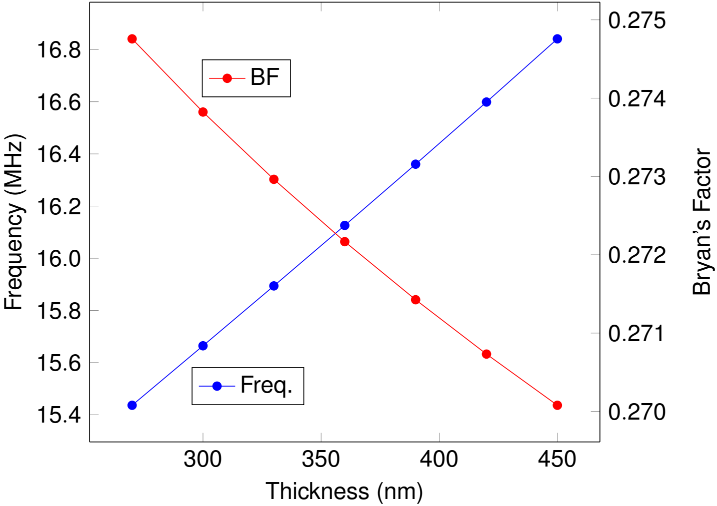
	D1	D2	D3
Hemisphere radius (microns)	10	32.5	40
Shell thickness (nm)	360	370	370
Lip outer radius (microns)	12	36	42.5
Anchor radius (microns)	4	20	20
Anchor height (microns)	2	10	10

Example sensitivity results (D3)

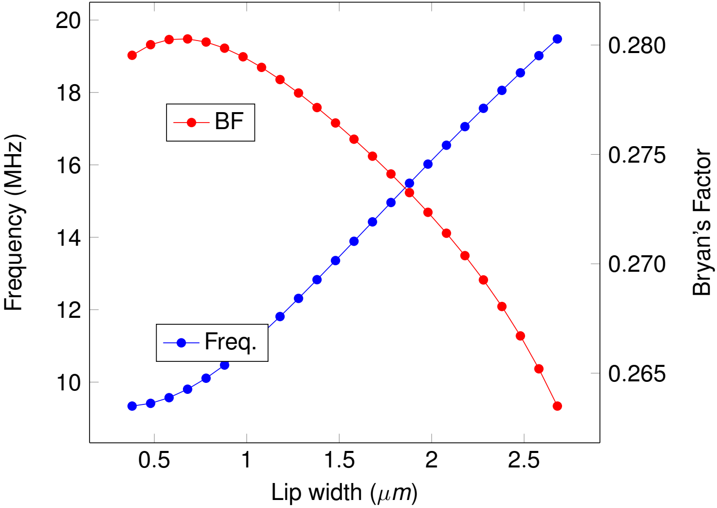
ω	2.4795 MHz
Q	11000
BF	0.28377

	σ_ω	σ_{QTED}	σ_{BF}
Hemisphere radius (40 microns)	-5.469836	8.672640	-0.134090
Shell thickness (370 nm)	0.076943	-0.012849	-0.003516
Lip length (2.5 microns)	0.114765	-0.650651	0.007138
Anchor radius (20 microns)	2.446579	2.631051	0.016222
Anchor height (10 microns)	-0.004707	-0.009311	0.000054

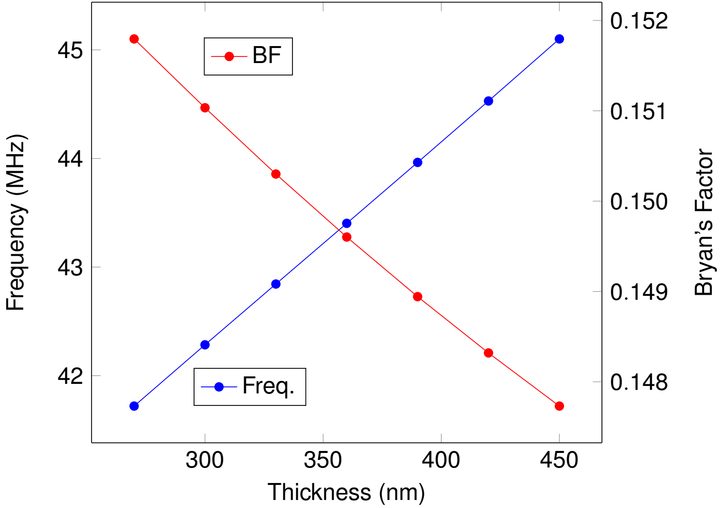
Frequency and BF vs. shell thickness ($m = 2$, D1)



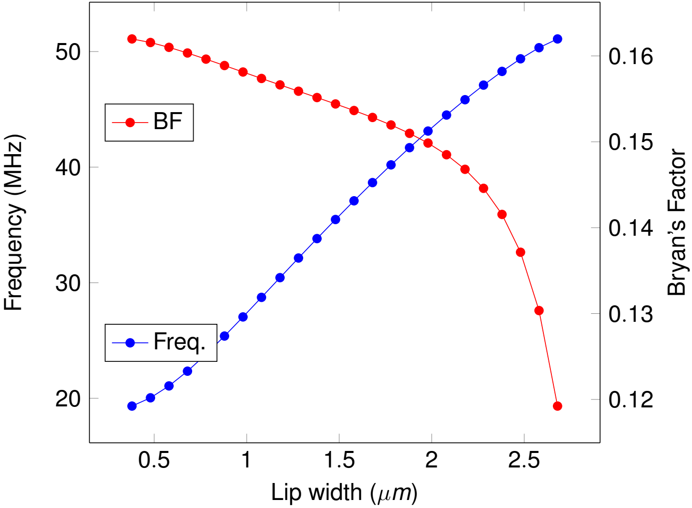
Frequency and BF vs lip width ($m = 2$, D1)



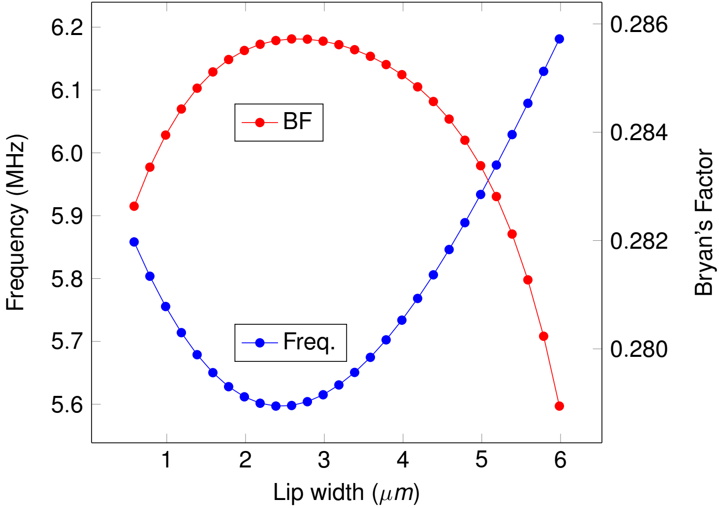
Frequency and BF vs. shell thickness ($m = 3$, D1)



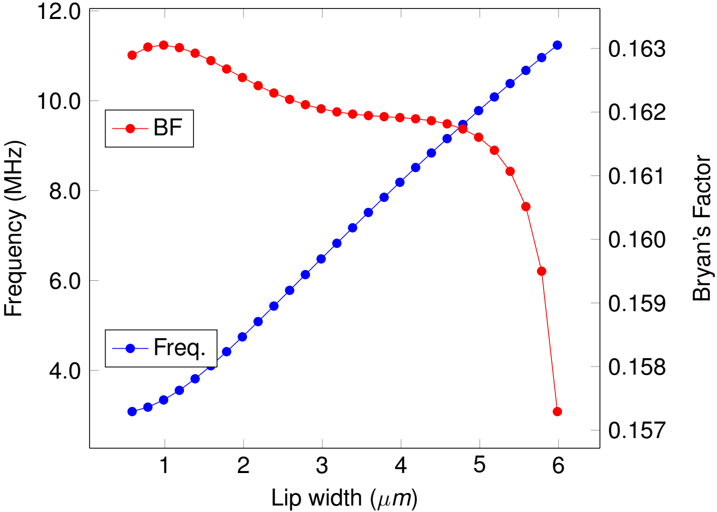
Frequency and BF vs. lip width ($m = 3$, D1)



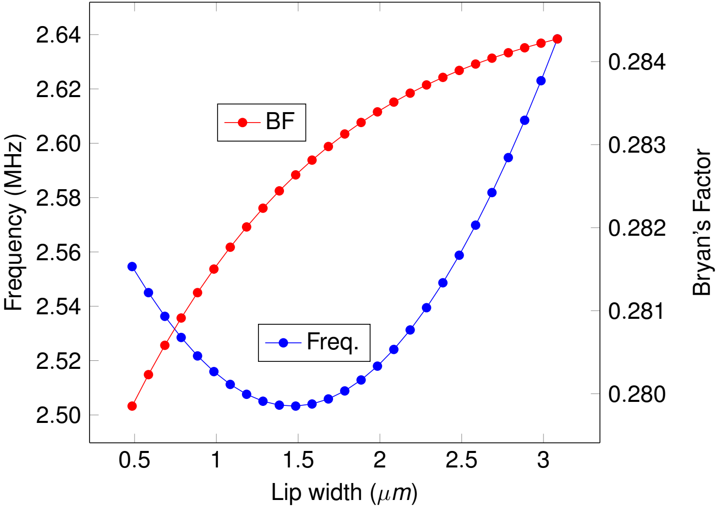
Frequency and BF vs. lip width ($m = 2$, D2)



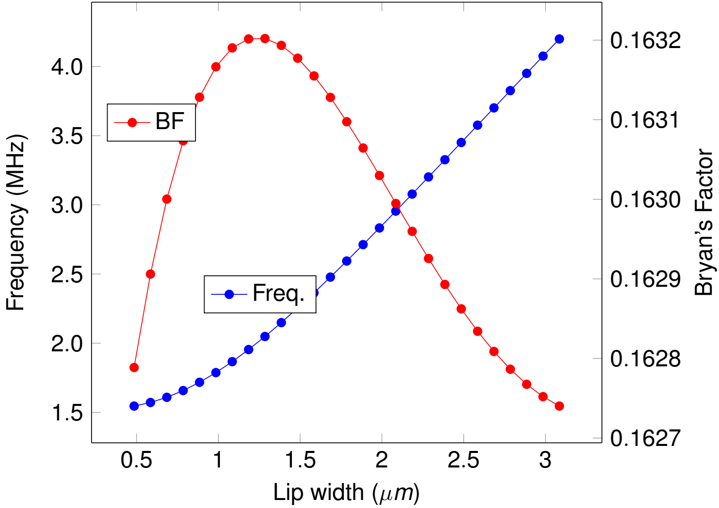
Frequency and BF vs. lip width ($m = 3$, D2)



Frequency and BF vs. lip width ($m = 2$, D3)



Frequency and BF vs. lip width ($m = 3$, D3)



BF vs. lip width for $m = 3$, large device (D3)

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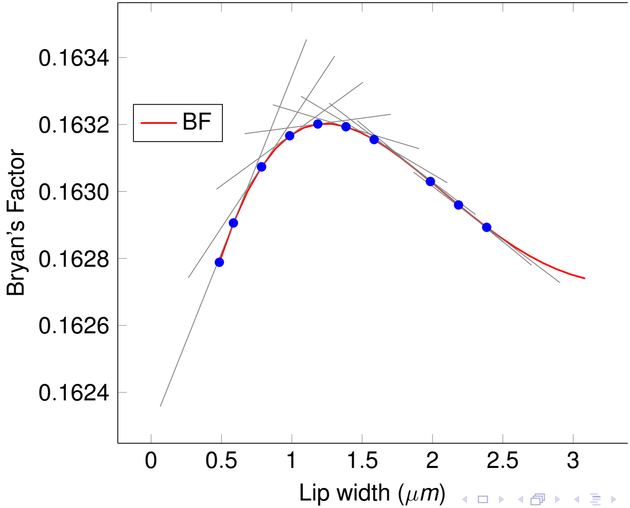

Summary

Initial code is working:

- Fast computation of Bryan's factor, Q_{TED}
- Anchor loss computations work separately
- Sensitivity analysis works
- Includes unit tests and validation test suite

Some things still needed:

- Full documentation
- Removal of some known performance bottlenecks
- Integration of anchor loss code into deal2lab
- Framework for surface loss modeling