Scientific Computing Group

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Connections

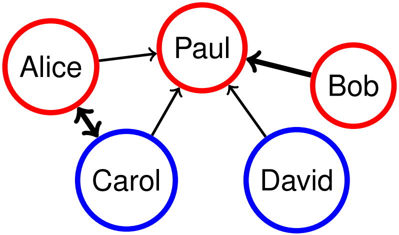
What we do:

- Matrix computations
- Fast transforms
- Model reduction
- Physical simulations
- Network modeling
- HPC

Who we talk to:

- Graphics and vision
- Machine learning
- Theory
- Computer systems
- Engineering
- Physical sciences

Example: Opinions in Networks



Modeling Opinion Formation

A basic model:

- A fixed *intrinsic* opinion s_i
- A variable *expressed* opinion x_i
- Equilibrium $x_i = \mathrm{argmin}_{z_i} c_i(z_i)$, where

$$c_i(z_i) \equiv (s_i - z_i)^2 + \sum_{j \in N(i)} w_{ij}(z_i - x_j)^2$$

- Define a *social cost* $c(z) = \sum_i c_i(z_i)$

From Networks to Numerical Linear Algebra

Methodology: Graph problem $\mapsto$ linear algebra problem.

Nash equilibrium:	$(L+I)x = s$
Social optimum:	$(A+I)y = s$
Cost at equilibrium:	$c(x) = s^T Cs$
Optimal social cost:	$c(y) = s^T Bs$

Price of anarchy is a ratio of quadratics:

$$\mathrm{PoA}(s) = \frac{c(x)}{c(y)} = \frac{s^T Cs}{s^T Bs}$$

Find worst case through a *a generalized eigenvalue problem*:

$$Cs_* = \lambda Bs_*$$

Tensor Computations

What is a tensor? Think of it as a higher dimensional matrix.

A fourth-order tensor...

$$A = A(1:n_1, 1:n_2, 1:n_3, 1:n_4)$$

They are typically very large data objects...

$$N = n_1 \cdot n_2 \cdot n_3 \cdots n_d$$

Why?

Make it possible for scientists to extract information from high-dimensional datasets that arise from modeling...

$$\mathcal{A}(i,j,k,\ell) \;=\; \textit{a measurement that results by setting the value of four independent variables}$$

or discretization...

$$\mathcal{A}(i,j,k,\ell) \;=\; f(w_i, x_j, y_k, z_\ell)$$

How?

A tensor

$$\mathcal{A} = \mathcal{A}(1\!:\!4, 1\!:\!3, 1\!:\!n_3, 1\!:\!n_4)$$

can be “flattened” into a block matrix:

$$A = \begin{bmatrix} \mathcal{A}(1,1,:,:) & \mathcal{A}(1,2,:,:) & \mathcal{A}(1,3,:,:) \\ \mathcal{A}(2,1,:,:) & \mathcal{A}(2,2,:,:) & \mathcal{A}(2,3,:,:) \\ \mathcal{A}(3,1,:,:) & \mathcal{A}(3,2,:,:) & \mathcal{A}(3,3,:,:) \\ \mathcal{A}(4,1,:,:) & \mathcal{A}(4,2,:,:) & \mathcal{A}(4,3,:,:) \end{bmatrix}$$

Methodology: Extract information from A using “classical” matrix computations. Then draw conclusions about tensor $\mathcal{A}$.

Focus: Low Rank Approximation

Given: $\mathcal{A}(1:n, 1:n, 1:n, 1:n, 1:n, 1:n)$.

Find: n-by-n matrices $B_1, \ldots, B_p$, $C_1, \ldots, C_p$, and $D_1, \ldots, D_p$ so that

$$\mathcal{A}(i_1, i_2, i_3, i_4, i_5, i_6) \approx \sum_{s=1}^{p} B_s(i_1, i_2) C_s(i_3, i_4) D_s(i_5, i_6)$$

Approximating an $O(n^6)$ data object with $3pn^2$ numbers.

Vehicle: Multilinear optimization

Goal: Make intractable problems tractable through approximation.