Structure-preserving model reduction for MEMS

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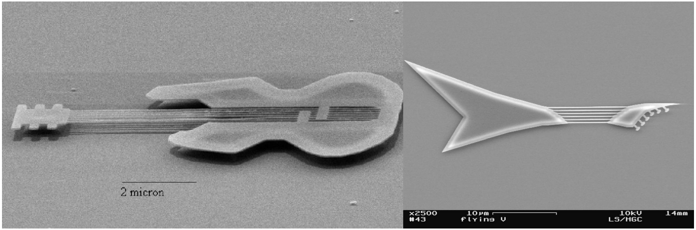
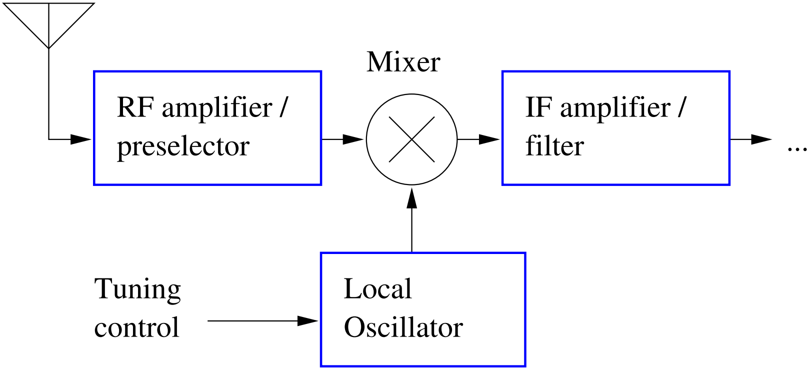
Resonant MEMS



Microguitars from Cornell University (1997 and 2003)

- kHz-GHz mechanical resonators
- *Lots* of applications:
  - Inertial sensors (in phones, airbag systems, ...)
  - Chemical sensors
  - Signal processing elements
  - Really high-pitch guitars!

Application: The Mechanical Cell Phone



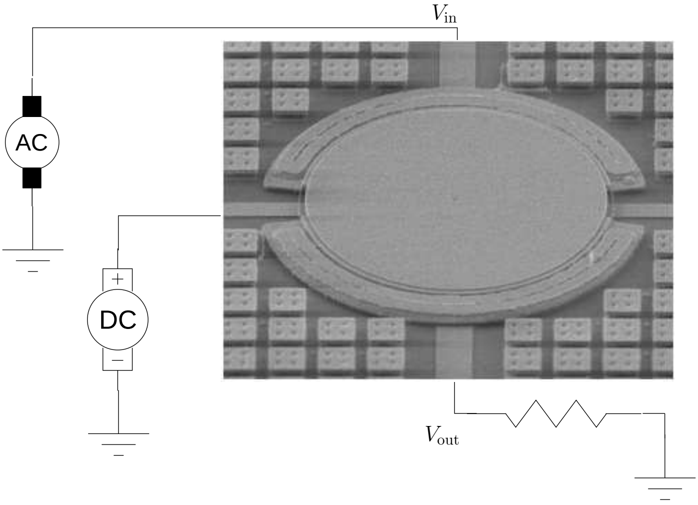
- Your cell phone has many moving parts!
- What if we replace them with integrated MEMS?

Ultimate Success

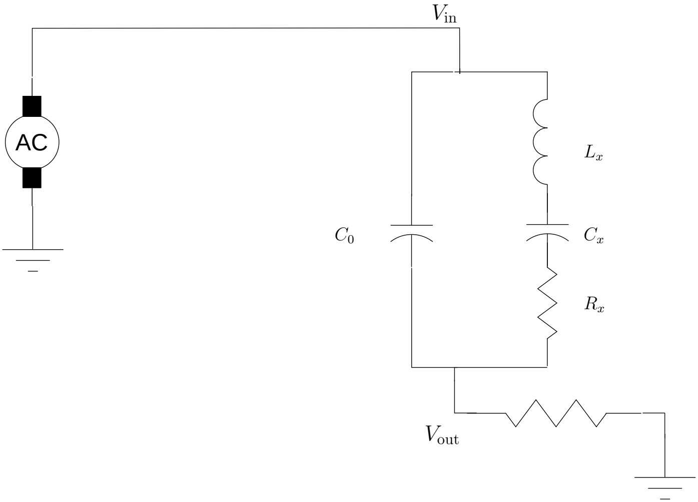
"Calling Dick Tracy!"

- Old dream: a Dick Trace watch phone!
- New dream: long battery life for smart phones

Example Resonant System



Example Resonant System



The Designer's Dream

Ideally, would like

- Simple models for behavioral simulation
- Interpretable degrees of freedom
- Including all relevant physics
- Parameterized for design optimization
- With reasonably fast and accurate set-up
- Backed by error analysis

We aren't there yet.

The Hero of the Hour

Major theme: use problem structure for better models

- Algebraic
  - Structure of ODEs (e.g. second-order structure)
  - Structure of matrices (e.g. complex symmetry)
- Analytic
  - Perturbations of physics (thermoelastic damping)
  - Perturbations of geometry (near axisymmetry)
  - Perturbations of boundary conditions (clamping)
- Geometric
  - Simplified models: planar motion, axisymmetry, ...
  - Substructures

SOAR and ODE structure

Damped second-order system:

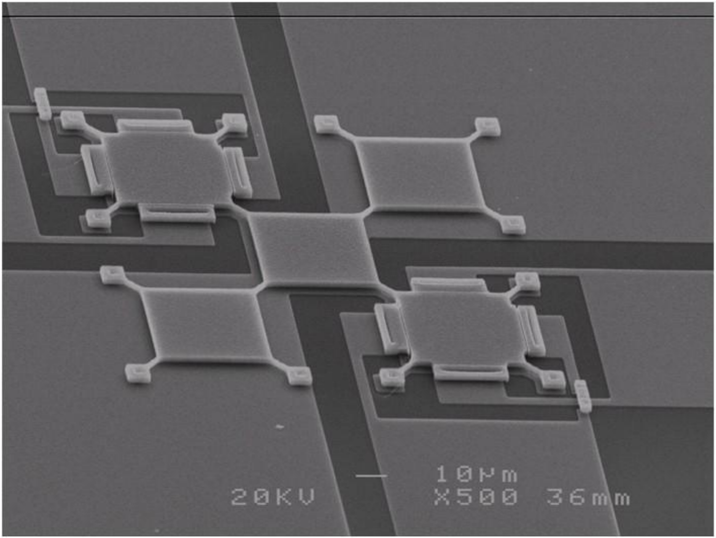
$$\begin{aligned} Mu'' + Cu' + Ku &= P\phi \\ y &= V^T u. \end{aligned}$$

Projection basis Q_n with Second Order ARnoldi (SOAR):

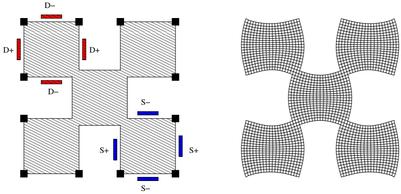
$$\begin{aligned} M_n u_n'' + C_n u_n' + K_n u_n &= P_n \phi \\ y &= V_n^T u \end{aligned}$$

where $P_n = Q_n^T P$, $V_n = Q_n^T V$, $M_n = Q_n^T M Q_n, \ldots$

Checkerboard Resonator



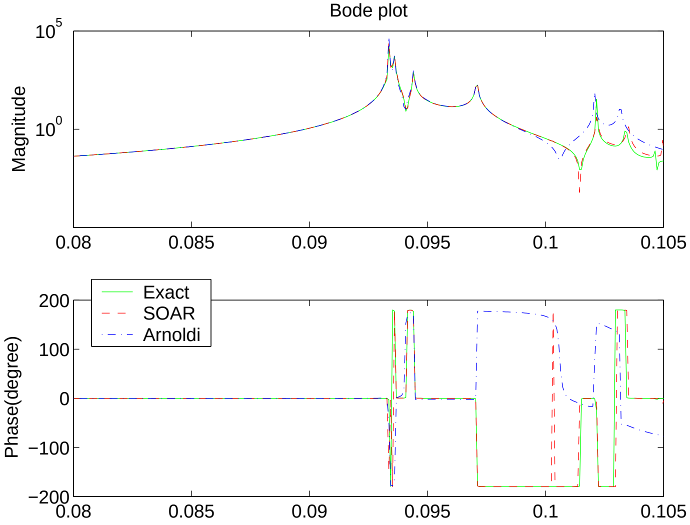
Checkerboard Resonator



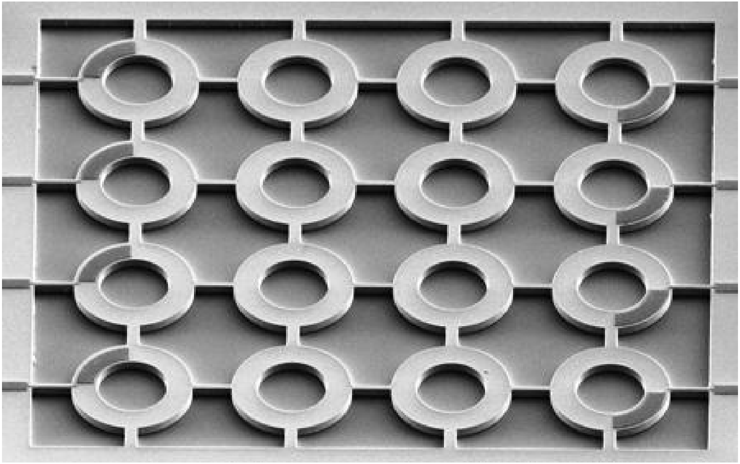
- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers

Performance of SOAR vs Arnoldi

$$N = 2154 \to n = 80$$



Aside: Next generation



Complex Symmetry

Model with radiation damping (PML) gives complex problem:

$$(K - \omega^2 M)u = f, \text{ where } K = K^T, M = M^T$$

Forced solution u is a stationary point of

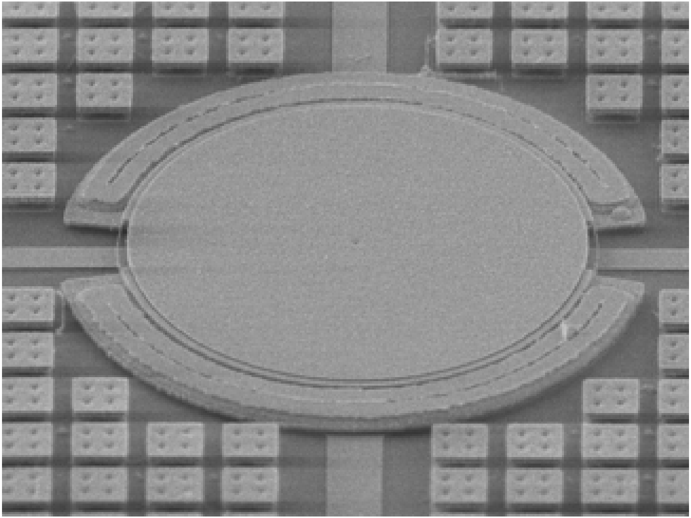
$$I(u) = \frac{1}{2}u^T(K - \omega^2 M)u - u^T f.$$

Eigenvalues of (K, M) are stationary points of

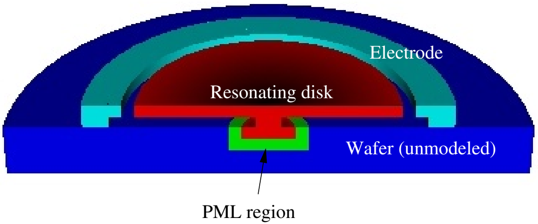
$$\rho(u) = \frac{u^T K u}{u^T M u}$$

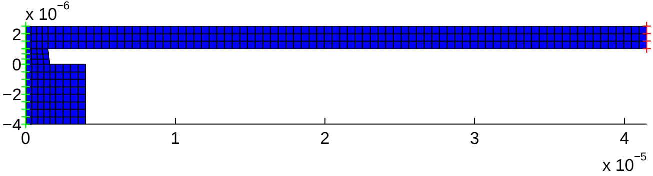
First-order accurate vectors $\implies$
second-order accurate eigenvalues.

Disk Resonator Simulations



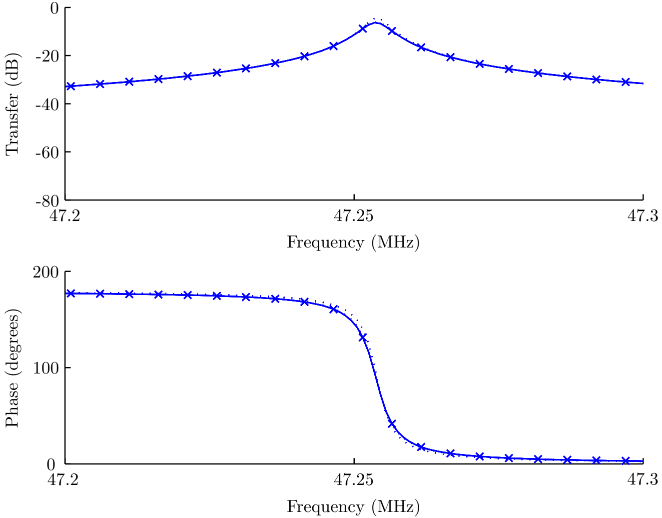
Disk Resonator Mesh





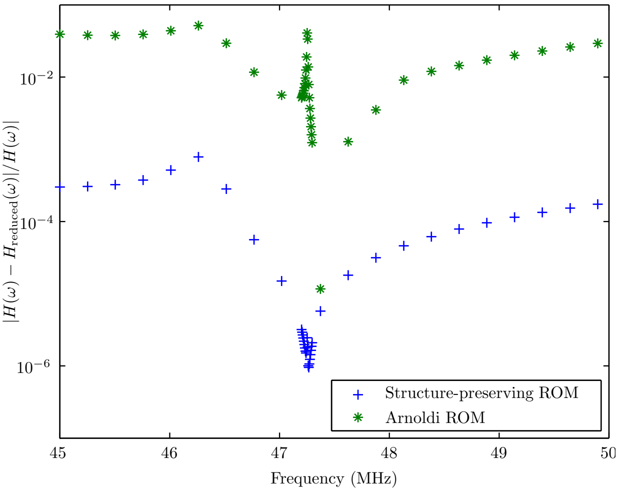
- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

Symmetric ROM Accuracy



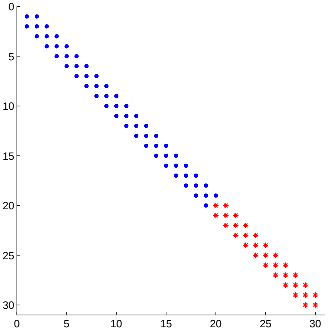
Results from ROM (solid and dotted lines) near indistinguishable from full model (crosses)

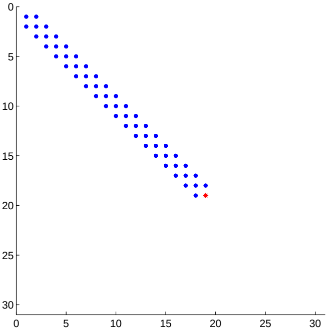
Symmetric ROM Accuracy



Preserve structure $\Longrightarrow$
get twice the correct digits

Aside: Model Expansion?





PML adds variables so that the Schur complement

$$\hat{A}(k)\psi_1 = \left(K_{11} - k^2 M_{11} - \hat{C}(k)\right)\psi_1 = 0$$

has a term $\hat{C}(k)$ to approximate a radiation boundary condition.

Perturbative Structure

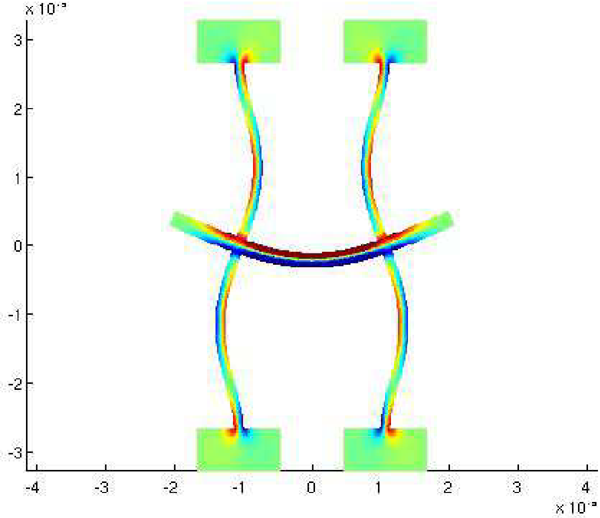
Dimensionless continuum equations for thermoelastic damping:

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ \ddot{u} &= \nabla \cdot \sigma \\ \dot{\theta} &= \eta\nabla^2\theta - \mathrm{tr}(\dot{\epsilon}) \end{aligned}$$

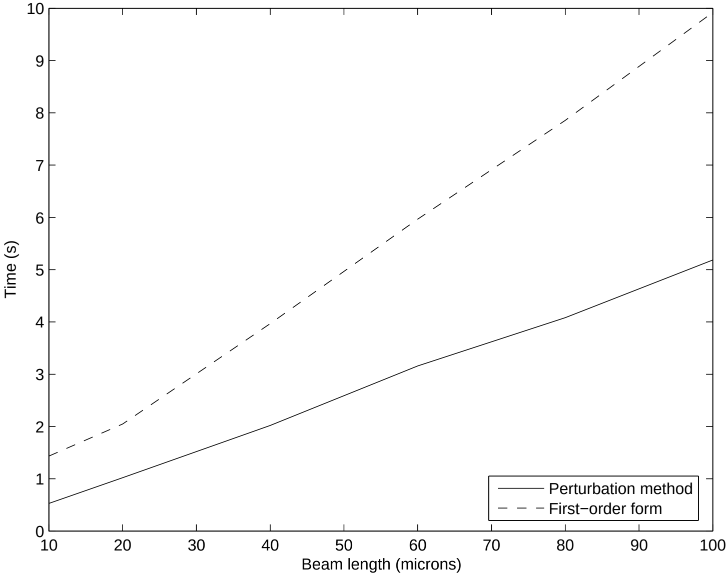
Dimensionless coupling ξ and heat diffusivity η are $10^{-4} \implies$ perturbation method (about $\xi = 0$).

Large, non-self-adjoint, first-order coupled problem $\rightarrow$ Smaller, self-adjoint, mechanical eigenproblem + symmetric linear solve.

Thermoelastic Damping Example



Performance for Beam Example



Aside: Effect of Nondimensionalization

100 μm beam example, first-order form.

Before nondimensionalization

- Time: 180 s
- $\mathrm{nnz}(L) = 11\mathrm{M}$

After nondimensionalization

- Time: 10 s
- $\mathrm{nnz}(L) = 380\mathrm{K}$

Semi-Analytical Model Reduction

We work with hand-build model reduction all the time!

- Circuit elements: Maxwell equation + field assumptions
- Beam theory: Elasticity + kinematic assumptions
- Axisymmetry: 3D problem + kinematic assumption

Idea: Provide *global shapes*

- User defines shapes through a callback
- Mesh serves defines a quadrature rule
- *Reduced equations fit known abstractions*

Global Shape Functions

Normally:

$$u(X) = \sum_j N_j(X)\hat{u}_j$$

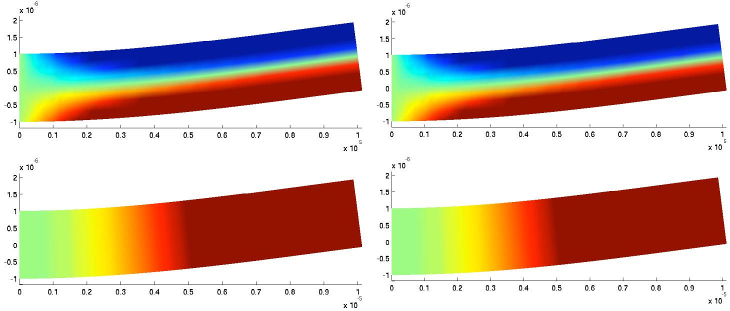
Global shape functions:

$$\hat{u} = \hat{u}^l + G(\hat{u}^g)$$

Then constrain values of some components of $\hat{u}^l$, $\hat{u}^g$.

"Hello, World!"

Which mode shape comes from the reduced model (3 dof)?



(Left: 28 MHz; Right: 31 MHz)

Latest widgets

- This is a gyroscope!
- HRG is widely used
- What about MEMS?

Simplest model

Two degree of freedom model:

$$m \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \Omega g \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + k \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = f.$$

- Multiple eigenvalues when at rest (symmetry)
- Rotation splits the eigenvalues — get a beat frequency
- Want dynamics of energy transfer between two modes
- Except there are more than two modes!

Structured models for wineglass gyros

We want to understand everything together!

- Axisymmetry is critical
  - Need to understand manufacturing defects!
  - Robustness through design and post-processing
- Low damping is critical
  - Need thermoelastic effects (perturbation)
  - Need coupling to substrate (??)
- Need optical-mechanical coupling for drive/sense

The moral of the preceding:

- Bad idea: 3D model in ANSYS + model reduction
- Better idea: Do some reduction by hand first!

Conclusions

Essentially, all models are wrong, but some are useful.
– George Box

Questions?

`http://www.cs.cornell.edu/~bindel`