Structure-preserving model reduction for MEMS modeling

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Resonant MEMS

Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Favorite application: radio on chip
- Close second: really high-pitch guitars
Your cell phone has many moving parts!
What if we replace them with integrated MEMS?
Ultimate Success

“Calling Dick Tracy!”

- Old dream: a Dick Trace watch phone!
- New dream: long battery life for smart phones
Example Resonant System

$V_{in}$

$V_{out}$
Example Resonant System
The Designer’s Dream

Ideally, would like

- Simple models for behavioral simulation
- Parameterized for design optimization
- Including all relevant physics
- With reasonably fast and accurate set-up
- Backed by error analysis

We aren’t there yet.
The Hero of the Hour

Major theme: use problem structure for better models

▶ Algebraic
  ▶ Structure of ODEs (e.g. second-order structure)
  ▶ Structure of matrices (e.g. complex symmetry)

▶ Analytic
  ▶ Perturbations of physics (thermoelastic damping)
  ▶ Perturbations of geometry (near axisymmetry)
  ▶ Perturbations of boundary conditions (clamping)

▶ Geometric
  ▶ Simplified models: planar motion, axisymmetry, ...
  ▶ Substructures
Damped second-order system:

\[ Mu'' + Cu' + Ku = P\phi \]
\[ y = V^T u. \]

Projection basis \( Q_n \) with Second Order ARnoldi (SOAR):

\[ M_n u_n'' + C_n u_n' + K_n u_n = P_n\phi \]
\[ y = V_n^T u \]

where \( P_n = Q_n^T P \), \( V_n = Q_n^T V \), \( M_n = Q_n^T M Q_n \), \ldots
Checkerboard Resonator
Checkerboard Resonator

- Anchored at outside corners
- Excited at **northwest** corner
- Sensed at **southeast** corner
- Surfaces move only a few nanometers
Performance of SOAR vs Arnoldi

\[ N = 2154 \rightarrow n = 80 \]

Bode plot

- Magnitude
- Phase (degree)

- Exact
- SOAR
- Arnoldi
Aside: Next generation
Complex Symmetry

Model with radiation damping (PML) gives complex problem:

\[(K - \omega^2 M)u = f\], where \[K = K^T, M = M^T\]

Forced solution \(u\) is a stationary point of

\[I(u) = \frac{1}{2} u^T (K - \omega^2 M) u - u^T f.\]

Eigenvalues of \((K, M)\) are stationary points of

\[\rho(u) = \frac{u^T Ku}{u^T Mu}\]

First-order accurate vectors \(\implies\) second-order accurate eigenvalues.
Disk Resonator Simulations
Disk Resonator Mesh

- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation
Symmetric ROM Accuracy

Results from ROM (solid and dotted lines) near indistinguishable from full model (crosses)
Symmetric ROM Accuracy

| Frequency (MHz) | \(|H(\omega) - H_{\text{reduced}}(\omega)|/|H(\omega)|\) |
|----------------|--------------------------------------------------|
| 45             | 10^{-6}                                          |
| 46             | 10^{-4}                                          |
| 47             | 10^{-2}                                          |
| 48             | 10^{-6}                                          |
| 49             | 10^{-4}                                          |
| 50             | 10^{-2}                                          |

Preserve structure \(\implies\) get twice the correct digits
Perturbative Structure

Dimensionless continuum equations for thermoelastic damping:

\[
\sigma = \hat{C}\varepsilon - \xi\theta 1 \\
\ddot{u} = \nabla \cdot \sigma \\
\dot{\theta} = \eta \nabla^2 \theta - \text{tr}(\dot{\varepsilon})
\]

Dimensionless coupling $\xi$ and heat diffusivity $\eta$ are $10^{-4} \implies$ perturbation method (about $\xi = 0$).

Large, non-self-adjoint, first-order coupled problem $\rightarrow$
Smaller, self-adjoint, mechanical eigenproblem + symmetric linear solve.
Thermoelastic Damping Example
Aside: Effect of Nondimensionalization

100 $\mu m$ beam example, first-order form.

Before nondimensionalization

- Time: 180 s
- $\text{nnz}(L) = 11\text{M}$

After nondimensionalization

- Time: 10 s
- $\text{nnz}(L) = 380\text{K}$
Semi-Analytical Model Reduction

We work with hand-build model reduction all the time!

- Circuit elements: Maxwell equation + field assumptions
- Beam theory: Elasticity + kinematic assumptions
- Axisymmetry: 3D problem + kinematic assumption

Idea: Provide *global shapes*

- User defines shapes through a callback
- Mesh serves defines a quadrature rule
- Reduced equations fit known abstractions
Global Shape Functions

Normally:

\[ u(X) = \sum_{j} N_j(X) \hat{u}_j \]

Global shape functions:

\[ \hat{u} = \hat{u}^l + G(\hat{u}^g) \]

Then constrain values of some components of \( \hat{u}^l, \hat{u}^g \).
Which mode shape comes from the reduced model (3 dof)?

(Left: 28 MHz; Right: 31 MHz)
The latest

http://www.cs.cornell.edu/~bindel