Resonances and Nonlinear Eigenvalue Problems

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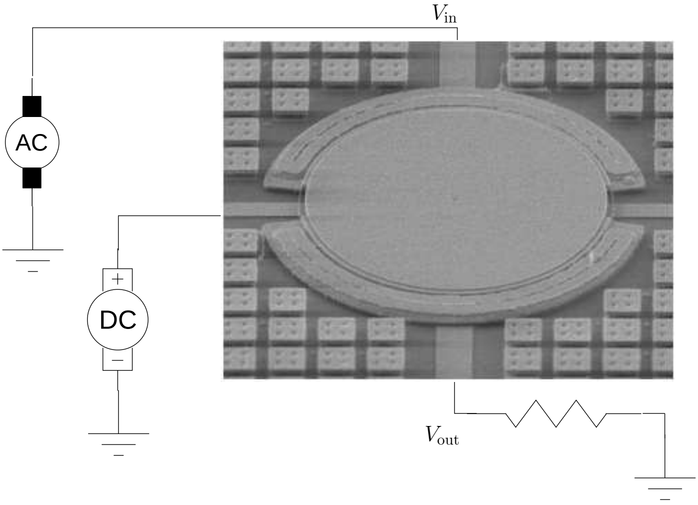
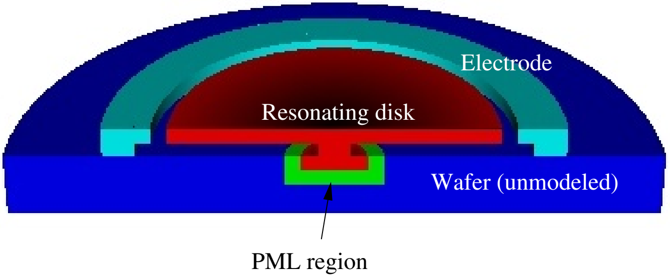
A very tiny problem



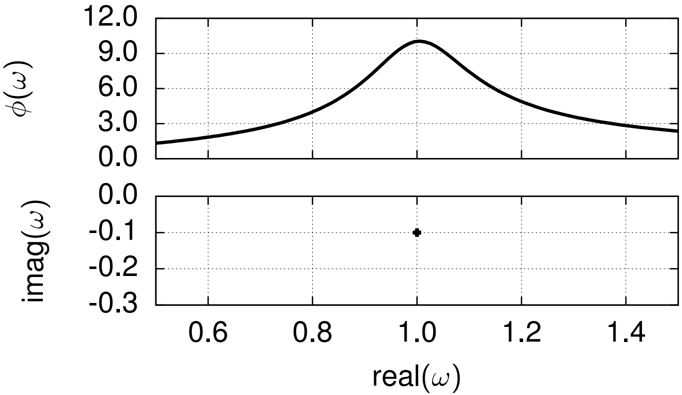
Modeling anchor losses



Finite element eigenvalue problem with absorbing layer.
Matches experimental results, gives useful predictions.

"How do you know this is right?"

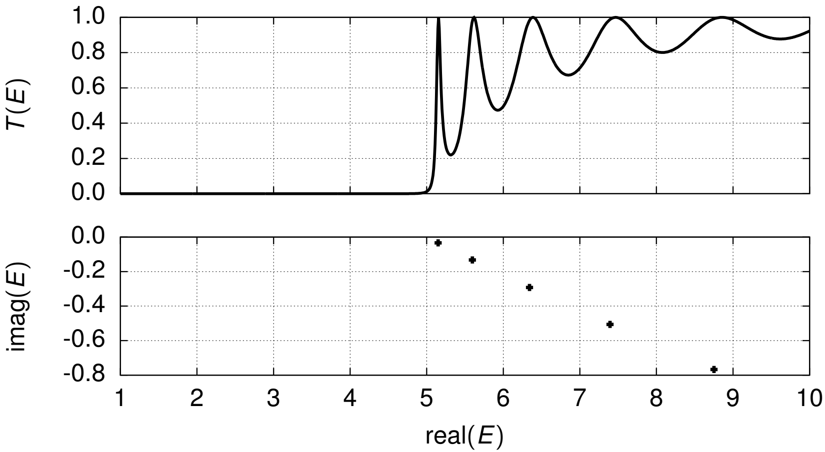
Resonances



- Closed system: steady-state analysis via eigenvalues.
- Open system: continuous spectrum, scattering states.
- Open systems with “almost” closed components?

Example: Resonances and transmission

$$\left(-\frac{d^2}{dx^2} + V(x)\right)\psi = 0, \quad V(x) = -5\,\chi_{[0,L]}(x).$$



Simple 1D Problem

Consider 1D Schrödinger:

$$\left(-\frac{d^2}{dx^2} + V(x)\right)\psi = E\psi.$$

How do we:

1. Quickly compute resonances (nice enough V)?
2. Make sure the computations are correct?

Resonance via nonlinear eigenproblems

$$\left(-\frac{d^2}{dx^2} + V(x)\right)\psi = E\psi.$$

If $\operatorname{supp}(V) \subset [a, b]$, write

$$\begin{aligned}\left(-\frac{d^2}{dx^2} + V(x) - k^2\right)\psi &= 0, x \in (a, b)\\ \left(\frac{d}{dx} - ik\right)\psi &= 0, x = b\\ \left(\frac{d}{dx} + ik\right)\psi &= 0, x = a\end{aligned}$$

$E = k^2$, $\operatorname{Im} k \geq 0$ for eigenvalues, $\operatorname{Im} k < 0$ for resonances.

Problem is *nonlinear* in E or k.

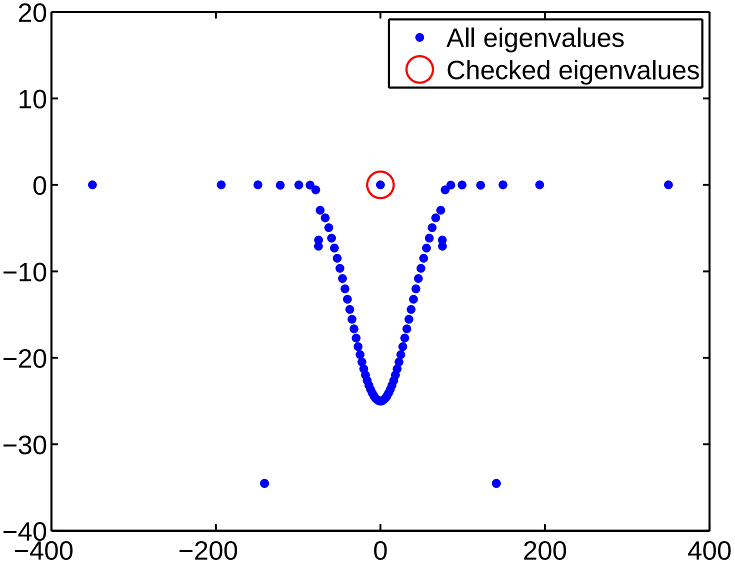
Pseudospectral discretization

Sample ψ at Chebyshev nodes and approximate $d\psi/dx$ by differentiating the interpolant:

$$\left(-D^2 + V(x) - k^2\right)\psi = 0, x \in (a, b)$$
$$(D - ik)\psi = 0, x = b$$
$$(D + ik)\psi = 0, x = a$$

Now linearize (introduce auxiliary variable $\phi = k\psi$) to get an ordinary generalized eigenvalue problem.

Is it that easy?



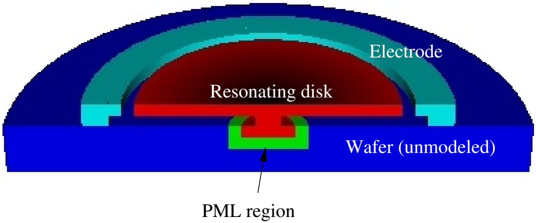
Backward error analysis

1. If $(\hat{\psi}, \hat{E})$ is a numerical solution with above scheme, then there is some $\hat{V}$ s.t. for $x \in (a, b)$,

$$(H_{\hat{V}} - \hat{E})\hat{\psi} = \left(-\frac{d^2}{dx^2} + \hat{V}(x) - \hat{E}\right)\hat{\psi} = 0$$

   together with corresponding radiation conditions.
2. Estimate $\hat{V}$ explicitly by remapping residual to finer mesh
3. Original problem is a perturbation of computed problem.
4. Use first-order perturbation theory to correct $\hat{E}$.
   Useful to take a *variational* approach.

But wait — there's more!



- Exact DtN boundary conditions usually aren't so simple.
- Approximate to compute — local error analysis?
- Care about all resonances in a region – global bounds?
- What NA for eigenvalues carries over to resonances?

"How do you know this is right?"

Questions?

http://www.cs.cornell.edu/~bindel/