Designing Linear Algebra for Multicore

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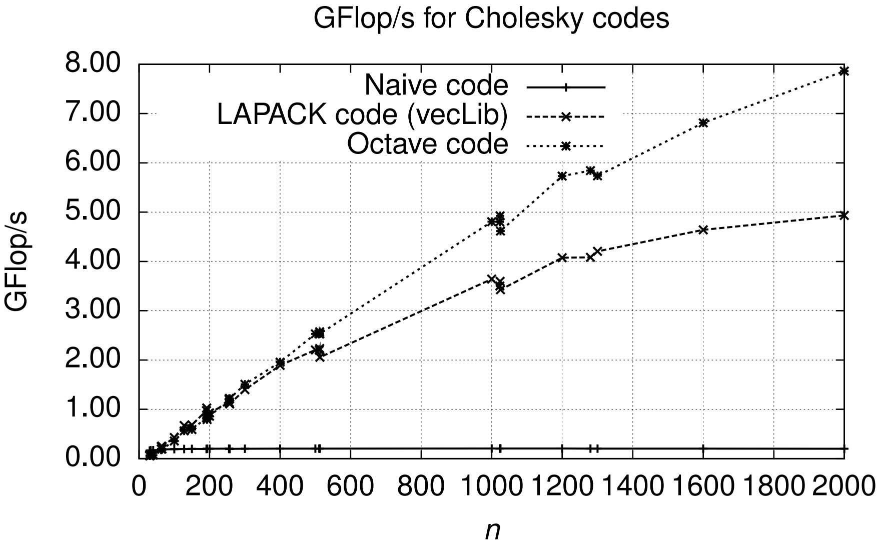
Illustrative example

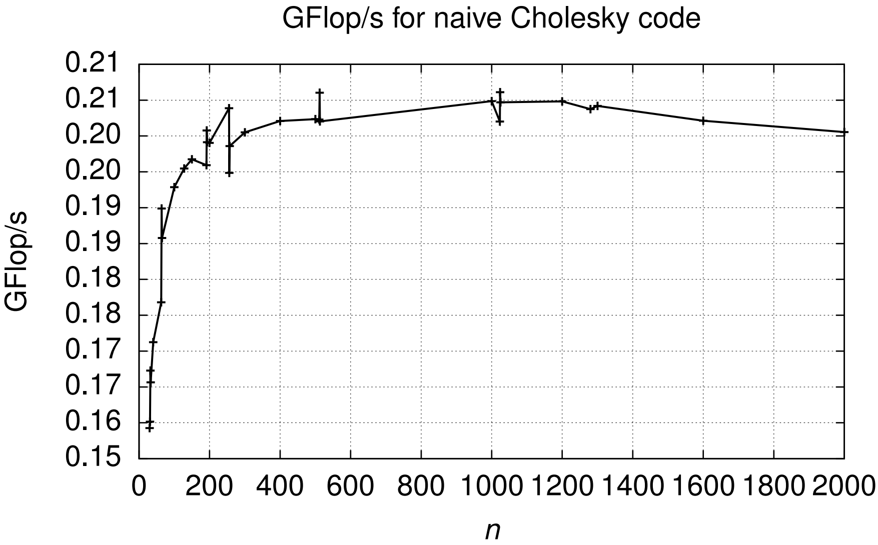
Consider Cholesky factorization: $A = LL^T$.

- Useful for solving linear SPD systems
- Straightforward algorithm, $n^2/2$ data, $n^3/3$ floating point ops
- Given as a programming exercise for my scientific computing class
- Also: compare to LAPACK / MATLAB / Octave

Timing for three versions of Cholesky



Timing for naive Cholesky



Question

Can one rationally focus on parallelism and claim that the serial performance is a secondary issue?

Issue: Memory communication

- Naive code is limited by memory latency (almost constant cache misses)
- On this machine, about 50$\times$ difference between L1 hit and main memory access.
- LAPACK uses blocking – most of the computation is shifted onto level 3 BLAS operations (i.e. matrix-matrix multiplies) which are tuned for cache efficiency.
- There are projects for automatically tuning such computational kernels (FFTW, FLAME, ATLAS, OSKI, etc).

Algorithmic tradeoffs

- Often have *mathematically* equivalent reorganizations of LA algorithms (i.e. same formulas, different arrangement)
  - *Not* equivalent with respect to cache!
  - *Not* equivalent with respect to communication!
  - *Not* equivalent with respect to approximate arithmetic!
- Also have arrangements that compute the same thing through different formulations.

Algorithm strategies

For *direct* (dense or sparse direct) linear algebra:

- Blocking to avoid memory overhead
- Redundant computations to avoid communication
- Randomization to avoid need for pivoting
- Iterative refinement to clean up fast-but-sloppy computations

For *iterative* linear algebra:

- Trading time to precondition against iteration count
- Redundant computations to avoid communication

Shameless theft!

"Lesser artists borrow. Great artists steal."
– Picasso, Dali, Stravinsky?

Demmel: "Avoiding Communication in Linear Algebra."
`http://www.stanford.edu/group/mmds/slides2008/demmel.pdf`

Programmer's task

Find an algorithm that

1. Yields the right answer.
2. Uses structure to minimize computation.
3. Is organized to minimize memory / communication burdens.

How languages can help

For novices:

- Make it easy to build on work of experts!

For experts:

- Make it “easy” to write cache-aware algorithms.
- Make it possible to tune those algorithms.
- Make it possible to express tradeoffs (e.g. scoped optimizations like “license associativity”).

Unfortunately, line between “novice” and “expert” is unclear.