Numerical and semi-analytical structure-preserving model reduction for MEMS

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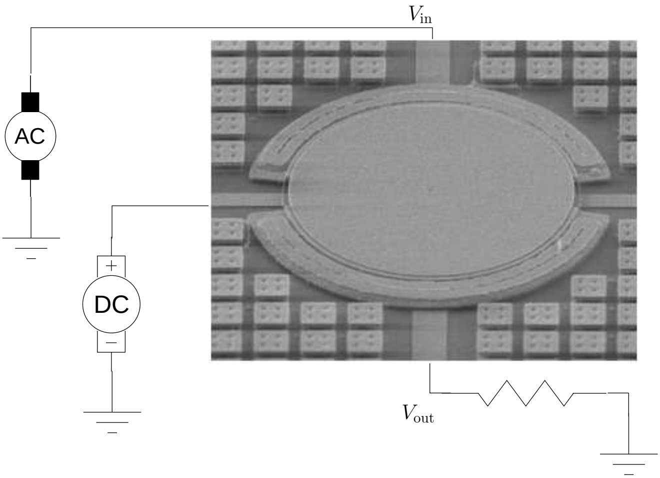
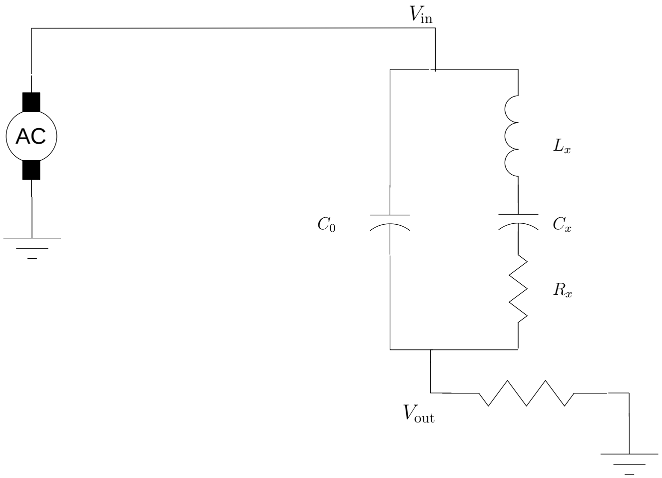
Example Resonant System



Example Resonant System



The Dream

Ideally, would like

- Simple models for behavioral simulation
- Parameterized for design optimization
- Including all relevant physics
- With reasonably fast and accurate set-up

We aren't there yet.

Major theme: use problem structure for better models

Complex Symmetry

Model with radiation damping (PML) gives complex problem:

$$(K - \omega^2 M)u = f, \text{ where } K = K^T, M = M^T$$

Forced solution u is a stationary point of

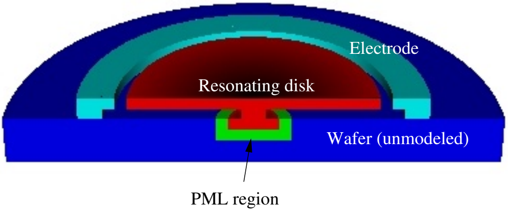
$$I(u) = \frac{1}{2}u^T(K - \omega^2 M)u - u^T f.$$

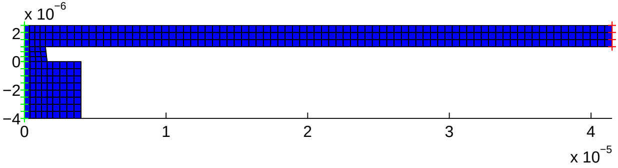
Eigenvalues of (K, M) are stationary points of

$$\rho(u) = \frac{u^T K u}{u^T M u}$$

First-order accurate vectors $\implies$
second-order accurate eigenvalues.

Disk Resonator Mesh





- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

Symmetric ROM Accuracy

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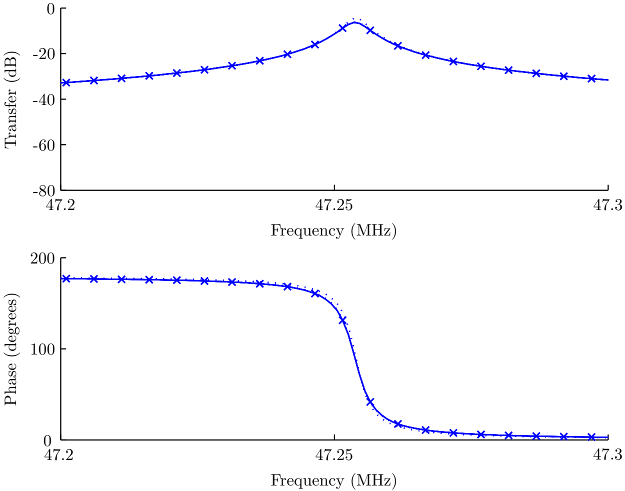
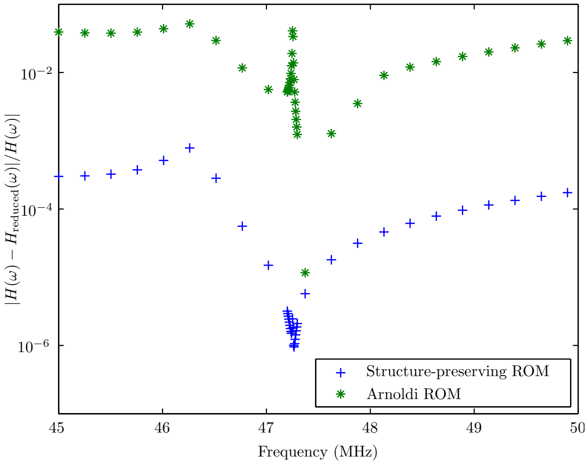


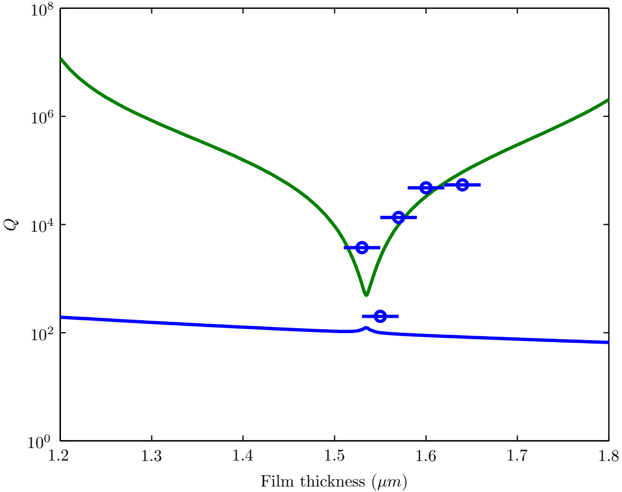
Results from ROM (solid and dotted lines) near indistinguishable from full model (crosses)

Symmetric ROM Accuracy



Preserve structure $\Longrightarrow$
get twice the correct digits

Variation in Quality of Resonance



Simulation and lab measurements vs. disk thickness

Perturbative Structure

Dimensionless continuum equations for thermoelastic damping:

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ \ddot{u} &= \nabla \cdot \sigma \\ \dot{\theta} &= \eta\nabla^2\theta - \text{tr}(\dot{\epsilon}) \end{aligned}$$

Discrete equations:

$$\begin{aligned} M_{uu}\ddot{u} + K_{uu}u &= \xi K_{u\theta}\theta + f \\ C_{\theta\theta}\ddot{\theta} + \eta K_{\theta\theta}\theta &= -C_{\theta u}\dot{u} \end{aligned}$$

Dimensionless coupling ξ and heat diffusivity η are 10^{-4} $\Longrightarrow$ perturbation method (about $\xi = 0$).

Perturbative Mode Calculation

Discretized mode equation:

$$\begin{aligned} (-\omega^2 M_{uu} + K_{uu})u &= \xi K_{u\theta}\theta \\ (i\omega C_{\theta\theta} + \eta K_{\theta\theta})\theta &= -i\omega C_{\theta u} u \end{aligned}$$

First approximation about $\xi = 0$:

$$\begin{aligned} (-\omega_0^2 M_{uu} + K_{uu})u_0 &= 0 \\ (i\omega_0 C_{\theta\theta} + \eta K_{\theta\theta})\theta_0 &= -i\omega_0 C_{\theta u} u_0 \end{aligned}$$

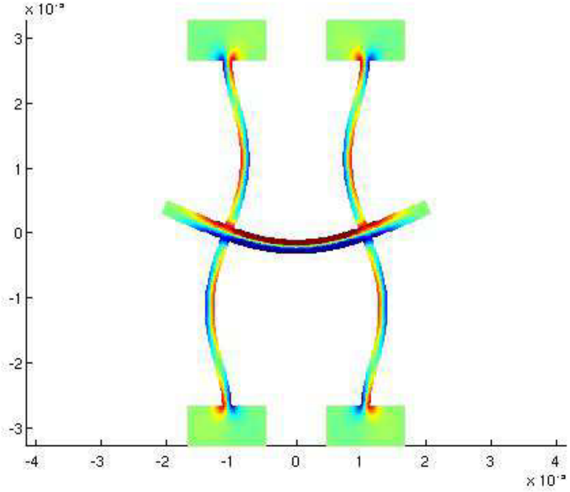
First-order correction in ξ:

$$-\delta(\omega^2) M_{uu} u_0 + (-\omega_0^2 M_{uu} + K_{uu})\delta u = \xi K_{u\theta}\theta_0$$

Multiply by u_0^T:

$$\delta(\omega^2) = -\xi \left(\frac{u_0^T K_{u\theta}\theta_0}{u_0^T M_{uu} u_0} \right)$$

Thermoelastic Damping Example



Semi-Analytical Model Reduction

We work with hand-build model reduction all the time!

- Circuit elements: Maxwell equation + field assumptions
- Beam theory: Elasticity + kinematic assumptions
- Axisymmetry: 3D problem + kinematic assumption

Idea: Provide *global shapes*

- User defines shapes through a callback
- Mesh serves defines a quadrature rule
- Reduced equations fit known abstractions

Global Shape Functions

Normally:

$$u(X) = \sum_j N_j(X)\hat{u}_j$$

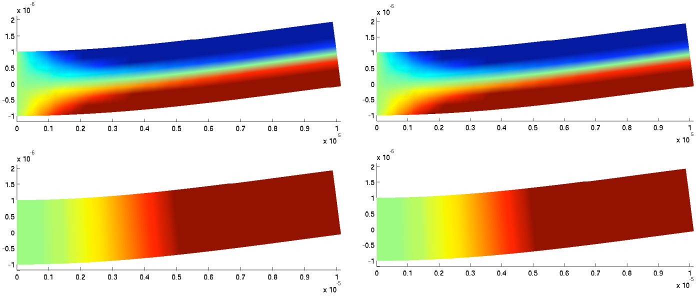
Global shape functions:

$$\hat{u} = \hat{u}^l + G(\hat{u}^g)$$

Then constrain values of some components of $\hat{u}^l$, $\hat{u}^g$.

“Hello, World!”

Which mode shape comes from the reduced model (3 dof)?



(Left: 28 MHz; Right: 31 MHz)

Conclusions

Respecting problem structure is a Good Thing!

- ODE structure
- Complex symmetric structure
- Perturbative structure
- Geometric structure

Result:
Better accuracy, faster set-up, better understanding.