Modeling Resonant Microsystems

Toward Cell Phones on a Chip?

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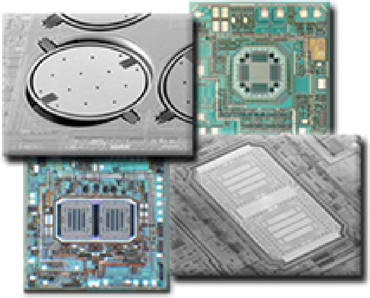
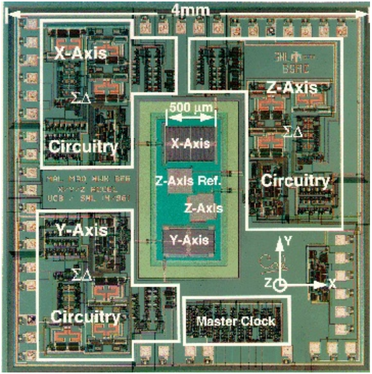
What are MEMS?

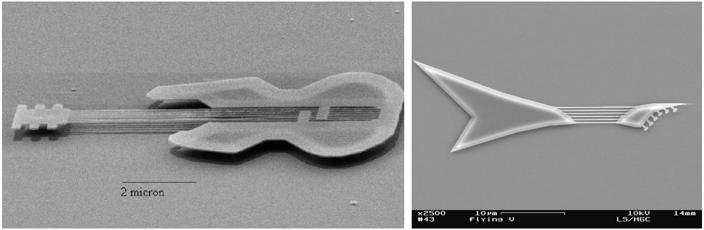




MEMS Basics

- Micro-Electro-Mechanical Systems
    - Chemical, fluid, thermal, optical (MECFTOMS?)
- Applications:
    - Sensors (inertial, chemical, pressure)
    - Ink jet printers, biolab chips
    - Radio devices: cell phones, inventory tags, pico radio
- Use integrated circuit (IC) fabrication technology
- Tiny, but still classical physics

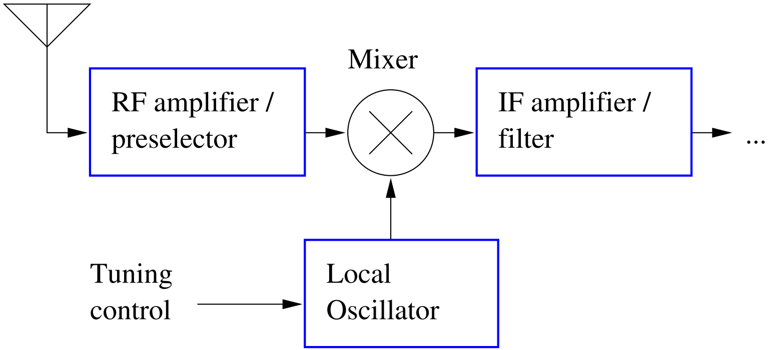
Resonant MEMS



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Favorite application: radio on chip
- Close second: really high-pitch guitars

The Mechanical Cell Phone

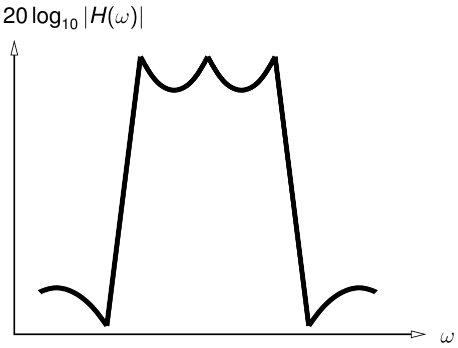


- Your cell phone has many moving parts!
- What if we replace them with integrated MEMS?

Ultimate Success

"Calling Dick Tracy!"

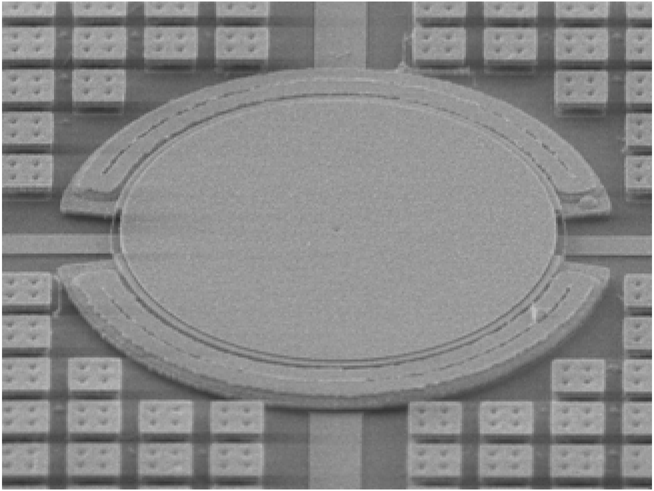
Narrowband Filter Needs



Want building blocks with:

- High frequency
- Low damping
- Tunability

Disk Resonator



Disk Resonator

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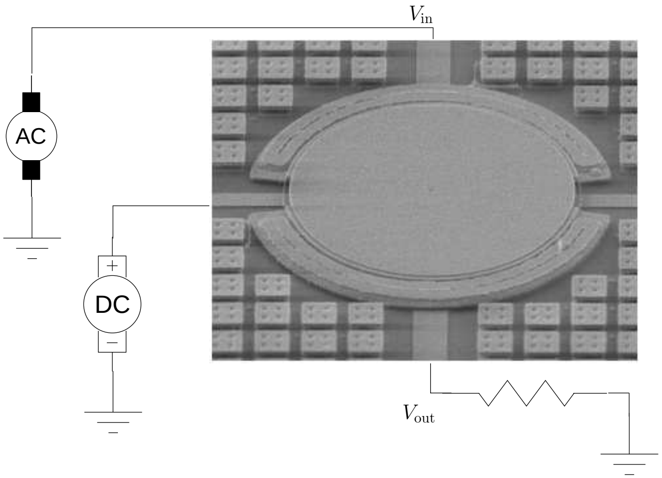
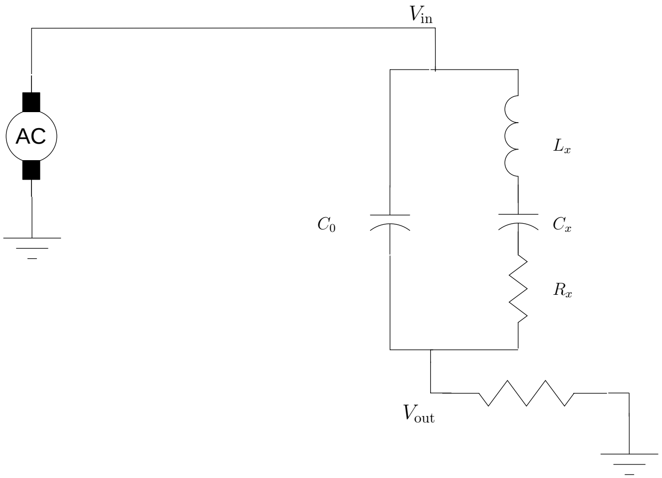


Disk Resonator



Electromechanical Model

Kirchoff's current law and balance of linear momentum:

$$
\begin{aligned}
\frac{d}{dt}(C(u)V) + GV &= I_{\text{external}} \\
Mu_{tt} + Ku - \nabla_u \left(\frac{1}{2} V^* C(u_0) V \right) &= F_{\text{external}}
\end{aligned}
$$

Linearize about static equilibium (V_0, u_0):

$$
\begin{aligned}
C(u_0)\,\delta V_t + G\,\delta V + (\nabla_u C(u_0) \cdot \delta u_t)\, V_0 &= \delta I_{\text{external}} \\
M\,\delta u_{tt} + \tilde{K}\,\delta u + \nabla_u (V_0^* C(u_0)\,\delta V) &= \delta F_{\text{external}}
\end{aligned}
$$

where

$$
\tilde{K} = K - \frac{1}{2} \frac{\partial^2}{\partial u^2} (V_0^* C(u_0) V_0)
$$

Electromechanical Model

Assume time-harmonic steady state, no external forces:

$$\begin{bmatrix} i\omega C + G & i\omega B \\ -B^T & \tilde{K} - \omega^2 M \end{bmatrix} \begin{bmatrix} \delta \hat{V} \\ \delta \hat{u} \end{bmatrix} = \begin{bmatrix} \delta \hat{I}_{\text{external}} \\ 0 \end{bmatrix}$$

Eliminate the mechanical terms:

$$\left(i\omega C + G + i\omega B^T (\tilde{K} - \omega^2 M)^{-1} B \right) \delta \hat{V} = \delta \hat{I}_{\text{external}}$$

Give a name to the coupling transfer function:

$$H(\omega) = B^T (\tilde{K} - \omega^2 M)^{-1} B$$

Goal: Understand electromechanical piece ($i\omega H(\omega)$).

- As a function of geometry and operating point
- Preferably as a simple circuit

Damping and Q

Designers want high *quality of resonance* (Q)

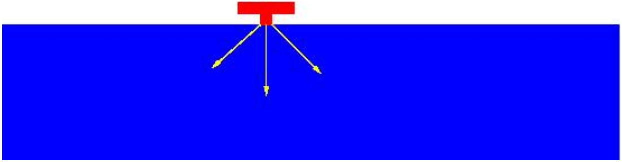
- Dimensionless damping in a one-dof system

$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

- For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{|\omega|}{2\,\mathrm{Im}(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

Damping Mechanisms



Possible loss mechanisms:

- Fluid damping
- Material losses
- Thermoelastic damping
- Anchor loss

Model substrate as semi-infinite with a

Perfectly Matched Layer (PML).

Perfectly Matched Layers

- Complex coordinate transformation
- Generates a “perfectly matched” absorbing layer
- Idea works with general linear wave equations
  - Electromagnetics (Berengér, 1994)
  - Quantum mechanics – *exterior complex scaling* (Simon, 1979)
  - Elasticity in standard finite element framework (Basu and Chopra, 2003)

Model Problem

- Domain: $x \in [0, \infty)$
- Governing eq:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

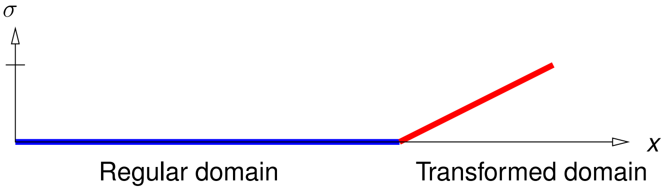
- Fourier transform:

$$\frac{d^2 \hat{u}}{dx^2} + k^2 \hat{u} = 0$$

- Solution:

$$\hat{u} = c_{\text{out}} e^{-ikx} + c_{\text{in}} e^{ikx}$$

Model with Perfectly Matched Layer

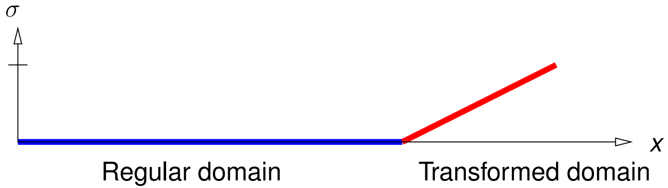


$$\frac{d\tilde{x}}{dx} = \lambda(x) \text{ where } \lambda(s) = 1 - i\sigma(s)$$

$$\frac{d^2\hat{u}}{d\tilde{x}^2} + k^2\hat{u} = 0$$

$$\hat{u} = c_{\text{out}} e^{-ik\tilde{x}} + c_{\text{in}} e^{ik\tilde{x}}$$

Model with Perfectly Matched Layer



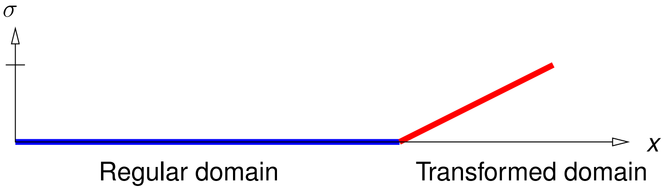
$$\frac{d\tilde{x}}{dx} = \lambda(x) \text{ where } \lambda(s) = 1 - i\sigma(s),$$

$$\frac{1}{\lambda}\frac{d}{dx}\left(\frac{1}{\lambda}\frac{d\hat{u}}{dx}\right) + k^2\hat{u} = 0$$

$$\hat{u} = c_{\text{out}}e^{-ikx-k\Sigma(x)} + c_{\text{in}}e^{ikx+k\Sigma(x)}$$

$$\Sigma(x) = \int_0^x \sigma(s)\,ds$$

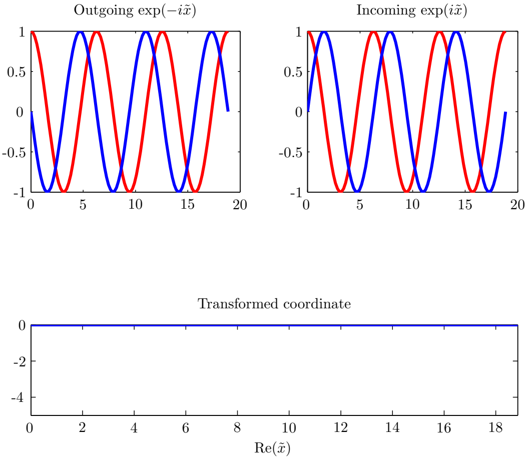
Model with Perfectly Matched Layer



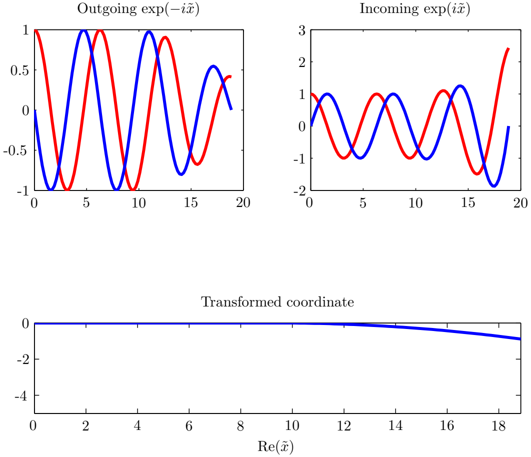
If solution clamped at $x = L$ then

$$\frac{c_{\text{in}}}{c_{\text{out}}} = O(e^{-k\gamma}) \text{ where } \gamma = \Sigma(L) = \int_0^L \sigma(s)\, ds$$

Model Problem Illustrated



Model Problem Illustrated



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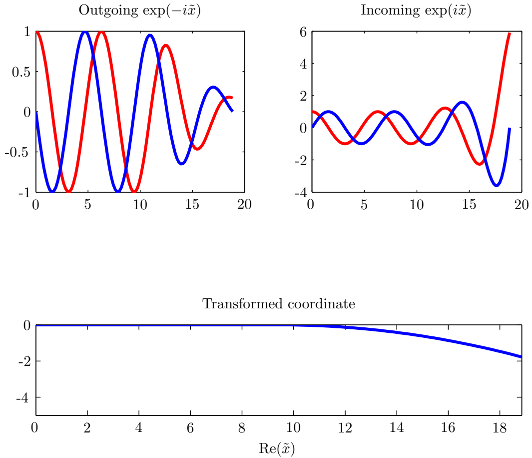
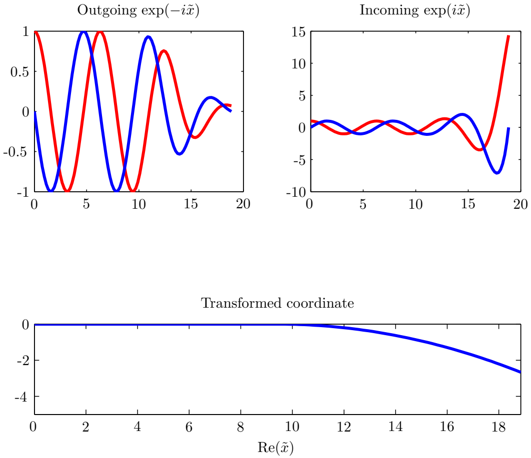


Model Problem Illustrated



Model Problem Illustrated

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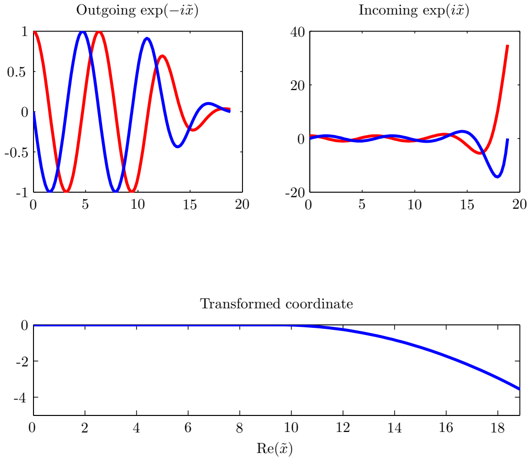
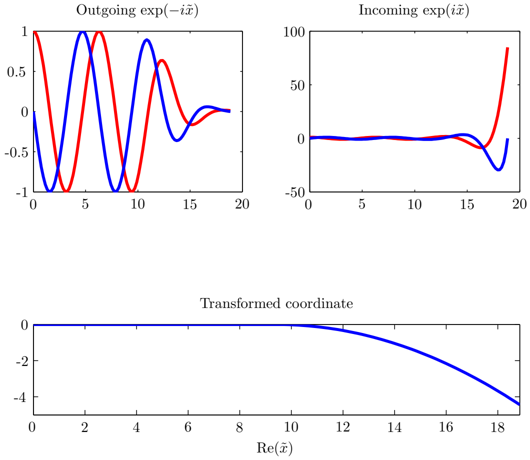
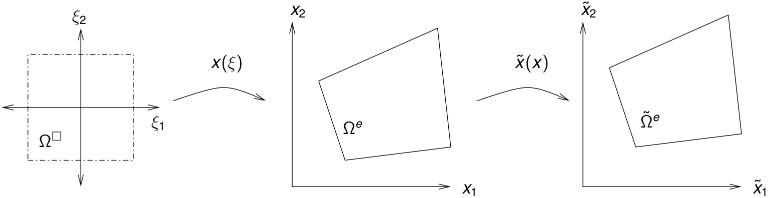


Model Problem Illustrated



Finite Element Implementation



- Combine PML and isoparametric mappings

$$\mathbf{k}^e = \int_{\Omega^\square} \tilde{\mathbf{B}}^T \mathbf{D} \tilde{\mathbf{B}} \tilde{J} \, d\Omega^\square$$

$$\mathbf{m}^e = \int_{\Omega^\square} \rho \mathbf{N}^T \mathbf{N} \tilde{J} \, d\Omega^\square$$

- Matrices are *complex symmetric*

Eigenvalues and Model Reduction

Want to know about the transfer function $H(\omega)$:

$$H(\omega) = B^T(K - \omega^2 M)^{-1} B$$

Can either

- Locate poles of H (eigenvalues of (K, M))
- Plot H in a frequency range (Bode plot)

Usual tactic: subspace projection

- Build an Arnoldi basis V
- Compute with much smaller $V^* K V$ and $V^* M V$

Can we do better?

Variational Principles

- Variational form for complex symmetric eigenproblems:
  - Hermitian (Rayleigh quotient):

$$\rho(v) = \frac{v^* K v}{v^* M v}$$

  - Complex symmetric (modified Rayleigh quotient):

$$\theta(v) = \frac{v^T K v}{v^T M v}$$

- First-order accurate eigenvectors $\implies$ Second-order accurate eigenvalues.
- Key: relation between left and right eigenvectors.

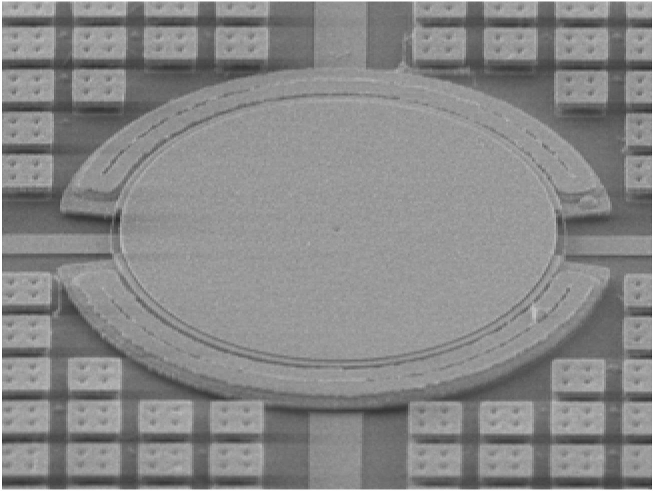
Accurate Model Reduction

- Build new projection basis from V:

$$W = \text{orth}[\text{Re}(V), \text{Im}(V)]$$

- span(W) contains both $\mathcal{K}_n$ and $\bar{\mathcal{K}}_n$
  $\Longrightarrow$ double digits correct vs. projection with V
- W is a real-valued basis
  $\Longrightarrow$ projected system is complex symmetric

Disk Resonator Simulations



Disk Resonator Mesh

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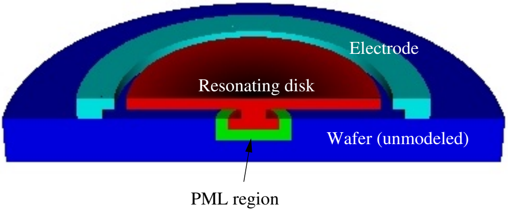
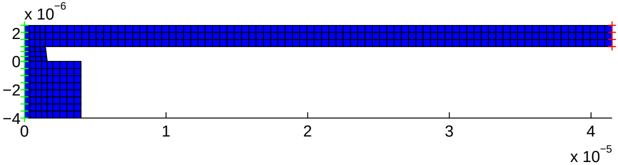
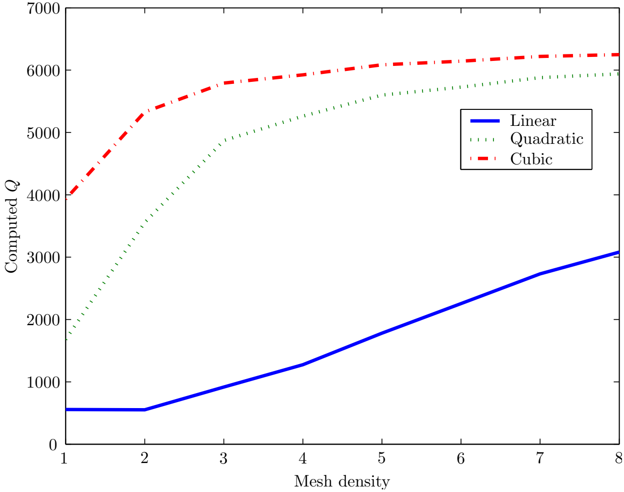




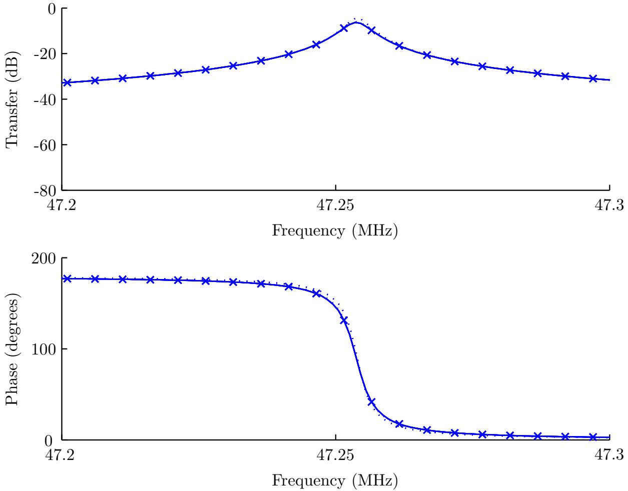
- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

Mesh Convergence



Cubic elements converge with reasonable mesh density

Model Reduction Accuracy



Results from ROM (solid and dotted lines) nearly indistinguishable from full model (crosses)

Model Reduction Accuracy

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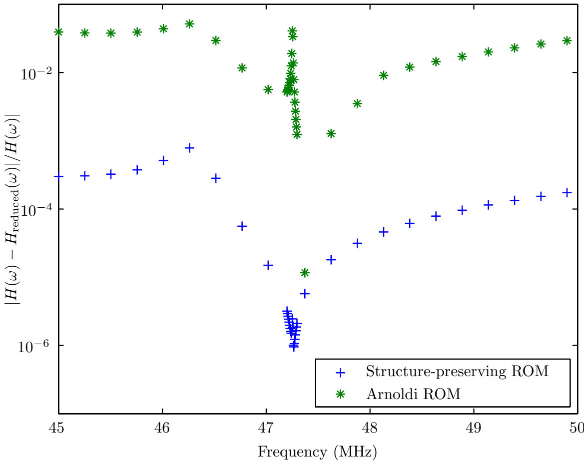
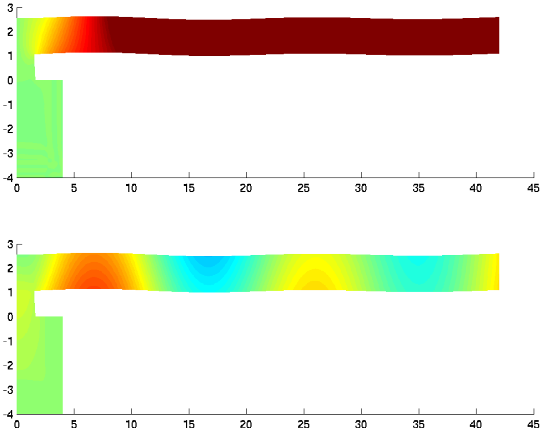


Preserve structure $\Longrightarrow$
get twice the correct digits

Response of the Disk Resonator



Variation in Quality of Resonance

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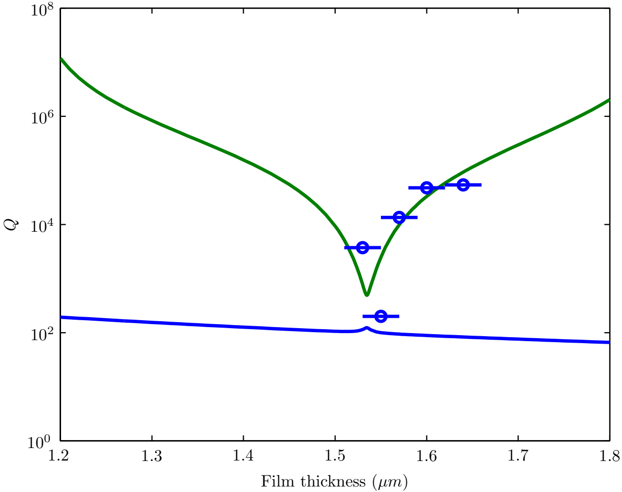

Simulation and lab measurements vs. disk thickness

Explanation of Q Variation

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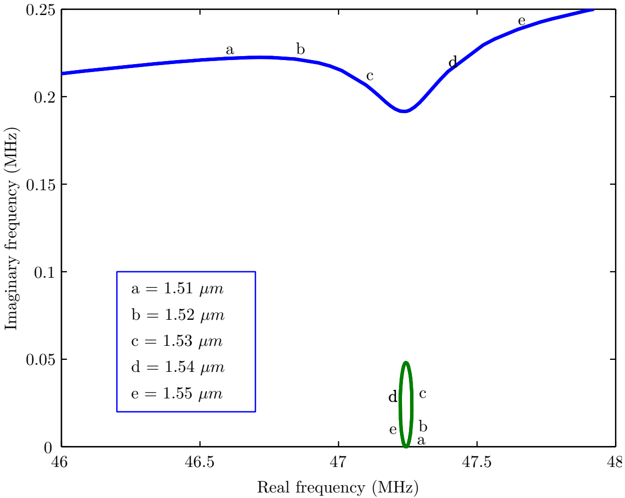

Interaction of two nearby eigenmodes

Onward!

What about:

- Modeling more geometrically complex devices?
- Modeling general dependence on geometry?
- Modeling general dependence on operating point?
- Computing nonlinear dynamics?
- Digesting all this to help designers?

Concluding Thoughts

The difference between art and science is that science is what we understand well enough to explain to a computer. Art is everything else.

Donald Knuth

The purpose of computing is insight, not numbers.

Richard Hamming