Computer-Aided Design for Micro-Electro-Mechanical Systems

Eigenvalues, Energy Losses, and Dick Tracy Watches

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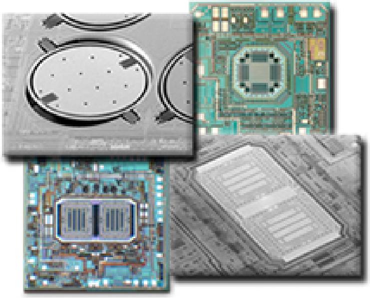
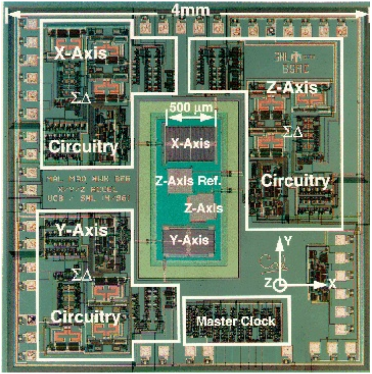
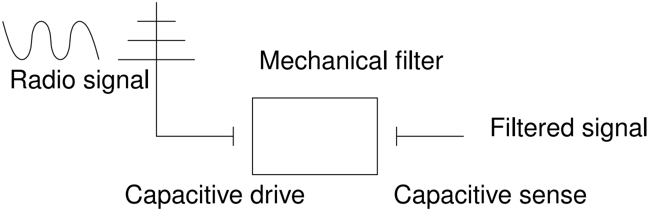
What are MEMS?





Micromechanical Filters



- Mechanical high-frequency (high MHz-GHz) filter
  - Your cell phone is mechanical!
  - New MEMS filters can be integrated with circuitry
    $\Longrightarrow$ smaller and lower power

Ultimate Success

"Calling Dick Tracy!"

Designing Transfer Functions

Time domain:

$$
\begin{aligned}
Mu'' + Cu' + Ku &= b\phi(t) \\
y(t) &= p^T u
\end{aligned}
$$

Frequency domain:

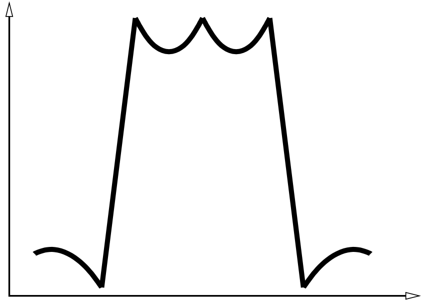
$$
\begin{aligned}
-\omega^2 M\hat{u} + i\omega C\hat{u} + K\hat{u} &= b\hat{\phi}(\omega) \\
\hat{y}(\omega) &= p^T \hat{u}
\end{aligned}
$$

Transfer function:

$$
\begin{aligned}
H(\omega) &= p^T(-\omega^2 M + i\omega C + K)^{-1} b \\
\hat{y}(\omega) &= H(\omega)\hat{\phi}(\omega)
\end{aligned}
$$

Narrowband Filter Needs

CAD for MEMS



- Want "sharp" poles for narrowband filters
- $\Longrightarrow$ Want to minimize damping

Damping and Q

Designers want high *quality of resonance* (Q)

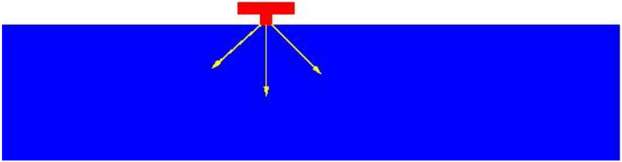
- Dimensionless damping in a one-dof system

$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

- For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{|\omega|}{2\,\mathrm{Im}(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

Damping Mechanisms



Possible loss mechanisms:

- Fluid damping
- Material losses
- Thermoelastic damping
- Anchor loss

Model substrate as semi-infinite with a

Perfectly Matched Layer (PML).

Perfectly Matched Layers

- Complex coordinate transformation
- Generates a “perfectly matched” absorbing layer
- Idea works with general linear wave equations
  - Electromagnetics (Berengér, 1994)
  - Quantum mechanics – *exterior complex scaling* (Simon, 1979)
  - Elasticity in standard finite element framework (Basu and Chopra, 2003)

Model Problem

- Domain: $x \in [0, \infty)$
- Governing eq:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

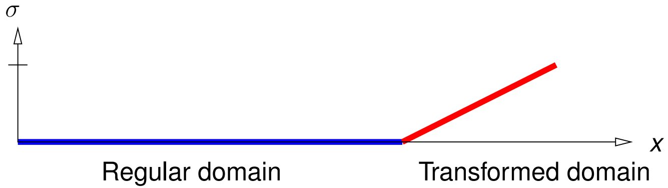
- Fourier transform:

$$\frac{d^2 \hat{u}}{dx^2} + k^2 \hat{u} = 0$$

- Solution:

$$\hat{u} = c_{\mathrm{out}} e^{-ikx} + c_{\mathrm{in}} e^{ikx}$$

Model with Perfectly Matched Layer

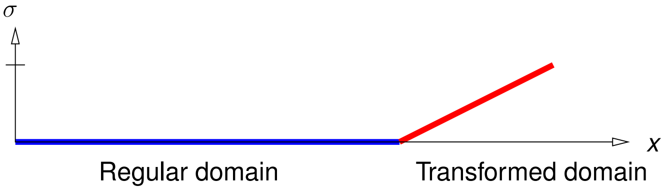


$$\frac{d\tilde{x}}{dx} = \lambda(x) \text{ where } \lambda(s) = 1 - i\sigma(s)$$

$$\frac{d^2\hat{u}}{d\tilde{x}^2} + k^2\hat{u} = 0$$

$$\hat{u} = c_{\text{out}} e^{-ik\tilde{x}} + c_{\text{in}} e^{ik\tilde{x}}$$

Model with Perfectly Matched Layer



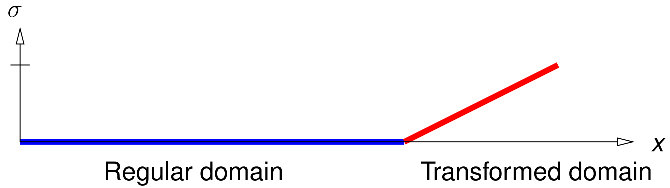
$$\frac{d\tilde{x}}{dx} = \lambda(x) \text{ where } \lambda(s) = 1 - i\sigma(s),$$

$$\frac{1}{\lambda}\frac{d}{dx}\left(\frac{1}{\lambda}\frac{d\hat{u}}{dx}\right) + k^2\hat{u} = 0$$

$$\hat{u} = c_{\text{out}} e^{-ikx - k\Sigma(x)} + c_{\text{in}} e^{ikx + k\Sigma(x)}$$

$$\Sigma(x) = \int_0^x \sigma(s)\, ds$$

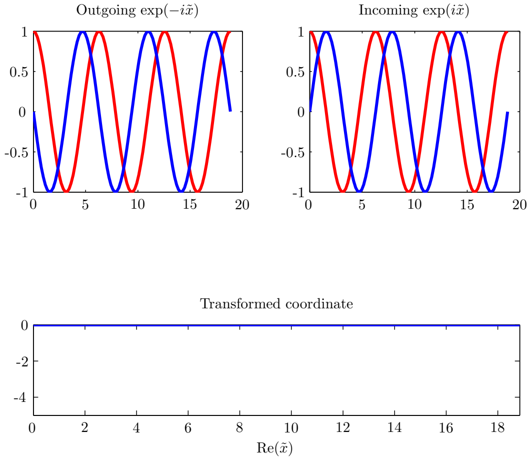
Model with Perfectly Matched Layer



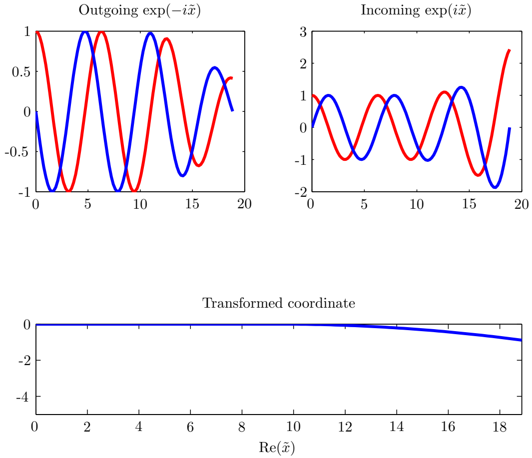
If solution clamped at $x = L$ then

$$\frac{c_{\text{in}}}{c_{\text{out}}} = O(e^{-k\gamma}) \text{ where } \gamma = \Sigma(L) = \int_0^L \sigma(s)\,ds$$

Model Problem Illustrated



Model Problem Illustrated



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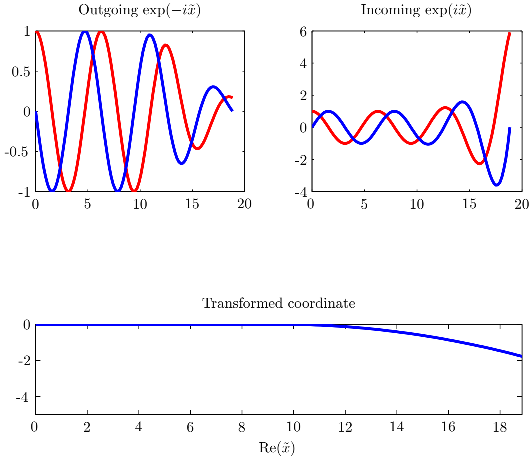
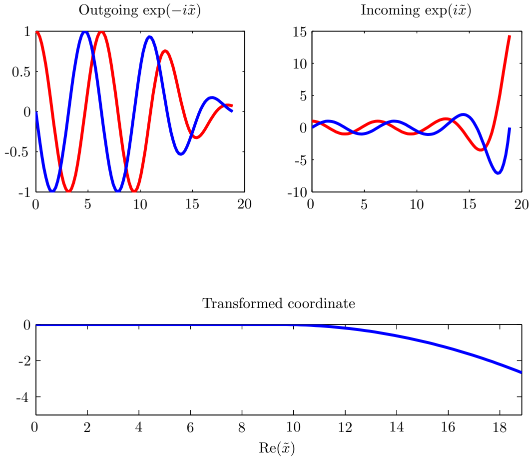
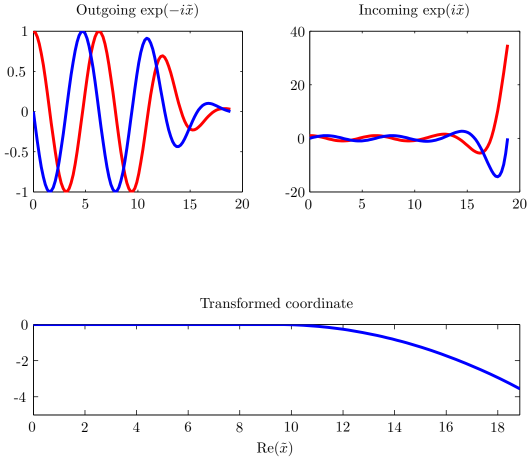


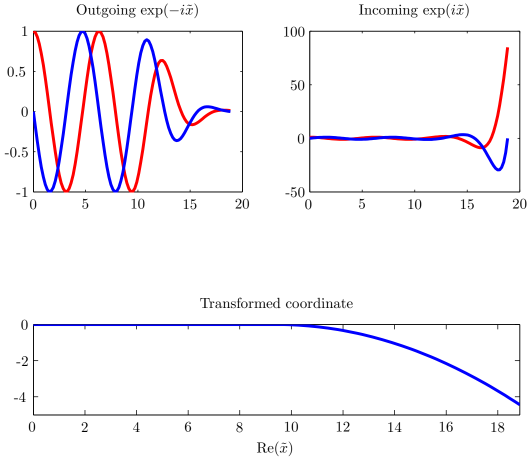
Model Problem Illustrated



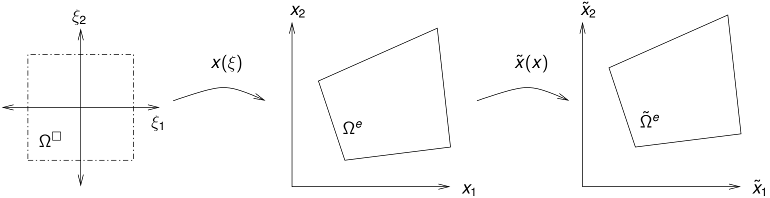
Model Problem Illustrated



Model Problem Illustrated



Finite Element Implementation



- Combine PML and isoparametric mappings

$$
\begin{aligned}
\mathbf{k}^e &= \int_{\Omega^\square} \tilde{\mathbf{B}}^T \mathbf{D} \tilde{\mathbf{B}} \tilde{J} \, d\Omega^\square \\
\mathbf{m}^e &= \int_{\Omega^\square} \rho \mathbf{N}^T \mathbf{N} \tilde{J} \, d\Omega^\square
\end{aligned}
$$

- Matrices are *complex symmetric*

Eigenvalues and Model Reduction

Want to know about the transfer function $H(\omega)$:

$$H(\omega) = p^T(K - \omega^2 M)^{-1} b$$

Can either

- Locate poles of H (eigenvalues of (K, M))
- Plot H in a frequency range (Bode plot)

Usual tactic: subspace projection

- Build an Arnoldi basis V
- Compute with much smaller $V^* K V$ and $V^* M V$

Can we do better?

Variational Principles

- Variational form for complex symmetric eigenproblems:
  - Hermitian (Rayleigh quotient):

$$\rho(v) = \frac{v^* K v}{v^* M v}$$

  - Complex symmetric (modified Rayleigh quotient):

$$\theta(v) = \frac{v^T K v}{v^T M v}$$

- First-order accurate eigenvectors $\implies$ Second-order accurate eigenvalues.
- Key: relation between left and right eigenvectors.

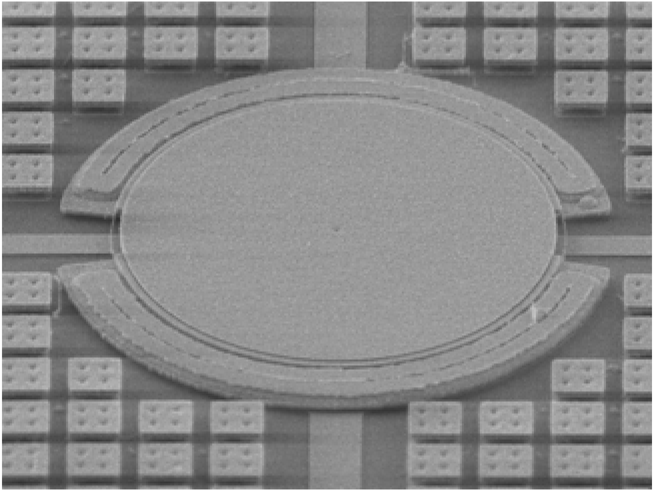
Accurate Model Reduction

- Build new projection basis from V:

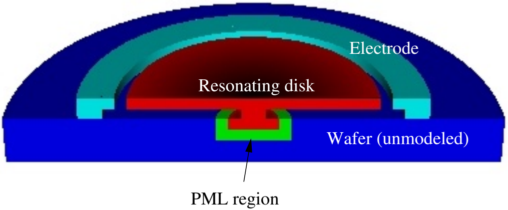
$$W = \text{orth}[\text{Re}(V), \text{Im}(V)]$$

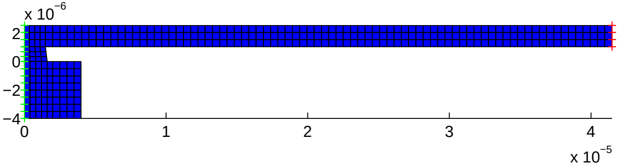
- span(W) contains both $\mathcal{K}_n$ and $\bar{\mathcal{K}}_n$
  $\implies$ double digits correct vs. projection with V
- W is a real-valued basis
  $\implies$ projected system is complex symmetric

Disk Resonator Simulations



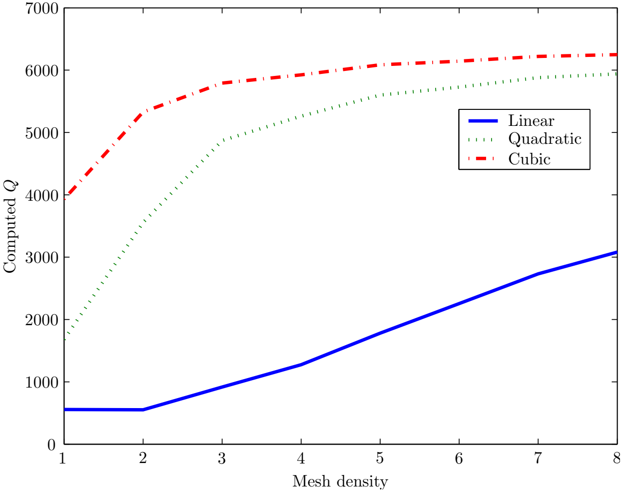
Disk Resonator Mesh





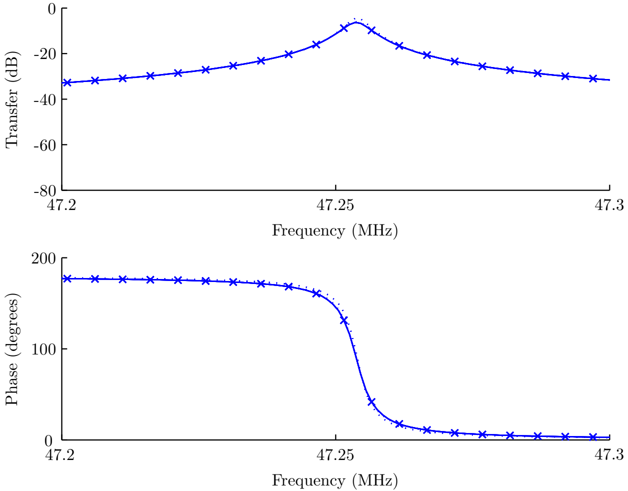
- Axisymmetric model with bicubic mesh
- About 10K nodal points in converged calculation

Mesh Convergence



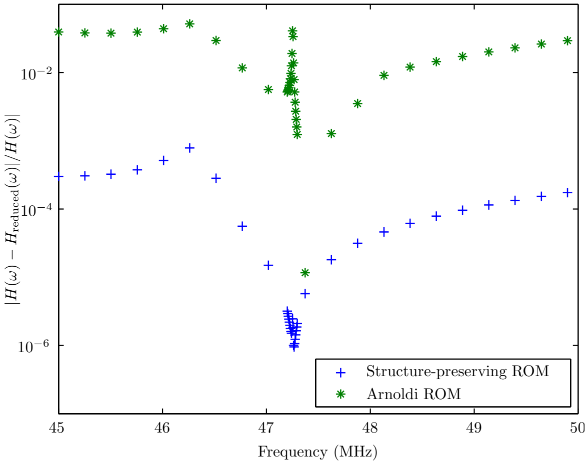
Cubic elements converge with reasonable mesh density

Model Reduction Accuracy



Results from ROM (solid and dotted lines) nearly indistinguishable from full model (crosses)

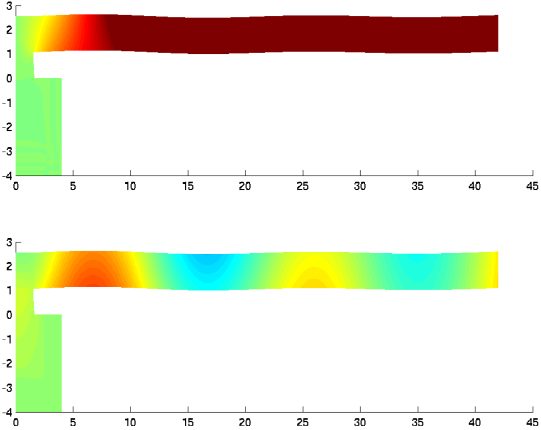
Model Reduction Accuracy



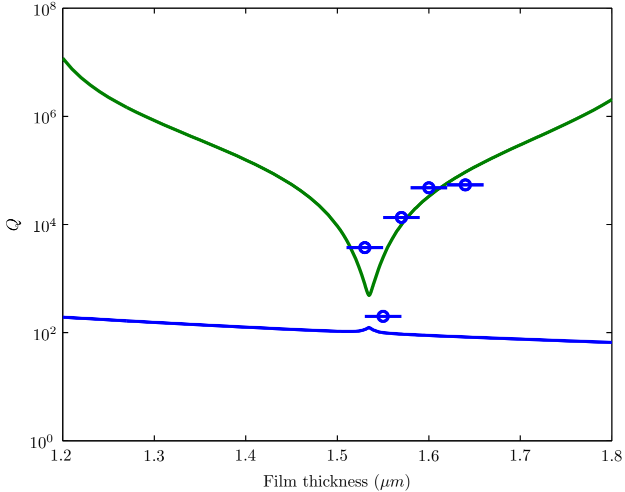
Preserve structure $\Longrightarrow$

get twice the correct digits

Response of the Disk Resonator

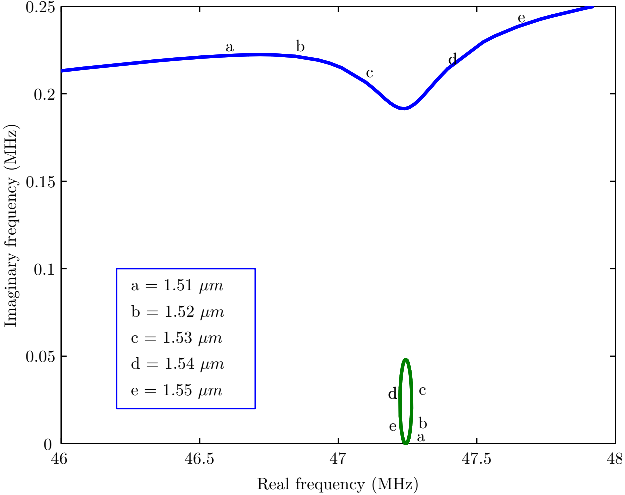


Variation in Quality of Resonance



Simulation and lab measurements vs. disk thickness

Explanation of Q Variation



Interaction of two nearby eigenmodes

Conclusions

- RF MEMS are a great source of problems
  - Interesting applications
  - Interesting physics (and not altogether understood)
  - Interesting numerical mathematics
- See also:
  - HiQLab: simulation of resonant MEMS
    `www.cs.berkeley.edu/~dbindel/hiqlab/`
  - Bindel and Govindjee. Elastic PMLs for resonator anchor loss simulations. *IJNME*, 64(6):789–818, October 2005.