Computer-Aided Design of MEMS

Eigenvalues, Energy Losses,
and Dick Tracy Watches

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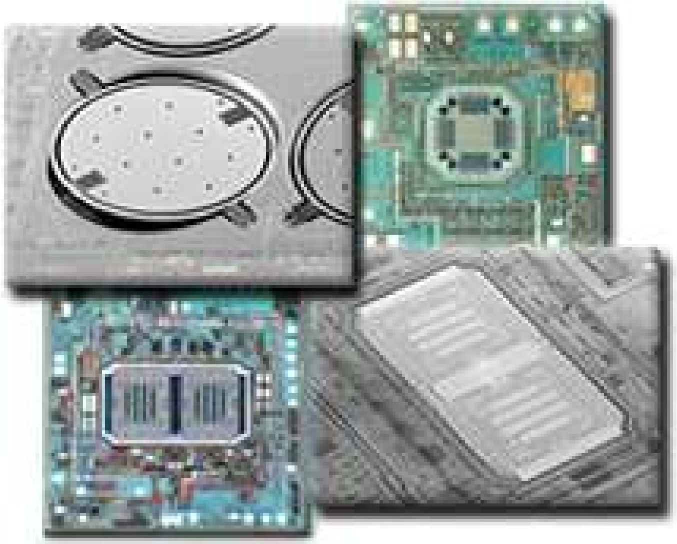
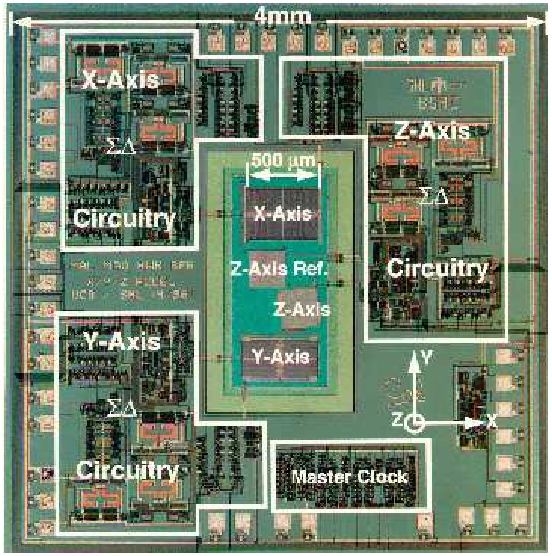
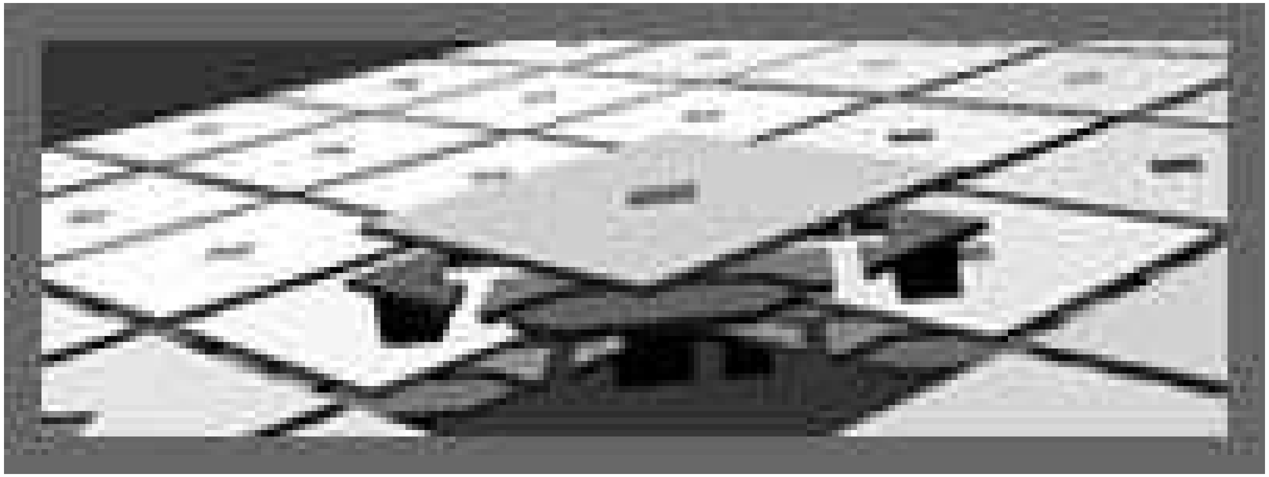
Outline

- MEMS (Micro-Electro-Mechanical Systems) basics and RF (Radio Frequency) MEMS
- Disk resonators and perfectly matched layers
- Beam resonators and thermoelastic damping
- Model reduction, mode tracking, and optimization
- Conclusions

What are MEMS?







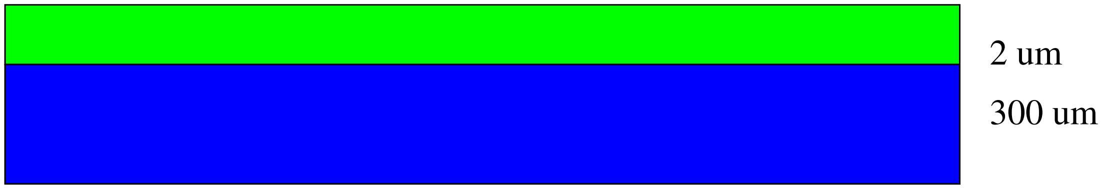
MEMS Basics

- Micro-electro-mechanical systems
  - Chemical, fluid, thermal, optical (MECFTOMS?)
- Applications:
  - Sensors (inertial, chemical, pressure)
  - Ink jet printers, biolab chips
  - RF devices: cell phones, inventory tags, pico radio
  - “Smart dust”
- Use integrated circuit (IC) fabrication technology
- Large surface area / volume ratio
- Still mostly classical (vs. nanosystems)

Fabrication outline

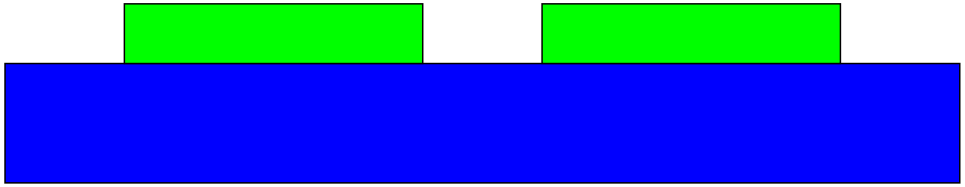
1. Si wafer

Fabrication outline



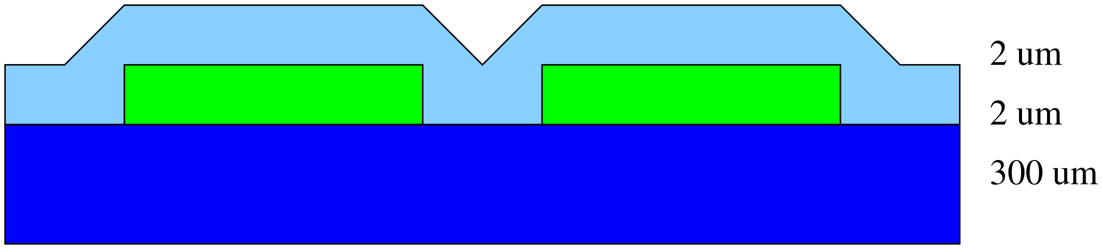
1. Si wafer
2. Deposit 2 microns SiO2

Fabrication outline



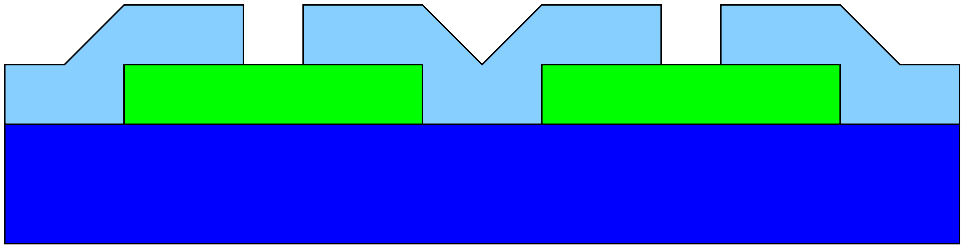
1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer

Fabrication outline



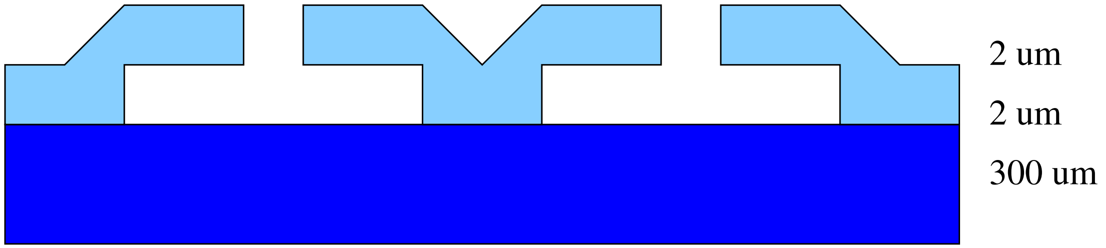
1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
4. Deposit 2 microns polycrystalline Si

Fabrication outline



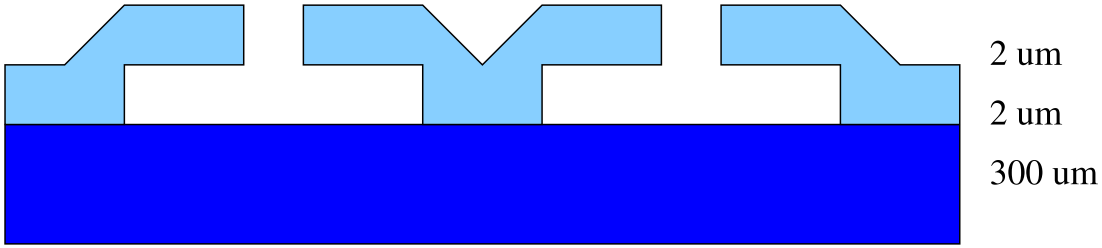
1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
4. Deposit 2 microns polycrystalline Si
5. Pattern and etch Si layer

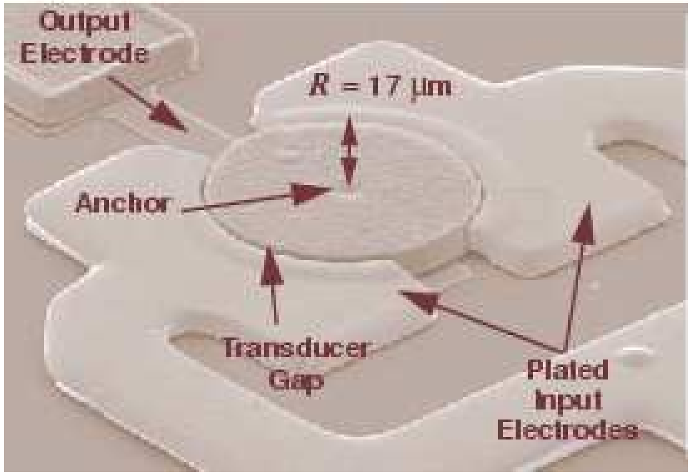
Fabrication outline



1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
4. Deposit 2 microns polycrystalline Si
5. Pattern and etch Si layer
6. Release etch remaining SiO2

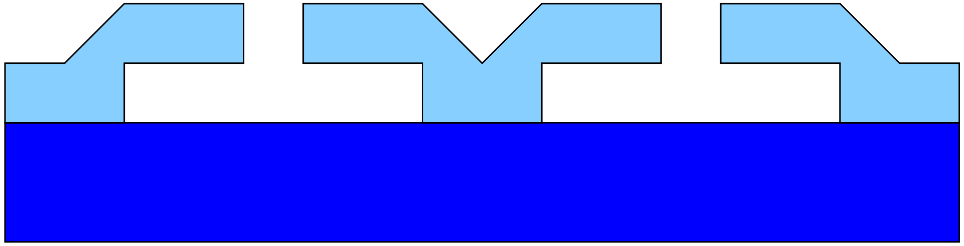
Fabrication result





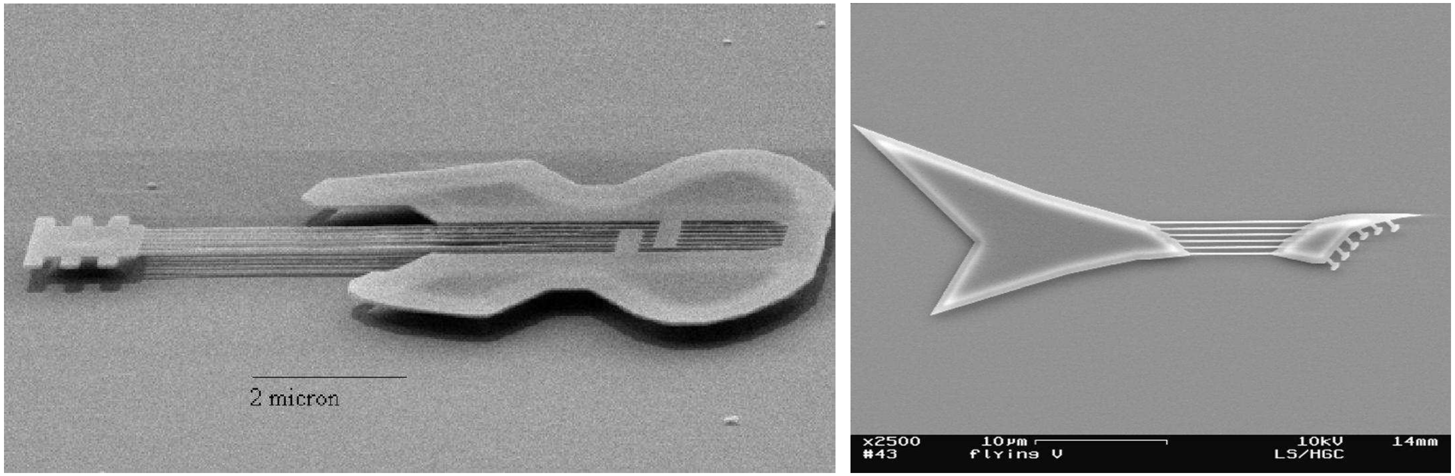
(C. Nguyen, iMEMS 01)

Fabrication characteristics



- Characteristic dimensions: microns
- Geometry is “2.5” dimensional
- Relatively loose fabrication tolerances
- Difficult to characterize

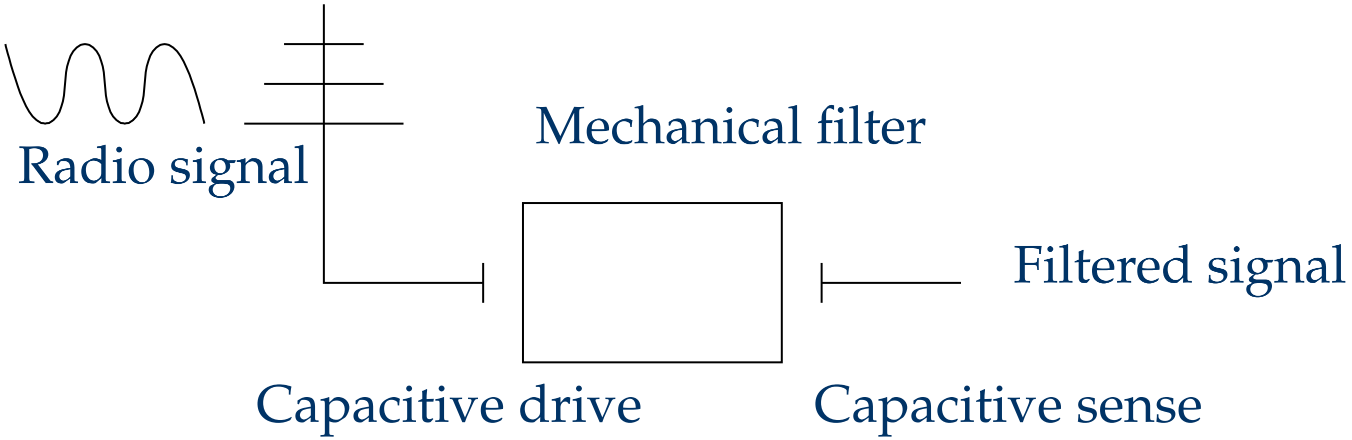
RF MEMS



Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Impact: smaller, lower-power cell phones
  - Replace quartz freq references, filter elements
  - Integrate into CMOS stack
- Other uses:
  - Sensing elements (e.g. chemical sensors)
  - Really high-pitch guitars

Micromechanical filters



- Mechanical high-frequency (high MHz-GHz) filter
  - Your cell phone is mechanical!
- Advantage over quartz surface acoustic wave filters
  - Integrated into chip
  - Low power

Success $\Longrightarrow$ "Calling Dick Tracy!"

Damping and Q

- Want to minimize damping
  - Electronic filters have too much
  - Understanding of damping in MEMS resonators is lacking
- Engineers want one number: Q
  - Non-dimensionalized damping in a one-variable system:

$$\frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)$$

  - For a resonant mode with frequency $\omega \in \mathbb{C}$:

$$Q := \frac{\Re(\omega)}{2\Im(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}$$

- Goal: Make Q big

Sources of damping

- Fluid damping
  - Air is a viscous fluid ($\mathrm{Re} \ll 1$)
  - Can operate in a vacuum
  - Shown not to dominate in many RF designs
- Anchor loss
  - Elastic waves radiate from structure
- Thermoelastic damping
  - Volume changes induce temperature change
  - Diffusion of heat leads to mechanical loss
- Material losses (catch-all)
  - Low intrinsic losses in silicon, diamond, germanium, etc.
  - Terrible material losses in metals

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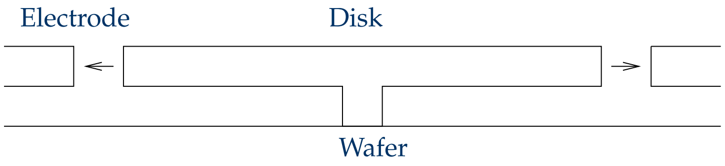
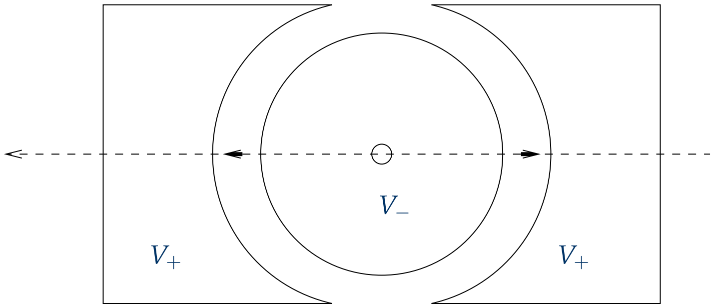
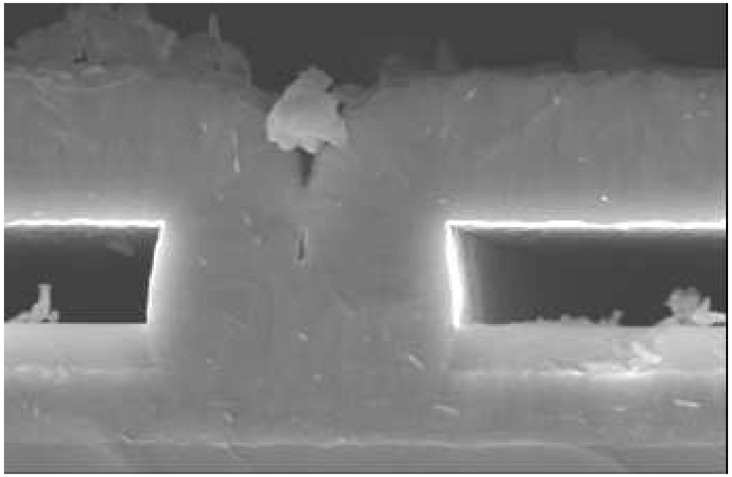
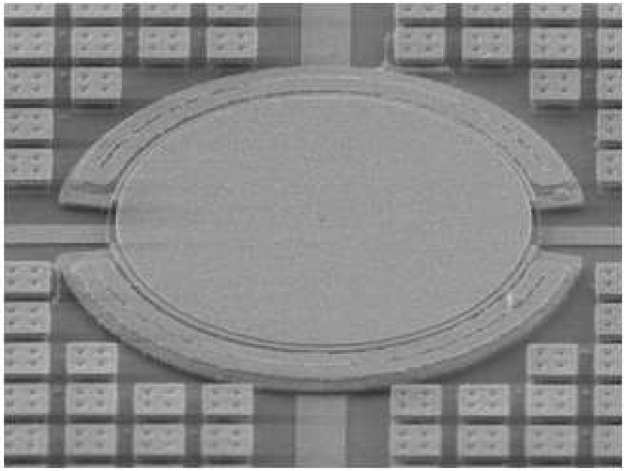
Disk resonator









- SiGe disk resonators built by E. Quévy
- Axisymmetric model with bicubic mesh, about 10K nodal points

Substrate model

Goal: Understand energy loss in this resonator

- Dominant loss is elastic radiation from anchor.
- Disk resonator is much smaller than substrate
- Very little energy leaving the post is reflected back
  - Substrate is semi-infinite from disk's perspective
- Possible semi-infinite models
  - Matched asymptotic modes
  - Dirichlet-to-Neumann maps
  - Boundary dampers
  - Perfectly matched layers

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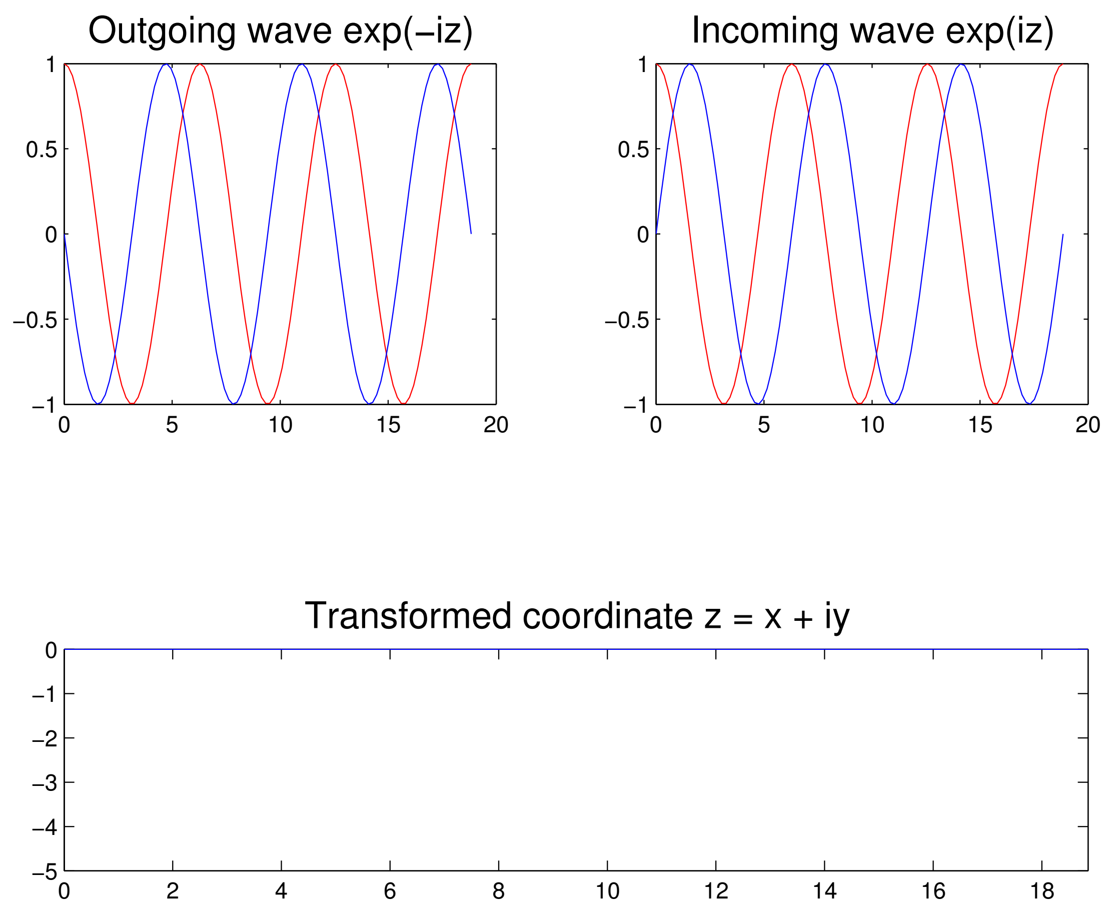
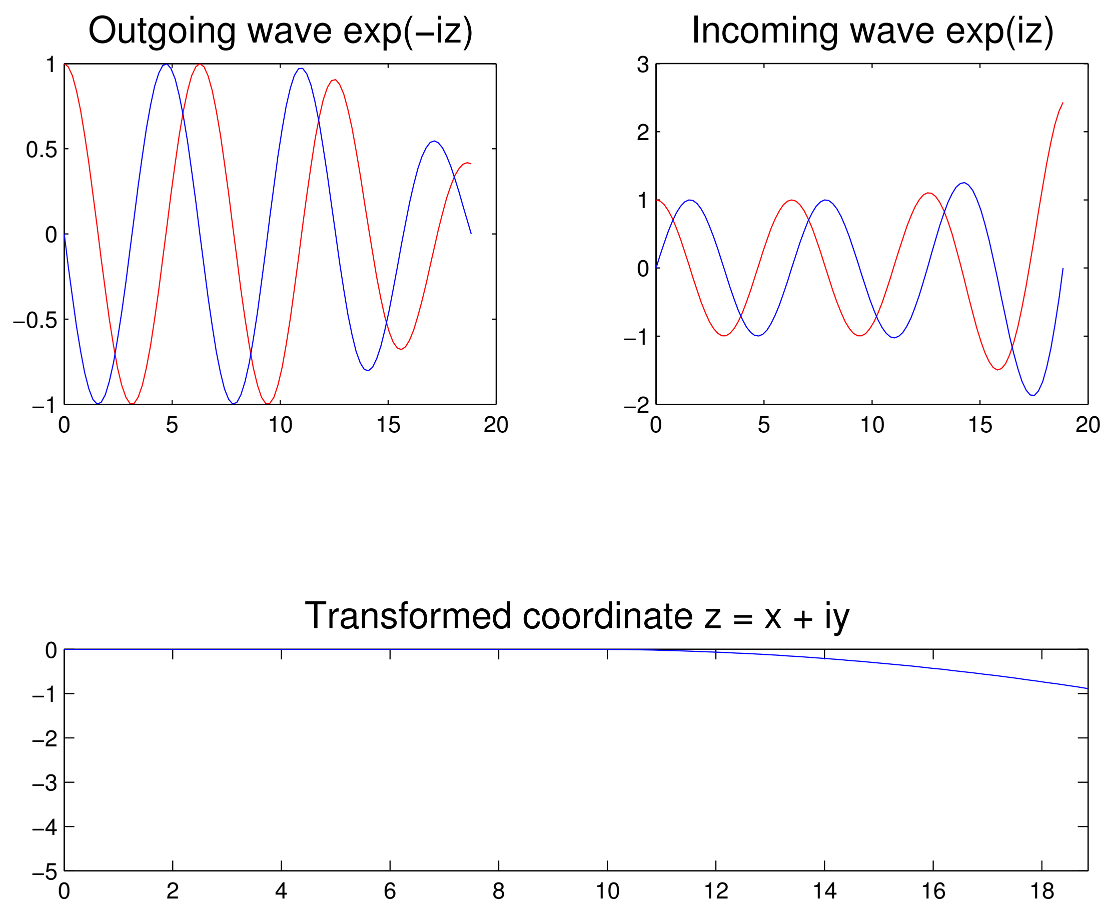
Perfectly matched layers

- Apply a complex coordinate transformation
- Generates a non-physical absorbing layer
- No impedance mismatch between the computational domain and the absorbing layer
- Idea works with general linear wave equations
  - First applied to Maxwell's equations (Berengér 95)
  - Similar idea introduced earlier in quantum mechanics (*exterior complex scaling*, Simon 79)

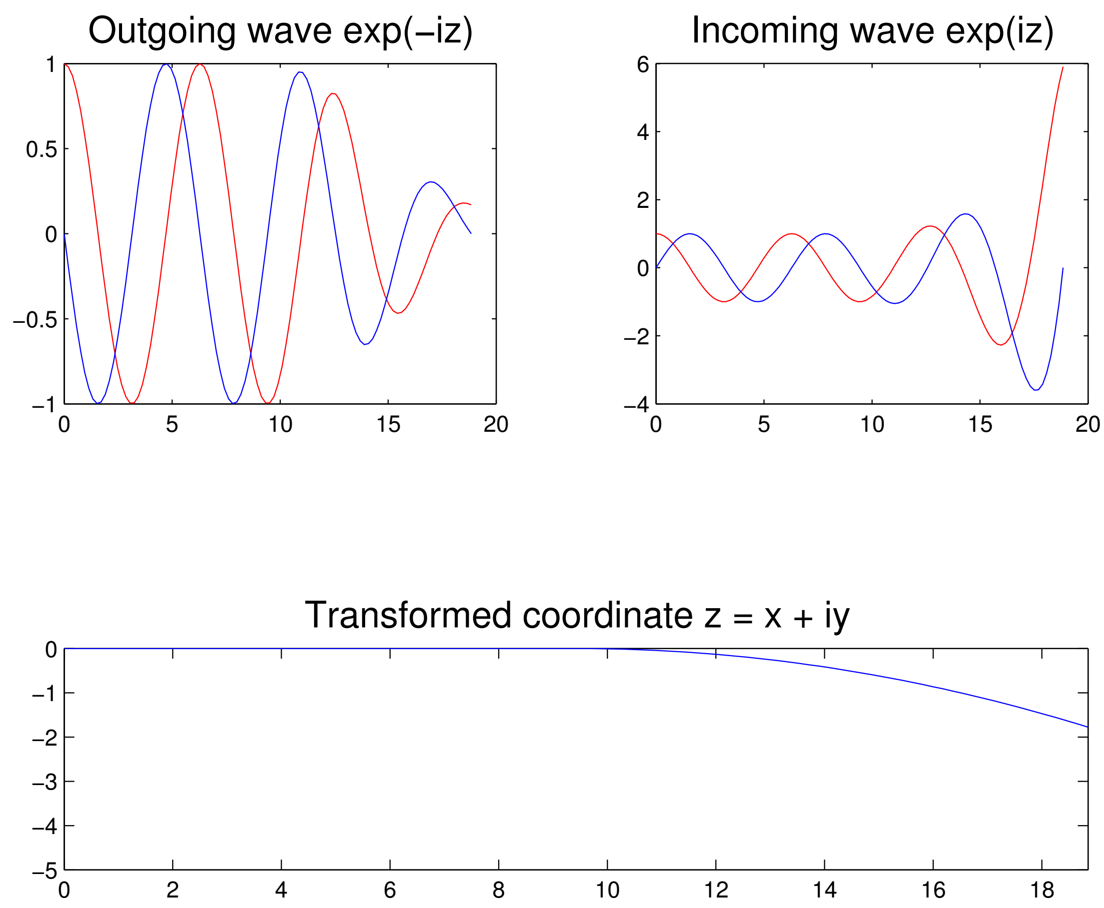
Scalar wave example



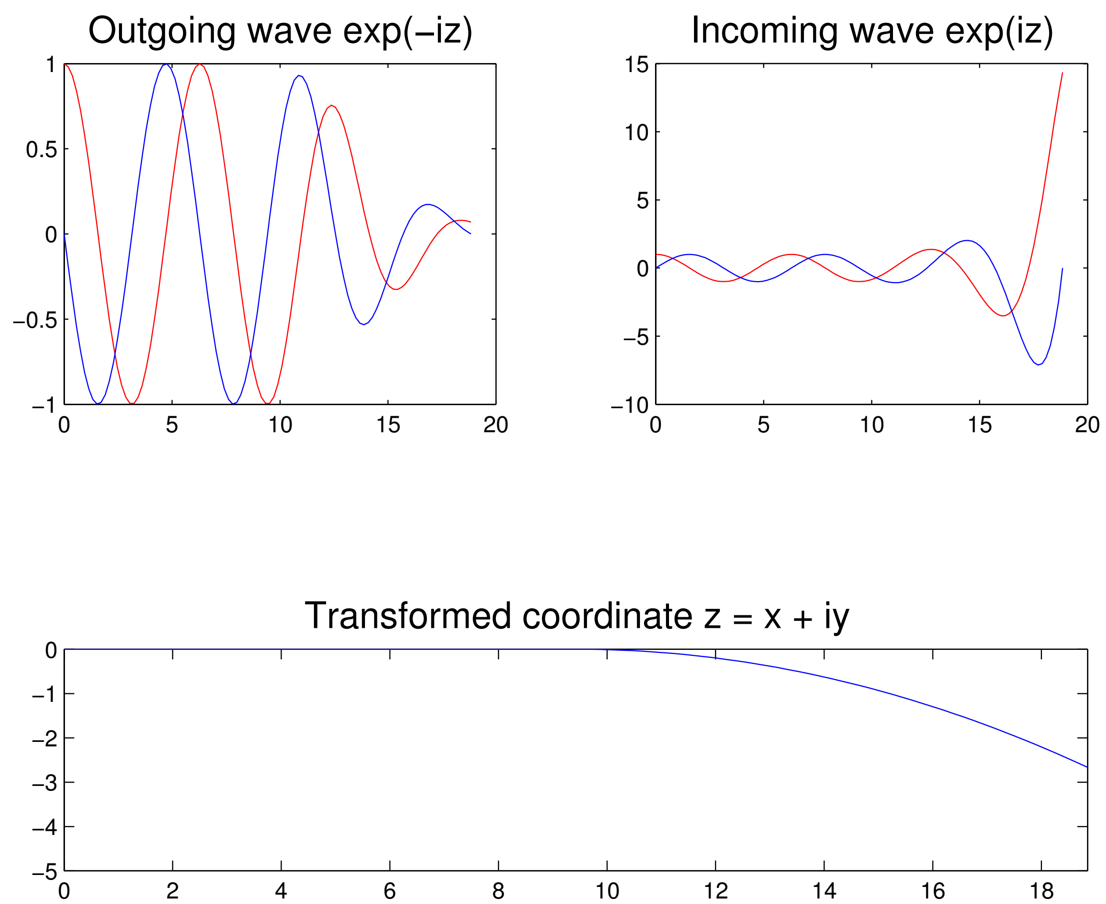
Scalar wave example



Scalar wave example



Scalar wave example



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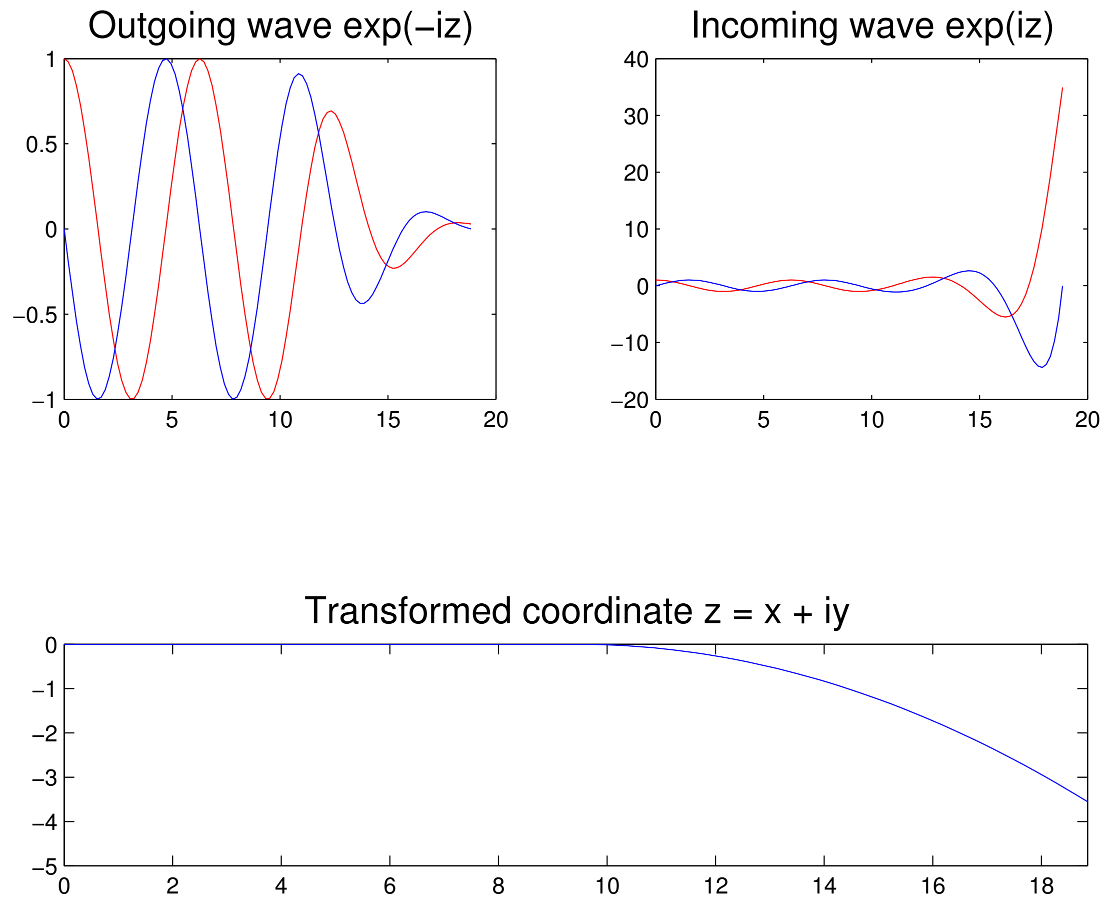
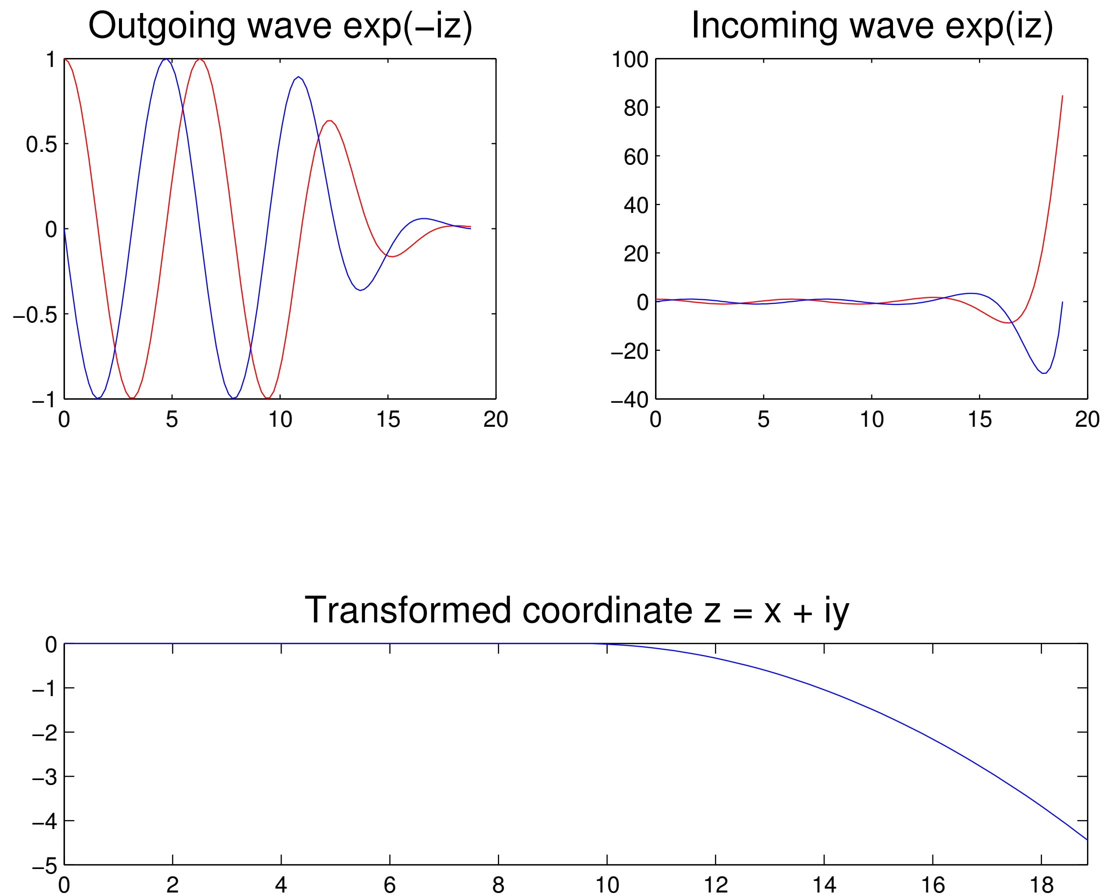


Scalar wave example



Clamp solution at transformed end to isolate outgoing wave.

PML weak form

Weak form of time-harmonic elasticity equations:

$$-\omega^2 \int_\Omega \rho w \cdot u \, d\Omega + \int_\Omega \epsilon(w) : \mathbf{C} : \epsilon(u) d\Omega = \int_\Gamma w \cdot t \, d\Gamma$$

Weak form of time-harmonic PML equation:

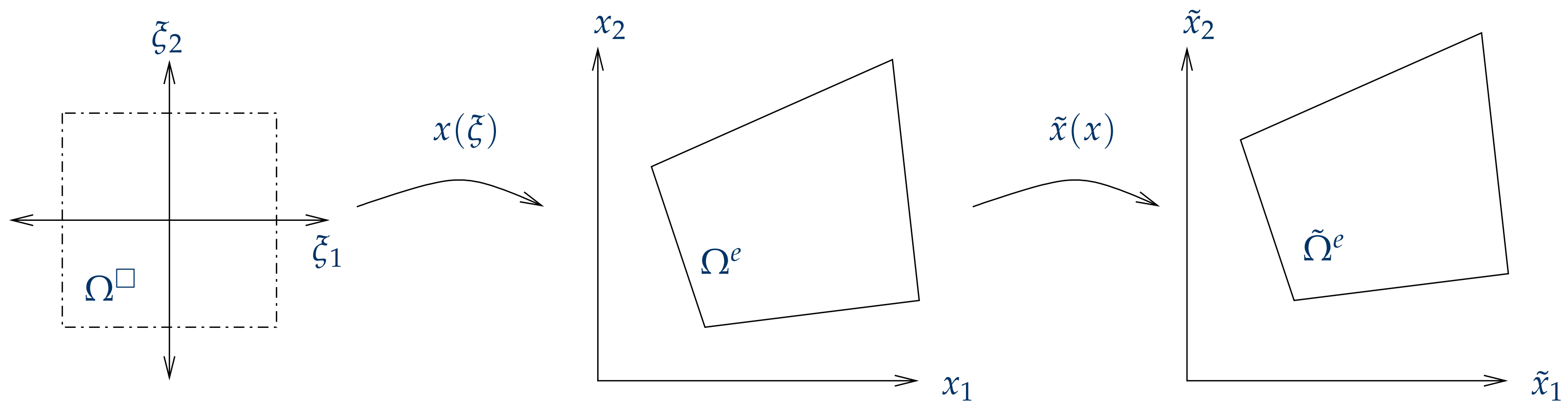
$$-\omega^2 \int_\Omega \rho w \cdot u \, J d\Omega + \int_\Omega \tilde{\epsilon}(w) : \mathbf{C} : \tilde{\epsilon}(u) \, J d\Omega = \int_\Gamma w \cdot \tilde{t} \, J d\Gamma$$

Bubnov-Galerkin finite element discretization leads to

$$-\omega^2 M u + K u = F$$

But in PML case, M and K are *complex symmetric*.

Finite elements



- Isoparametric elements already use mapped integration
- View PML as an added coordinate transformation
  - Requires little modification to existing elements
  - Just transform derivatives and Jacobian determinant

Eigenstructure

- Complex symmetry implies row and column eigenvectors are (non-conjugated) transposes.
- Can therefore achieve second-order accuracy with a modified Rayleigh quotient:

$$\theta(v) = (v^T K v) / (v^T M v)$$

- It *is* possible to have $v^T M v \approx 0$
  - Propagating modes (continuous spectrum)
  - Not the modes of interest for resonators

Perturbation analysis

Know first-order perturbation behavior of eigenvalues:

$$(\omega + \delta\omega)^2 = \frac{v^T(K + \delta K)v}{v^T(M + \delta M)v}$$

Useful for sensitivity analysis.

Model reduction

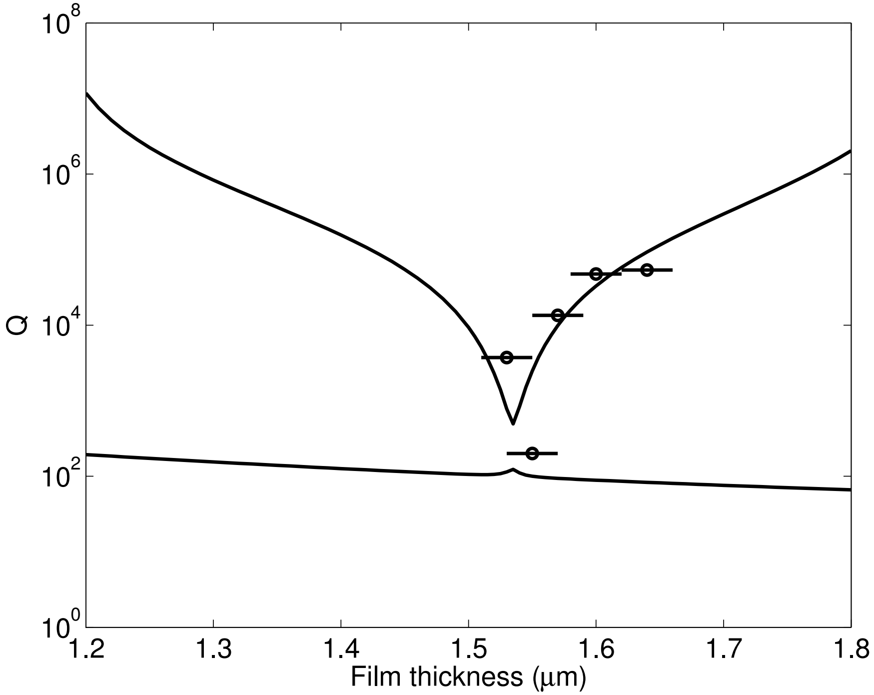
Would like a reduced model which

- Preserves second-order accuracy for converged eigs
- Keeps at least Arnoldi's accuracy otherwise
- Is physically meaningful

Idea:

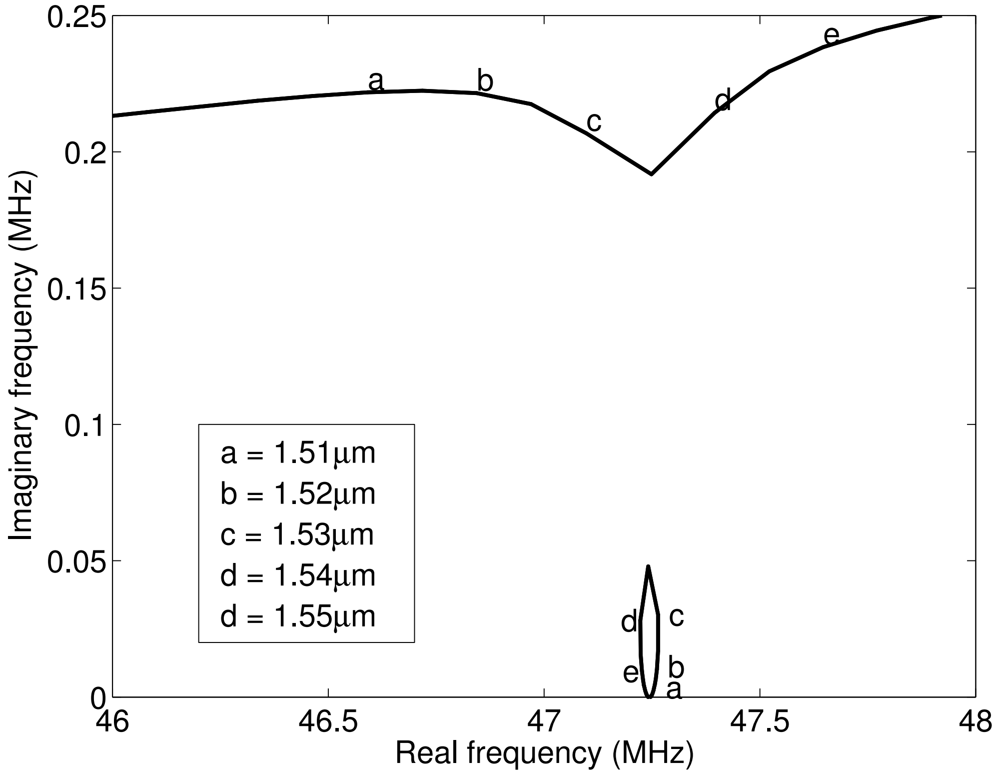
- Build an Arnoldi basis V
- Double the size: $W = \text{orth}([\Re(V), \Im(V)])$
- Use W as a projection basis
- Resulting system is still a Galerkin approximation with real shape functions for the continuum PML equations

Q variation



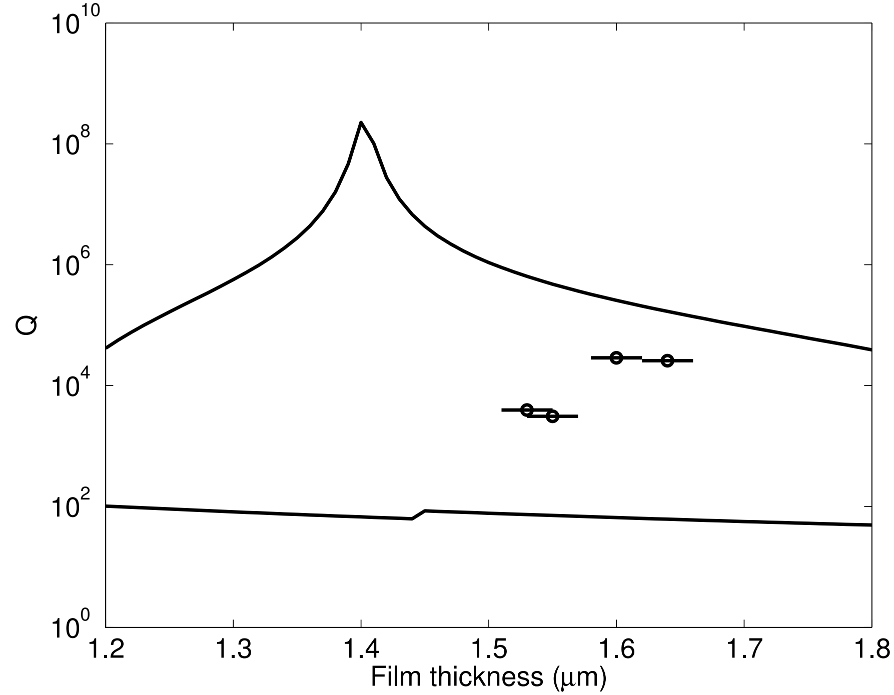
- Compute complex frequencies by shift-and-invert Arnoldi with an analytically determined shift
- Surprising variation – experimentally observed – in Q as film thickness changes!

Effect of varying film thickness



- Sudden dip in Q comes from an interaction between a (mostly) bending mode and a (mostly) radial mode
- Non-normal interaction between the modes

Truth in advertising



Data from a set of $30\mu m$ radius disks.

Thermoelastic damping (TED)

u is displacement and $T = T_0 + \theta$ is temperature

$$\begin{aligned} \sigma &= C\epsilon - \beta\theta 1 \\ \rho u_{tt} &= \nabla \cdot \sigma \\ \rho c_v \theta_t &= \nabla \cdot (\kappa \nabla \theta) - \beta T_0 \operatorname{tr}(\epsilon_t) \end{aligned}$$

- Volumetric strain rate drives energy transfer from mechanical to thermal domain
  - Irreversible diffusion $\implies$ mechanical damping
  - Not often an important factor at the macro scale
  - Recognized source of damping in microresonators
- Zener: semi-analytical approximation for TED in beams
- We consider the fully coupled system

Nondimensionalization

$$
\begin{aligned}
\sigma &= \hat{C}\epsilon - \xi\theta 1 \\
u_{tt} &= \nabla \cdot \sigma \\
\theta_t &= \eta \nabla^2 \theta - \operatorname{tr}(\epsilon_t)
\end{aligned}
$$

$$\xi := \left(\frac{\beta}{\rho c}\right)^2 \frac{T_0}{c_v} \text{ and } \eta := \frac{\kappa}{\rho c_v c L}$$

$$
\begin{aligned}
\text{Length} &\sim L \\
\text{Time} &\sim L/c, \text{ where } c = \sqrt{E/\rho} \\
\text{Temperature} &\sim T_0 \frac{\beta}{\rho c_v}
\end{aligned}
$$

Scaling analysis

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ u_{tt} &= \nabla \cdot \sigma \\ \theta_t &= \eta \nabla^2 \theta - \mathrm{tr}(\epsilon_t) \end{aligned}$$

$$\xi := \left(\frac{\beta}{\rho c}\right)^2 \frac{T_0}{c_v} \text{ and } \eta := \frac{\kappa}{\rho c_v c L}$$

- Micron-scale poly-Si devices: ξ and η are $\sim 10^{-4}$.
- Small η leads to thermal boundary layers
- Linearize about $\xi = 0$

Discrete mode equations

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ u_{tt} &= \nabla \cdot \sigma \\ \theta_t &= \eta \nabla^2 \theta - \mathrm{tr}(\epsilon_t) \end{aligned}$$

$$\begin{aligned} \sigma &= \hat{C}\epsilon - \xi\theta 1 \\ -\omega^2 u &= \nabla \cdot \sigma \\ i\omega\theta &= \eta \nabla^2 \theta - i\omega\, \mathrm{tr}(\epsilon) \end{aligned}$$

$$\begin{aligned} -\omega^2 M_{uu} u + K_{uu} u + K_{ut}\theta &= 0 \\ i\omega D_{tt}\theta + K_{tt}\theta + i\omega D_{tu} u &= 0 \end{aligned}$$

Perturbation computation

$$\begin{aligned} -\omega^2 M_{uu} u + K_{uu} u + K_{ut} \theta &= 0 \\ i\omega D_{tt} \theta + K_{tt} \theta + i\omega D_{tu} u &= 0 \end{aligned}$$

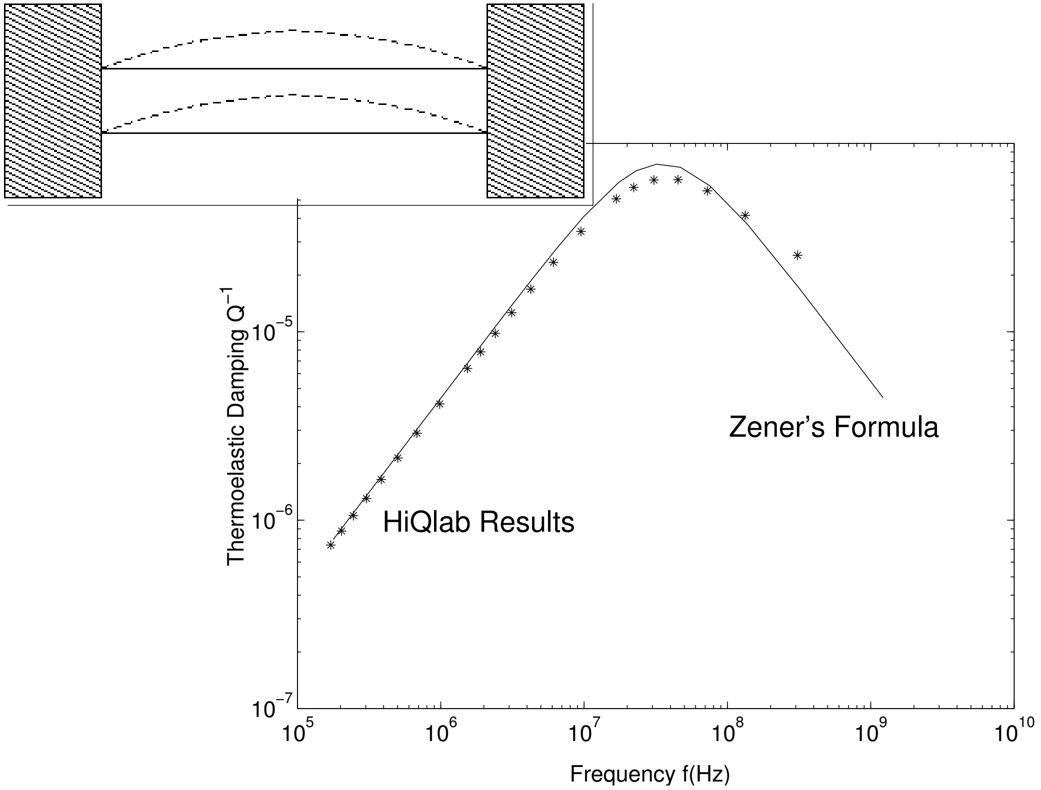
Approximate ω by perturbation about $K_{ut} = 0$:

$$\begin{aligned} -\omega_0^2 M_{uu} u_0 + K_{uu} u_0 &= 0 \\ i\omega_0 D_{tt} \theta_0 + K_{tt} \theta_0 + i\omega_0 D_{tu} u_0 &= 0 \end{aligned}$$

Choose $v : v^T u_0 \neq 0$ and compute

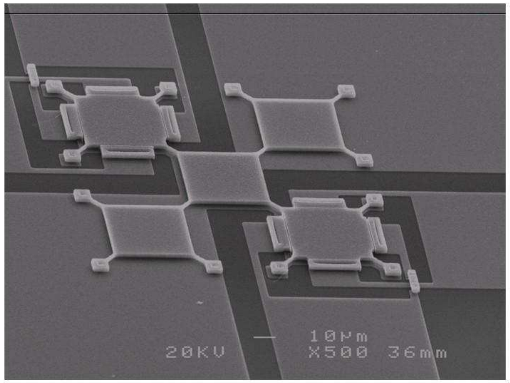
$$\begin{bmatrix} (-\omega_0^2 M_{uu} + K_{uu}) & -2\omega_0 M_{uu} u_0 \\ v^T & 0 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta \omega \end{bmatrix} = \begin{bmatrix} -K_{ut} \theta_0 \\ 0 \end{bmatrix}$$

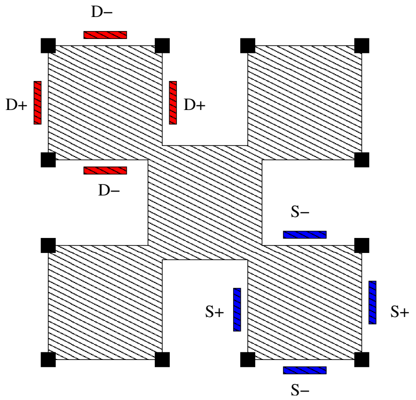
Comparison to Zener's model

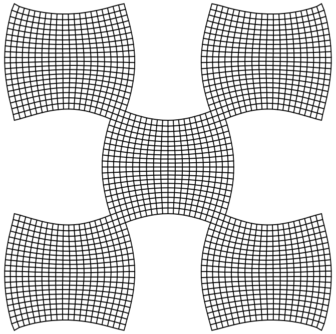


- Comparison of fully coupled simulation to Zener approximation over a range of frequencies
- Real and imaginary parts after first-order correction agree to about three digits with Arnoldi

Checkerboard resonator

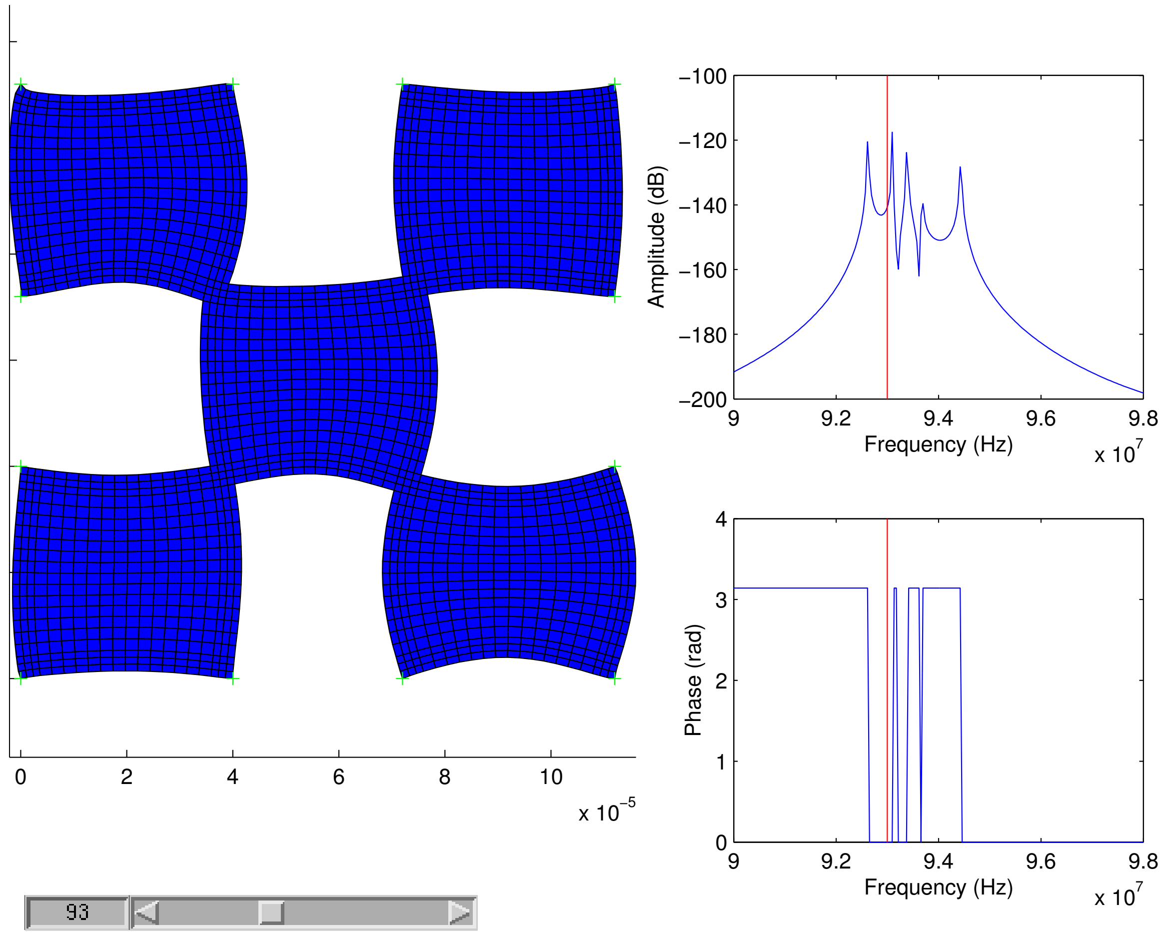




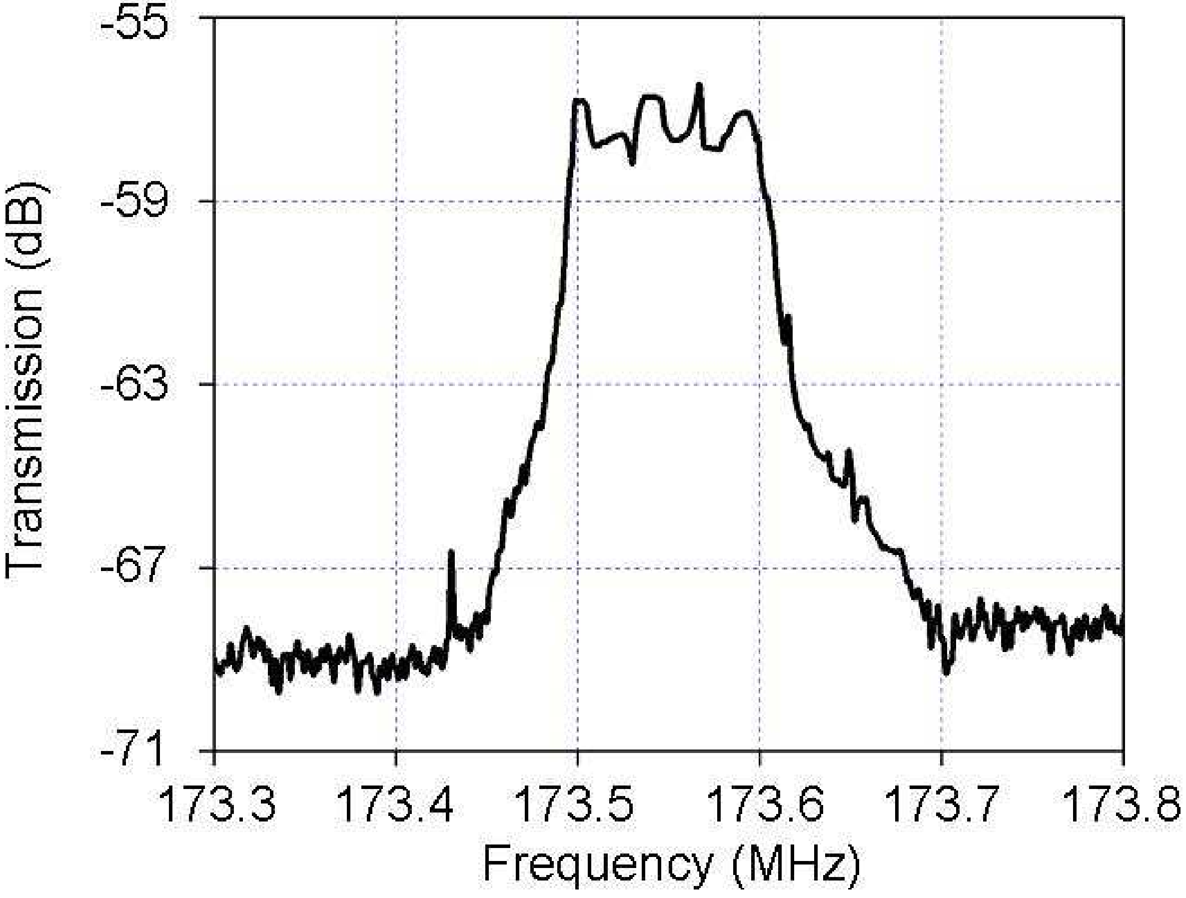


- Array of loosely coupled resonators
- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers

Checkerboard simulation



Checkerboard measurement



S. Bhave, MEMS 05

Transfer function optimization

- Choose geometry to make a good bandpass filter
- What is a "good bandpass filter?"
  - $|H(\omega)|$ is big on $[\omega_l, \omega_r]$
  - $|H(\omega)|$ is tiny outside this interval
- How do we optimize?
  - Overton's gradient sampling method
  - Use Byers-Boyd-Balikrishnan algorithm for distance to instability to minimize $|H(\omega)|$ on $[\omega_l, \omega_r]$
  - Small Hamiltonian eigenproblem (with ROM)

Contributions

- Mathematical
  - Reformulation of PML technology
  - Perturbation solution for thermoelastic damping
- Computer science
  - HiQLab, SUGAR, and FEAPMEX
- Engineering physics
  - Effects of mode interference in damping
  - Relative importance of anchor loss and TED

The other talk

- CLAPACK
- Finding roots of polynomials
- Continuation of invariant subspaces for sparse problems
- Computer network tomography
- OceanStore and distributed system security
- Pontificating about floating point arithmetic

Conclusions

- RF MEMS are a great source of problems
  - Interesting applications
  - Interesting physics (and not altogether understood)
  - Interesting numerical mathematics

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http://www.cs.berkeley.edu/~dbindel/feapmex.html
http://www.cs.berkeley.edu/~dbindel/hiqlab
http://bsac.berkeley.edu/cadtools/sugar/sugar/
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Role of simulation

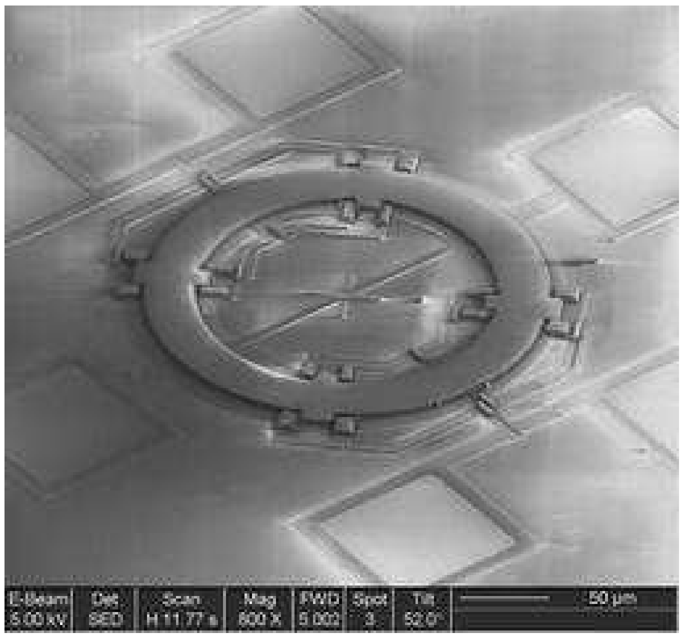
HiQLab: Modeling RF MEMS

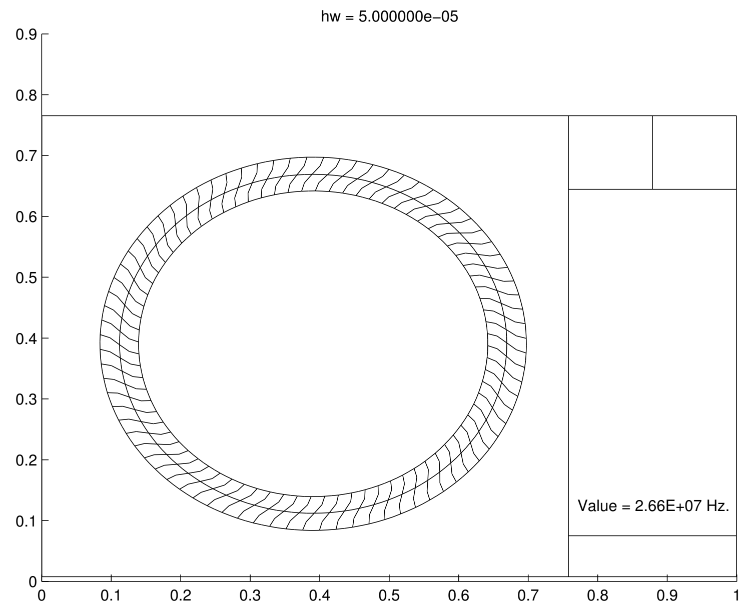
- Explore fundamental device physics
  - Particularly details of damping
- Detailed finite element modeling
- Reduced models eventually to go into SUGAR

SUGAR: “Be SPICE to the MEMS world”

- Fast enough for early design stages
- Simple enough to attract users
- Support design, analysis, optimization, synthesis
- Verify models by comparison to measurement

Shear ring resonator





- Ring is driven in a shearing motion
- Can couple ring to other resonators
- How do we track the desired mode?

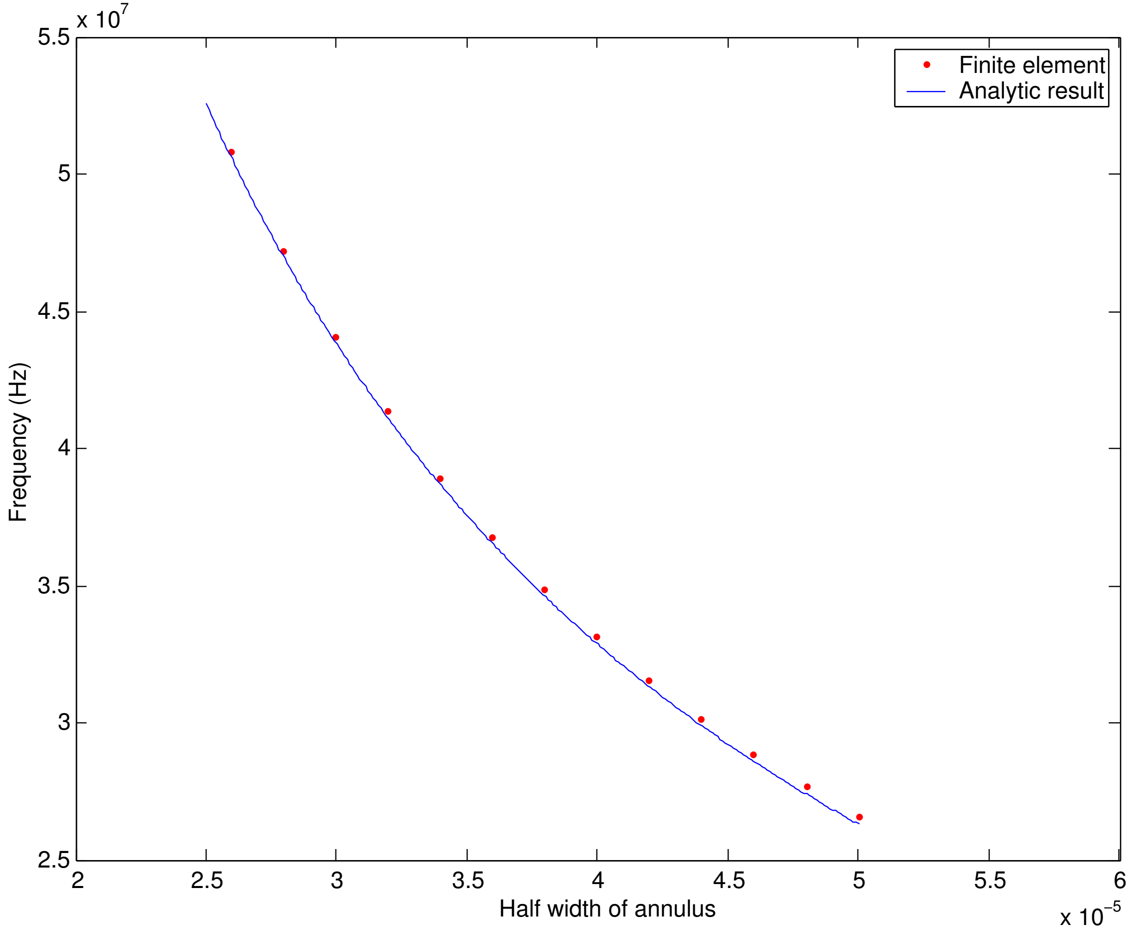
Mode tracking

Find a continuous solution to

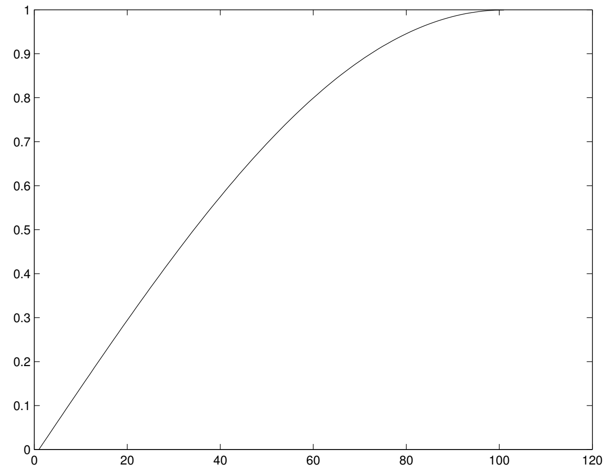
$$\left(K(s) - \omega(s)^2 M(s)\right) u(s) = 0.$$

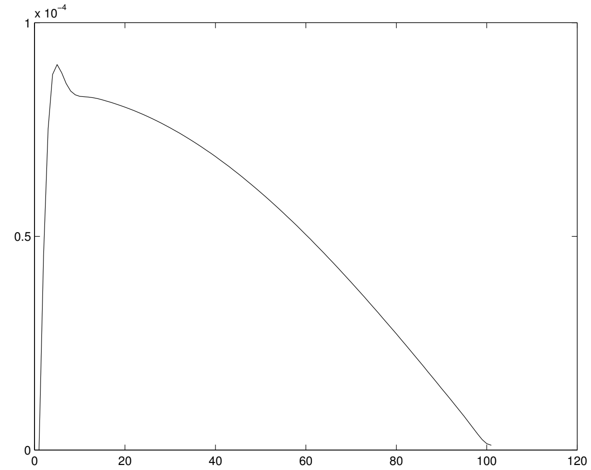
- K and M are symmetric and $M > 0$
- Eigenvectors are M-orthogonal
- Perturbation theory gives good shifts
- Look if $u(s+h)$ and $u(s)$ are on the same path by looking at $u(s+h)^T M(s+h) u(s)$
- Many more subtleties in the nonsymmetric case
  - Focus of the *CIS algorithm*

Mode tracking in a shear resonator



Thermoelastic boundary layer





- One-dimensional test problem (longitudinal mode in a bar)
- Fixed temperature and displacement at left
- Free at right