QLab, PMLs, and resonator loss

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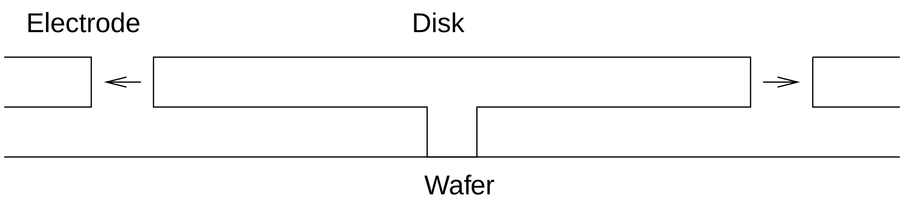
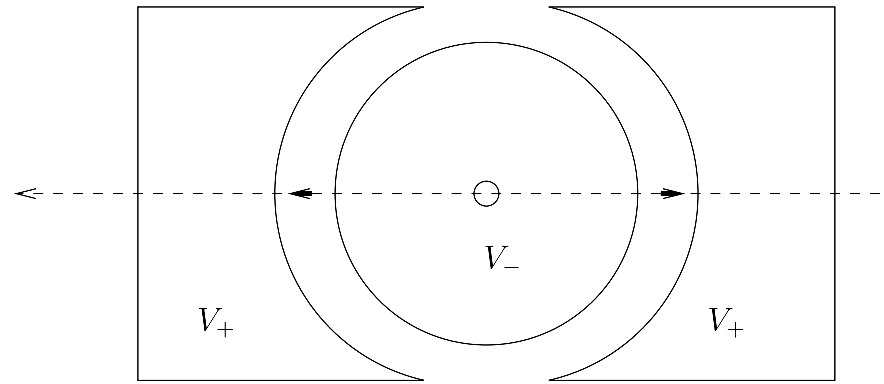
PMLs

- Absorbing layer for mimicing an infinite-domain
- Accomplished by a complex-valued coordinate transformation
- Interpretation: Impedance varies sufficiently smoothly that there are no interface reflections
- Generates *complex* symmetric M and K

Disk resonator model





Current model:

- Axisymmetric assumption.
- No details of refilled post geometry.
- No details of substrate stack materials.
- PML to model substrate as semi-infinite.

What affects Q?

How do the following affect Q?

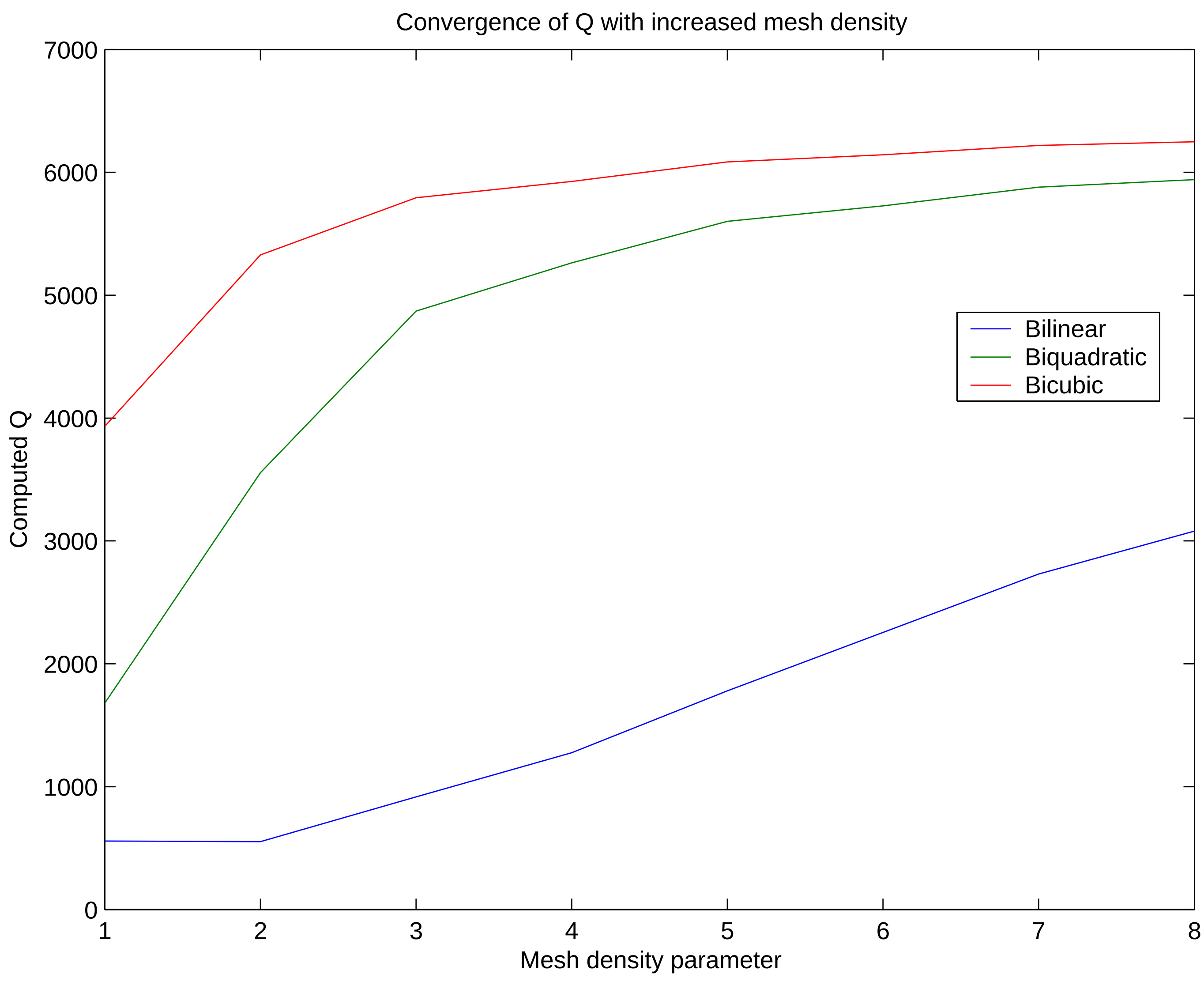
- Model idealizations (post geometry, substrate stack)?
- Model inaccuracies (material properties, geometric properties)?
- Loading from circuitry / electrostatics?

Can show through nondimensionalization that inaccurate E, ρ will affect computed frequency, but *not* Q (though inaccurate mismatch characterization might affect Q).

How well do we match?

- Well for some devices, less well for others (particularly second mode?)
- Experiments are difficult to run, so no statistics on the measurements
- Sweep data for diamond resonators

Simulation convergence



Simulation convergence

- Seem to need a fine mesh to resolve high Q
  - If error in real and imaginary parts are about the same, need five or six digits of convergence in ω to resolve $Q = 10000$
  - Physically, have a wave and a reflection which nearly perfectly interact to form a standing wave – have to resolve what's left
  - Seems to match numerical experiment (low Q simulations converge quickly)
- Three-dimensional effects seem important as well (2D BLR model gives cruddy results)
- $\Longrightarrow$ Gets expensive!

PML ROM: Preserving structure

Weak form of the PML equation (frequency domain) is

$$s^2 \int_\Omega \rho w \cdot u \, J d\Omega + \int_\Omega \tilde{\epsilon}(w) : \mathbf{C} \tilde{\epsilon}(u) \, J d\Omega = \int_\Gamma w \cdot p\tilde{n} \, J d\Gamma$$

Finite element discretization (Bubnov-Galerkin) yields complex-symmetric mass and stiffness.

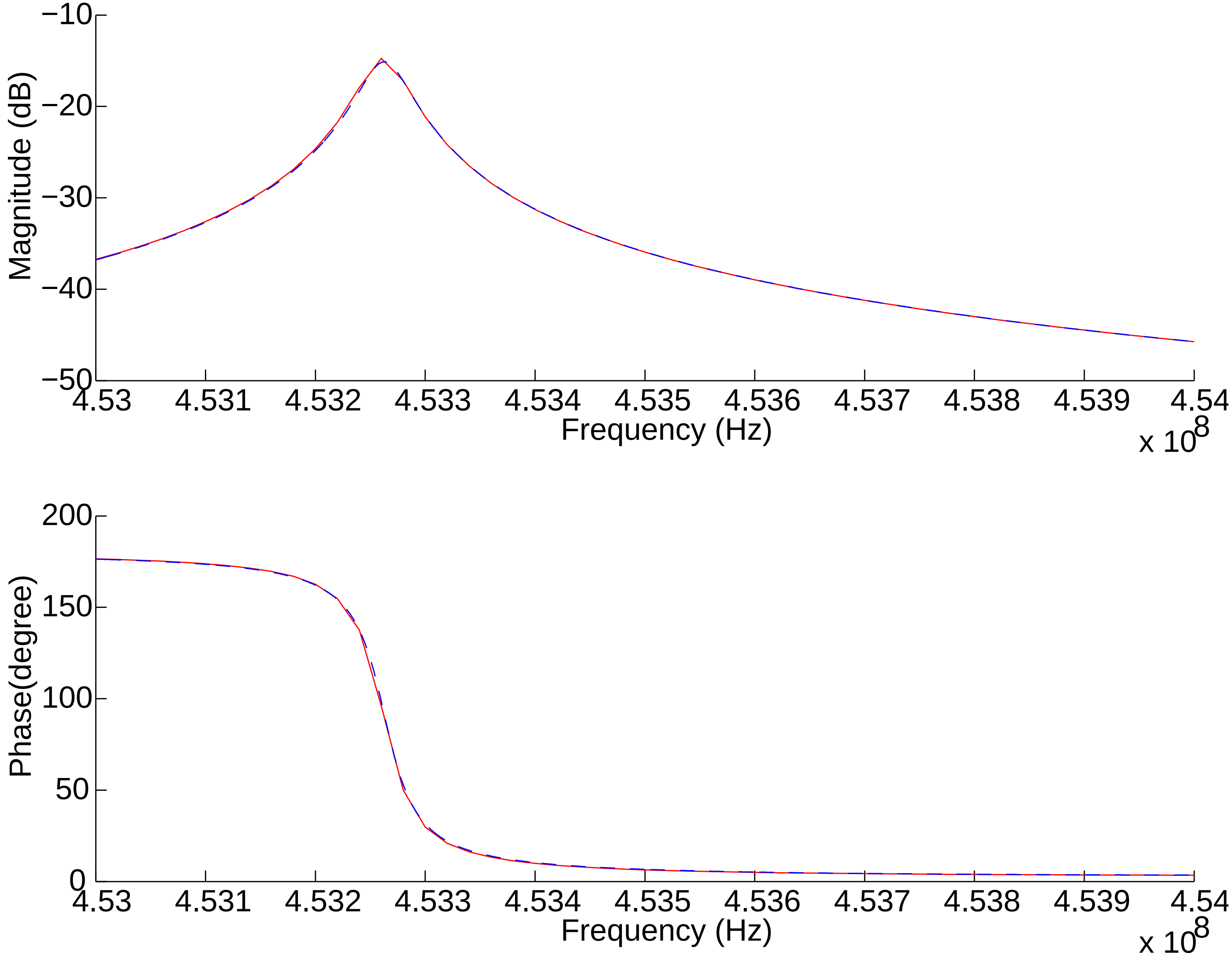
Reduction procedure:

- Run Arnoldi on $(s_0^2 M + K)^{-1}$ to get basis $\hat{Q}_n$
- $Q_n = \mathrm{orth}([\mathrm{Re}(\hat{Q}_n), \mathrm{Im}(\hat{Q}_n)])$
- $M_n = Q_n^T M Q_n$ and $K_n = Q_n^T K Q_n$ are symmetric

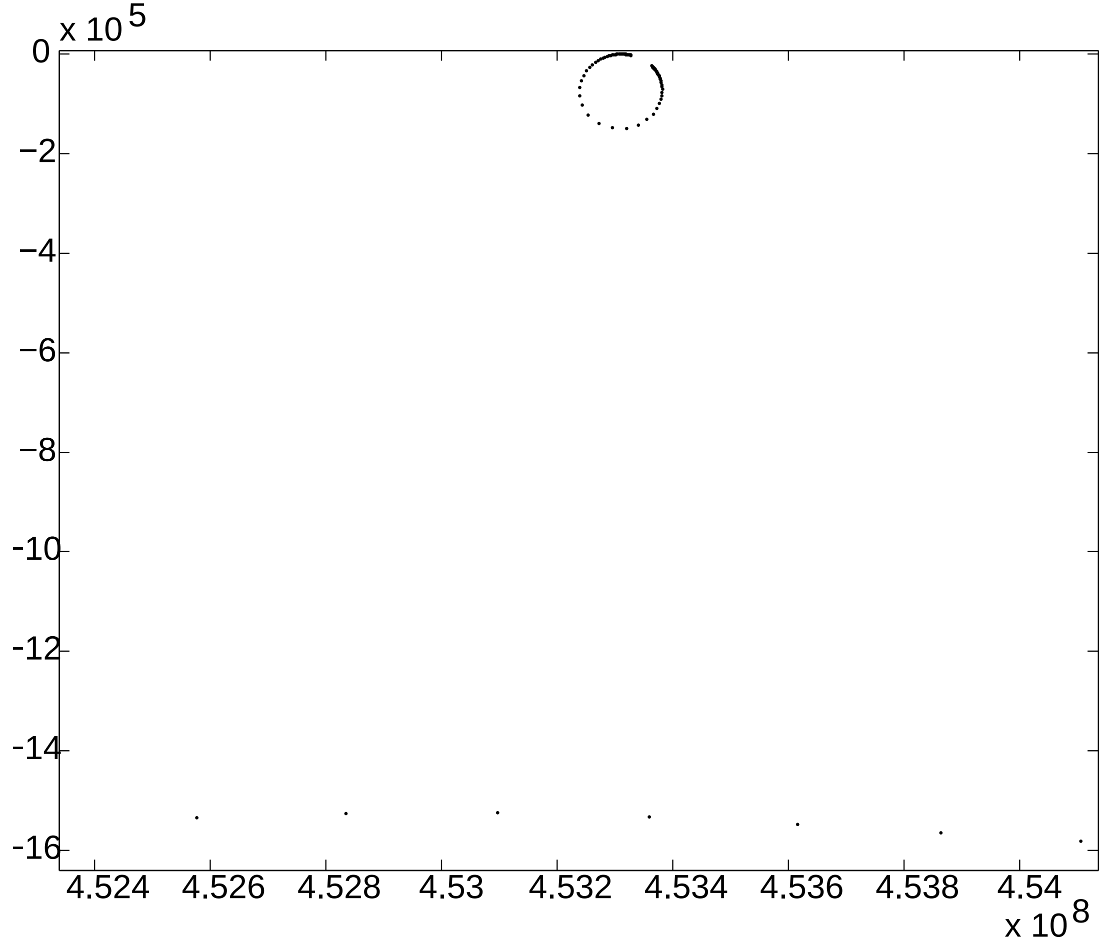
ROM is a Bubnov-Galerkin projection of original equation.

Anchor loss

$$N = 57475 \rightarrow n = 3$$

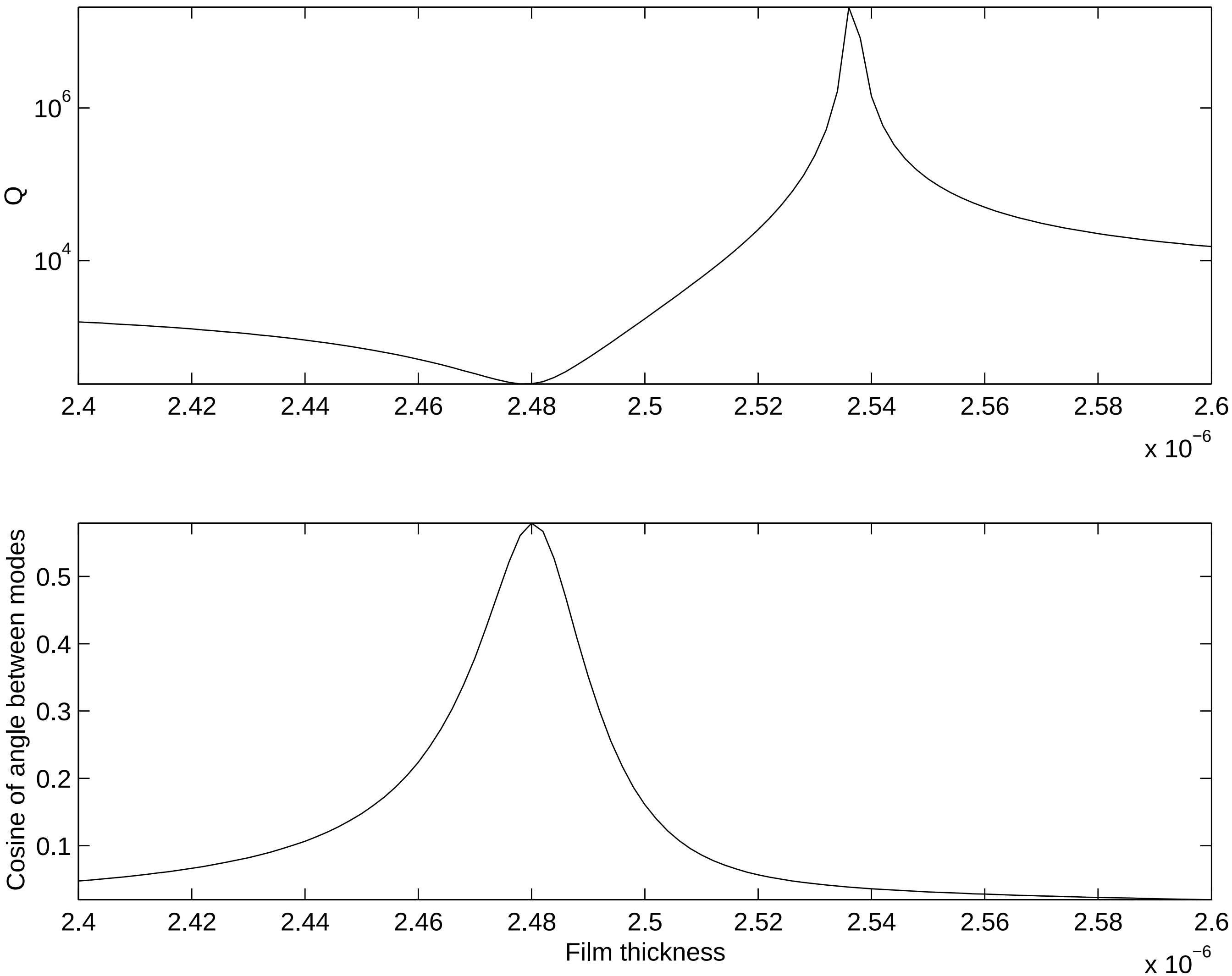


Why $n = 3$?



- Behavior in this region is dominated by two poles
- Mode “mixing” dramatically affects predicted resonance!
- Impact: Eliminate most anchor loss for disk resonator?

Mode-mixing



Q
10^6
10^4
2.4
2.42
2.44
2.46
2.48
2.5
2.52
2.54
2.56
2.58
2.6
x 10^{-6}
Cosine of angle between modes
0.5
0.4
0.3
0.2
0.1
2.4
2.42
2.44
2.46
2.48
2.5
2.52
2.54
2.56
2.58
2.6
Film thickness
x 10^{-6}

Plans and questions

- Testing of mode-interaction
  - What parts of device are SiGe, and what are polySi?
  - Using a self-aligned post?
- Semi-analytical models for anchor loss
  - Some qualitative explanation of mode mixing, etc?
  - Any quantitative hand models in the literature?
- More investigation of geometric sensitivity?

Plans and questions

- QLab software
  - A little awkward to fit into ANSYS (complex mass)
  - Currently a MATLAB / C hybrid
  - Put in TED, too
- Mode-tracking technology and accelerated solvers
- Comparison with approach of Park and Park
- Paper plans?
  - MEMS 05 – mode mixing?
  - JMEMS – anchor modeling?
  - IJNME – PML interpretation?