HW for 2019-06-13

(due: 2019-06-24)

1: Ring around the rosie Consider the undirected ring on n nodes. For $n = 5$, the Laplacian is

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix},$$

and similarly for larger n. For $n = 10$, plot the graph in 2D by placing each node according to the Laplacian eigenmap coordinates. Repeat with a tiny random change to the edge weights; what happens to the coordinates, and why? If you are using MATLAB, you may want to use the gplot command.

2: Almost reducible chains Consider an almost irreducible Markov chain with transition matrix

$$P = P^{\text{ref}} + E, \quad P^{\text{ref}} = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}$$

and suppose P_{11} and P_{22} are both ergodic with unique stationary vectors π_1^* and π_2^*, respectively. We can approximate the stationary distribution of P by

$$\hat{\pi} = \begin{bmatrix} \alpha\pi_1 \\ (1-\alpha)\pi_2 \end{bmatrix};$$

show how to choose α to minimize the residual error $\|(I - P)\hat{\pi}\|^2$.