HW for 2019-05-23

(due: 2019-05-30)

1: Regularized residual The normal equations can be written as a system of two equations in two block unknowns as

$$\begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} r \\ x \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

In this problem, we extend this approach to the least squares problem with regularization

1. Show that the solution to the problem

   $$\text{minimize } \|b - Ax\|^2 + \lambda \|x\|^2$$

   can be written in the form

   $$\begin{bmatrix} I & A \\ A^T & -\lambda I \end{bmatrix} \begin{bmatrix} r \\ x \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}.$$

2. Use Gaussian elimination on the preceding equations to get a linear equation in r alone.

3. Based on the equation in the previous step, compute the derivative $dr/d\lambda$ at $\lambda = 0$.

2: Modified Landweber Consider a modification to the Landweber iteration associated with gradually decreasing step sizes

$$x^{k+1} = x^k - \frac{\alpha}{k+1} A^T (Ax^k - b).$$

1. Show that this iteration converges for any positive α.

2. Argue that step k of the iteration is equivalent to applying the regularized least squares solve

   $$x^k = V f_k(\Sigma)^{-1} U^T$$

   Write a code to plot f_k on $[0, 1]$ for $\alpha = 1$ and $k = 1, 2, \ldots, 10$.