HW for 2019-05-22

(due: 2019-05-22)

1: Constrained least squares Consider the least squares problem

$$\text{minimize } \|Ax - b\|^2 \text{ s.t. } \sum_{j=1}^{n} x_i = 1.$$

Write down the KKT conditions for this problem and write a short code in your favorite language to solve the problem given an economy QR factorization $A = QR$.

2: Residual sensitivity In this exercise, we will analyze how sensitive the norm of the least squares residual is to perturbations in A.

1. Differentiate the expression $r^T r = \|r\|^2$ to show that

$$\delta\|r\| = \frac{r^T(\delta r)}{\|r\|}.$$

2. Differentiate the relationship $r = b - Ax$ and pre-multiply by r^T to get that

$$\delta\|r\| = \frac{-r^T(\delta A)x}{\|r\|}.$$

   Hint: It helps to recall that $A^T r = 0$.

From the last inequality, a standard norm bound shows that

$$\delta\|r\| \leq \|\delta A\|\|x\|,$$

i.e. the residual magnitude can only change significantly with a small change to A if the coefficients in the least squares solve were large.