Midterm

You may refer to your book, your notes, and the MATLAB help files, but not to your classmates. You may use any MATLAB functions you wish in your solutions, and you may use MATLAB to check correctness of your codes. You will be graded primarily on the correctness (and correct efficiency) of your solutions, but please include derivations, test cases, etc. that you think will help me evaluate your solutions. You should submit your solutions in class or electronically by the start of lecture on Monday, October 26.

1. Suppose $A \in \mathbb{R}^{3\times 3}$ is invertible. Write a MATLAB code to compute the singular values of A given $\|A\|_2$, $\|A^{-1}\|_2$, and $\|A\|_F$. Your code should look like

```
function s = p1sv(normA, normAinv, normAfro)
%
% Returns a vector of singular values for a 3-by-3 A s.t.
%   normA    = norm(A),
%   normAinv = norm(inv(A)),
%   normAfro = norm(A, 'fro');
```

2. Suppose $M \in \mathbb{R}^{m\times m}$ is symmetric and positive definite, so that M defines an inner product

$$\langle x, y\rangle_M = x^T M y$$

and a corresponding norm

$$\|x\|_M^2 = \langle x, x\rangle_M.$$

Write a routine to compute $A = UR$ where the columns of $U \in \mathbb{R}^{m\times n}$ are M-orthonormal ($U^T M U = I$) and $R \in \mathbb{R}^{n\times n}$ is upper triangular. Your code should look like

```
function [U,R] = p2ur(A,M)
%
% Compute A = U*R where U has M-orthonormal columns and
% R is upper triangular.
```

3. Suppose $A \in \mathbb{R}^{m \times n}$ has rank r, and `I` and `J` are length r index matrices such that the submatrix `A(I,J)` is nonsingular. Given `I`, `J`, `Arows = A(I,:)` and `Acols = A(J,:)`, write a MATLAB routine to reconstruct all of A. Your code should look like

```
function A = p3reconstruct(I,J,Arows,Acols)
%
% Reconstruct a rank r matrix A s.t. for I and J of length r,
%   Arows = A(I,:);
%   Acols = A(:,J);
%   A(I,J) is nonsingular
```

4. Suppose $A, B \in \mathbb{R}^{m \times n}$ where $m > n$ and A is full rank. Devise an $O(mn^2)$ algorithm to find the unit upper triangular $U \in \mathbb{R}^{n \times n}$ to minimize $\|AU - B\|_F^2$. Your code should look like

```
function [U] = p4solve(A,B)
%
% Find unit upper triangular U to minimize norm(A*U-B,'fro').
```

5. Suppose the economy QR decomposition $A = QR$ is given ($A \in \mathbb{R}^{m \times n}$), as are vectors $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$. Assuming both A and $A + uv^T$ are full rank, write an $O(mn)$ algorithm to minimize $\|(A + uv^T)x - b\|_2$. Your code should look like

```
function x = p5solve(Q, R, u, v, b)
%
% Given [Q,R] = qr(A,0), find x to minimize
% norm( (A+u*v')*x-b )
```