HW 3
Due: Fri, Mar 4

1: Pesky polynomials We would like the best quartic (degree 4) approximation to \(\cos(x)\) on \([-1, 1]\) in a least squares sense; that is,

\[
\text{minimize } \int_{-1}^{1} |p(x) - \cos(x)|^2 \, dx
\]

Set up and solve in MATLAB, and compare to a solution based on sampling at a uniform mesh of ten points. *Hint:* \(\int_{-1}^{1} x^k \cos(x) \, dx = 26 \sin(1) - 40 \cos(1)\) for \(k = 4\) and \(4 \cos(1) - 2 \sin(1)\) for \(k = 2\).

2: QR to SVD Suppose \(A = QR\) is an economy QR factorization. Show that the singular values of \(A\) are the same as those of \(R\).

3: Vector projector Suppose \(A \in \mathbb{R}^{m \times n}\) where \(m > n\) has full column rank. Given \(A\) and a vector \(b\), write one line of MATLAB to compute the element \(c\) in the range space of \(A\) that is nearest to \(b\) (in the Euclidean norm).

4: Generally speaking Often, we use least squares to construct models of the world. We assume that the "truth" is

\[Ax = b,\]

but what we measure is the first few rows of \(A\) and \(b\) (which we write as \(A_1\) and \(b_1\)), and those measurements are corrupted by noise. Suppose we have \(A\) exactly, but only get the noisy partial right hand side \(\hat{b}_1 = b_1 + e_1\), from which we form

\[
\text{minimize } \|A_1 \hat{x} - \hat{b}_1\|^2.
\]

Our goal in this problem is to use the error analysis ideas in Section 6.2 to figure out the inherited error in the reconstruction of \(\hat{b}_2 = A_2 \hat{x}\).

1. Let \(e_2 = \hat{b}_2 - b_2\). Argue briefly that \(e_2 = A_2 A_1^\dagger e_1\).

2. Show that

\[
\frac{\|e_2\|}{\|b_2\|} \leq \kappa(A_2 A_1^\dagger) \frac{\|e_1\|}{\|b_1\|}.
\]

Things get somewhat more complicated if we also allow the entries of \(A\) to be contaminated by error, though the same basic ingredients come into play.