PS 5
Due: Mon, Mar 2

Note: My solutions this week fit on the front and back of a page (typed), including the graph for problem 1 and problem text. Don't overthink these!

1: By the book   Book problem 6.4.1.

2: QR to SVD   Suppose $A = QR$ is an economy QR factorization. Show that the singular values of A are the same as those of R.

3: Vector projector   Suppose $A \in \mathbb{R}^{m \times n}$ where $m > n$ has full column rank. Given A and a vector b, write one line of MATLAB to compute the element c in the range space of A that is nearest to b (in the Euclidean norm).

4: Generally speaking   Often, we use least squares to construct models of the world. We assume that the "truth" is

$$Ax = b,$$

but what we measure is the first few rows of A and b (which we write as A_1 and b_1), and those measurements are corrupted by noise. Suppose we have A exactly, but only get the noisy partial right hand side $\hat{b}_1 = b_1 + e_1$, from which we form

$$\text{minimize } \|A_1 \hat{x} - \hat{b}_1\|^2.$$

Our goal in this problem is to use the error analysis ideas in Section 6.2 to figure out the inherited error in the reconstruction of $\hat{b}_2 = A_2 \hat{x}$.

1. Let $e_2 = \hat{b}_2 - b_2$. Argue *briefly* that $e_2 = A_2 A_1^\dagger e_1$.

2. Show that
$$\frac{\|e_2\|}{\|b_2\|} \leq \kappa(A_2 A_1^\dagger) \frac{\|e_1\|}{\|b_1\|}.$$

Things get somewhat more complicated if we also allow the entries of A to be contaminated by error, though the same basic ingredients come into play.