HW 5

Due by CMS by 11:59 on Monday, April 2

Remember that you may (and should!) talk about the problems amongst yourselves, or discuss them with me or the TA, providing attribution for any good ideas you might get – but your final write-up should be your own.

1: Differentiating an interpolant. The code `hw5newton` (available on CMS) computes a vector of divided differences for use in the Newton form of interpolation. Using these divided differences, write a routine `hw5neval` that evaluates the interpolating polynomial p and its derivative p':

```
% [pxx, dpxx] = hw5neval(x,fdd, xx)
%
% Evaluate the Newton form of the interpolant at points xx.
% Inputs:
%   x    – coordinates of interpolation nodes
%   fdd  – table of divided differences returned by hw5newton
%   xx   – evaluation points
%
% Outputs:
%   pxx  – interpolating polynomial evaluated at the points xx
%   dpxx – derivative of the interpolating polynomial evaluated at xx

function [pxx, dpxx] = hw5neval(x,fdd, xx)
```

Your code should only involve $O(n)$ work per evaluation point, where n is the number of interpolation points. You may want to test your code using the `hw5p1test` script (also on CMS).

2: Newton at work Compute one solution of the equations

$$\begin{aligned} (x_1 + 3)(x_2^2 - 7) + 18 &= 0 \\ \sin(x_2 \exp(x_1) - 1) &= 0 \end{aligned}$$

using Newton's method, with the initial guess

$$x^0 = \begin{bmatrix} -0.5 \\ 1.4 \end{bmatrix}$$

Use MATLAB's `semilogy` to show the norm of the update $\Delta x^k = x^{k+1} - x^k$ for the first five steps. Also, print

$$\phi_k = \frac{\log(\|\Delta x^{k+1}\|)}{\log(\|\Delta x^k\|)}$$

at each step. If you've implemented everything correctly, ϕ_k should approach 2 (up until you hit the roundoff floor). You should understand why this is true, but I'm not going to ask you to explain it for the homework.

Your submission for this problem should consist of a single MATLAB script `hw2p2.m` that runs the Newton iteration, produces the semi-logarithmic plot, and prints ϕ_k.