HW 2
Due by lecture on Wed, Feb 8

Remember that you may (and should!) talk about the problems amongst yourselves, or discuss them with me or the TA, providing attribution for any good ideas you might get – but your final write-up should be your own.

1: Mr. Fix-It
Consider the fixed point iteration

\[ x_{k+1} = \frac{x_k}{4} \left( 5 - ax_k^3 \right) \]

- What does the iteration converge to?
- Show the iteration converges linearly and compute the rate constant.

Note: You should be able to work this out purely analytically, but please do check your work against a numerical experiment.

2: Water, water
The dispersion relation for shallow water waves is

\[ \omega^2 = k \left( g + \frac{T}{\rho} k^2 \right) \tanh(kh) \]

where

- \( h \) = water depth
- \( k \) = spatial wave number (2\( \pi \) / wave length)
- \( \omega \) = frequency (2\( \pi \) / period)
- \( T \) = surface tension
- \( \rho \) = mass density
- \( g \) = gravitational acceleration.

For water at 25C, \( T/\rho = 7.2 \times 10^{-5} \) N/m\(^4\), and the acceleration due to gravity is \( g = 9.8 \) m/s\(^2\). Assuming these values, write a code using Newton’s method to find \( k \) given \( \omega \) and \( h \), assuming \( kh \ll 1 \). Your routine should take the form

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function k = hw2p2(omega, h)
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3: Devilish differences  Consider the function

\[ f(x) = \sin(x) + \text{erf}(x) \]

where \( \text{erf} \) denotes the error function

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) \, dt. \]

This function has an infinite sequence of positive roots \( 0 < r_1 < r_2 < r_3 < \ldots \). Write a function to compute the \( d_1 = r_2 - r_1 \) and \( d_2 = r_4 - r_3 \). Your function should have the interface

\[ \text{function } [d1, d2] = hw2p3() \]

Notes: MATLAB provides an \texttt{erf} function, but you will probably find the \texttt{erfc} function (\( \text{erfc}(x) = 1 - \text{erf}(x) \)) more useful if you want to rewrite \( f \) so that you can evaluate it more accurately in the regions of interest. You probably will not want to use \( x \) as the main variable in your computation. I changed variables, used a power series to estimate the relevant roots, and then applied Newton. You may choose another strategy, but you should believe your answers are correct to at least six significant figures.