HW 2

Due by lecture on Wed, Feb 8

Remember that you may (and should!) talk about the problems amongst yourselves, or discuss them with me or the TA, providing attribution for any good ideas you might get – but your final write-up should be your own.

1: Mr. Fix-It Consider the fixed point iteration

$$x_{k+1} = \frac{x_k}{4}\left(5 - a x_k^3\right)$$

- What does the iteration converge to?
- Show the iteration converges linearly and compute the rate constant.

Note: You should be able to work this out purely analytically, but please *do* check your work against a numerical experiment.

2: Water, water The dispersion relation for shallow water waves is

$$\omega^2 = k\left(g + \frac{T}{\rho}k^2\right)\tanh(kh)$$

where

$$\begin{aligned} h &= \text{water depth} \\ k &= \text{spatial wave number } (2\pi \text{ / wave length}) \\ \omega &= \text{frequency } (2\pi \text{ / period}) \\ T &= \text{surface tension} \\ \rho &= \text{mass density} \\ g &= \text{gravitational acceleration.} \end{aligned}$$

For water at 25C, $T/\rho = 7.2 \times 10^{-5}$ N/m^4, and the acceleration due to gravity is $g = 9.8$ m/s^2. Assuming these values, write a code using Newton's method to find k given ω and h, assuming $kh \ll 1$. Your routine should take the form

```
function k = hw2p2(omega, h)
```

3: Devilish differences Consider the function

$$f(x) = \sin(x) + \operatorname{erf}(x)$$

where erf denotes the *error function*

$$\operatorname{erf}(x)\frac{2}{\sqrt{\pi}} = \int_0^x \exp(-t^2)\,dt.$$

This function has an infinite sequence of positive roots $0 < r_1 < r_2 < r_3 < \ldots$. Write a function to compute the $d_1 = r_2 - r_1$ and $d_2 = r_4 - r_3$. Your function should have the inferface

function [d1, d2] = hw2p3()

Notes: MATLAB provides an `erf` function, but you will probably find the `erfc` function (erfc(x) = 1 − erf(x)) more useful if you want to rewrite f so that you can evaluate it more accurately in the regions of interest. You probably will not want to use x as the main variable in your computation. I changed variables, used a power series to estimate the relevant roots, and then applied Newton. You may choose another strategy, but you should believe your answers are correct to at least six significant figures.