Music of the Microspheres: Eigenvalue Problems from Micro-Gyro Design

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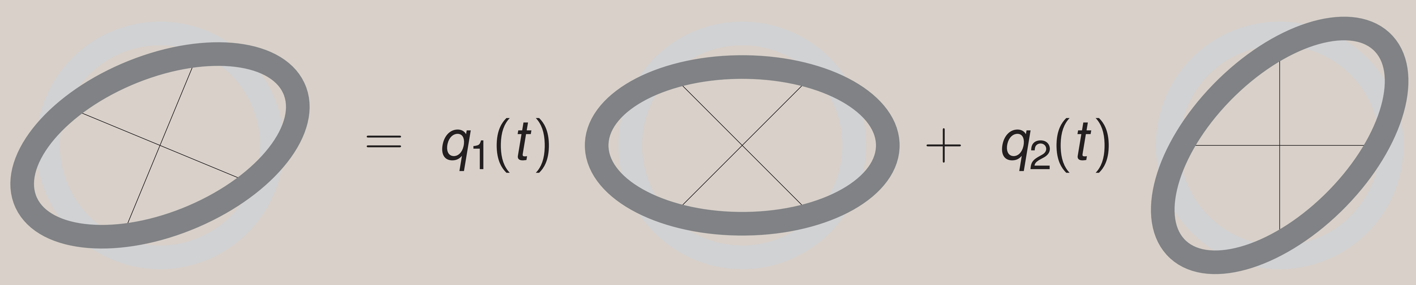
Hemispherical Rate Gyroscope (HRG)

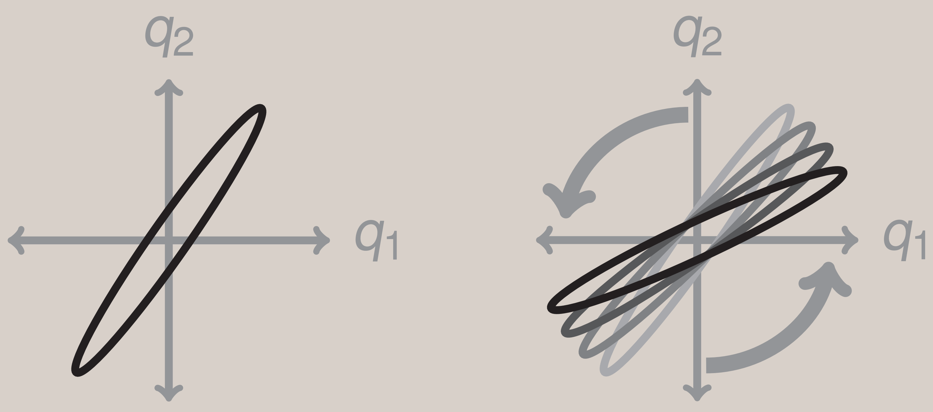
- Developed by Northrup-Grumman (c. 1965–early 1990s)
- Rate integrating, accurate, low power
- Gyro of choice for space applications
- Cost: ≈ \$100K / axis

Basic Dynamics: Solid-Wave Foucault Pendulum

Same 2D model describes the HRG and Foucault's pendulum.

Motion is described in a configuration space:





Stationary frame   Rotating frame

Oscillators coupled by Coriolis effect (rate of rotation is $\Omega \ll \omega_0$):

$$\ddot{\mathbf{q}} + 2\beta\Omega \mathbf{J}\dot{\mathbf{q}} + \omega_0^2 \mathbf{q} = 0, \qquad \mathbf{J} \equiv \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Solution $\mathbf{q}^0$ in stationary frame precesses at rate $\beta\Omega$:

$$\begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} \approx \begin{bmatrix} \cos(-\beta\Omega t) & -\sin(-\beta\Omega t) \\ \sin(-\beta\Omega t) & \cos(-\beta\Omega t) \end{bmatrix} \begin{bmatrix} q_1^0(t) \\ q_2^0(t) \end{bmatrix}.$$

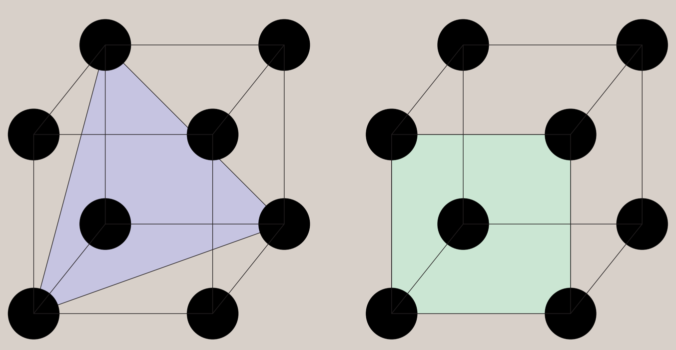
DARPA Micro-Rate Integrating Gyro (MRIG) project

Courtesy Laura Fegely and Sunil Bhave

- Goal: cheap micro-HRGs
- Fab at Cornell lead by Sunil Bhave
- Our work: simulation to aid design
  - What practically limits accuracy?
  - How do we solve relevant physics?
- Many questions involve *imperfections*

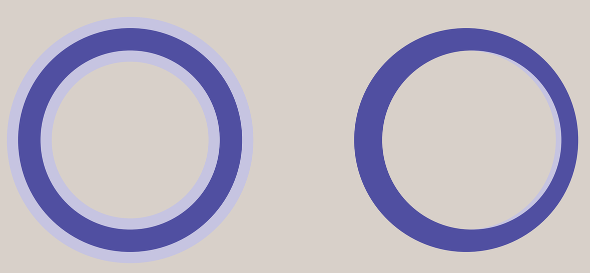
Imperfections

Micro-fabrication methods (etch and deposit) are not perfect.



Etch depend on crystal direction, distorts bowls etched in wafers:

- (111) wafers ⟹ triangular
- (100) wafers ⟹ square.



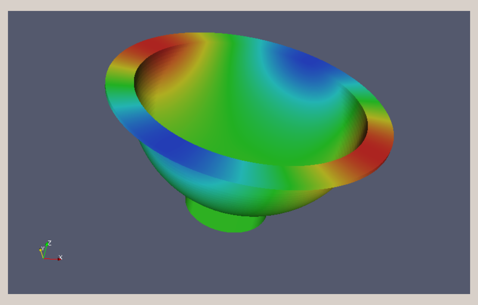
Might see changes in device thickness:

- Over/under etch errors
- Mask misalignment errors

Or thicker layers near deposition gas inflow.

These are major impediments to HRG miniaturization.

The Perturbation Picture

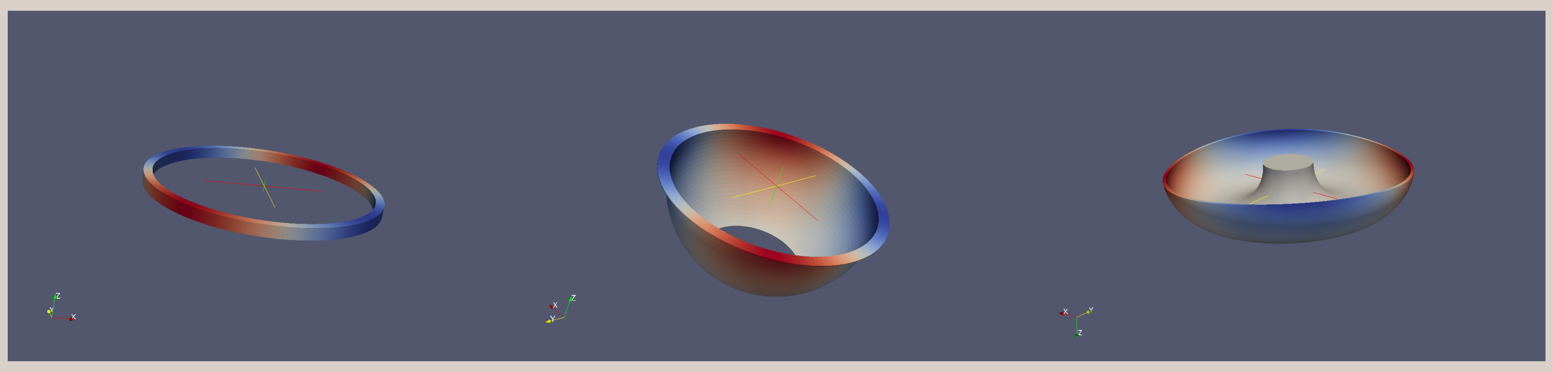


Perturbations split degenerate modes:

- Coriolis forces (good)
- Imperfect fab (bad, but physical)
- Discretization error (non-physical)

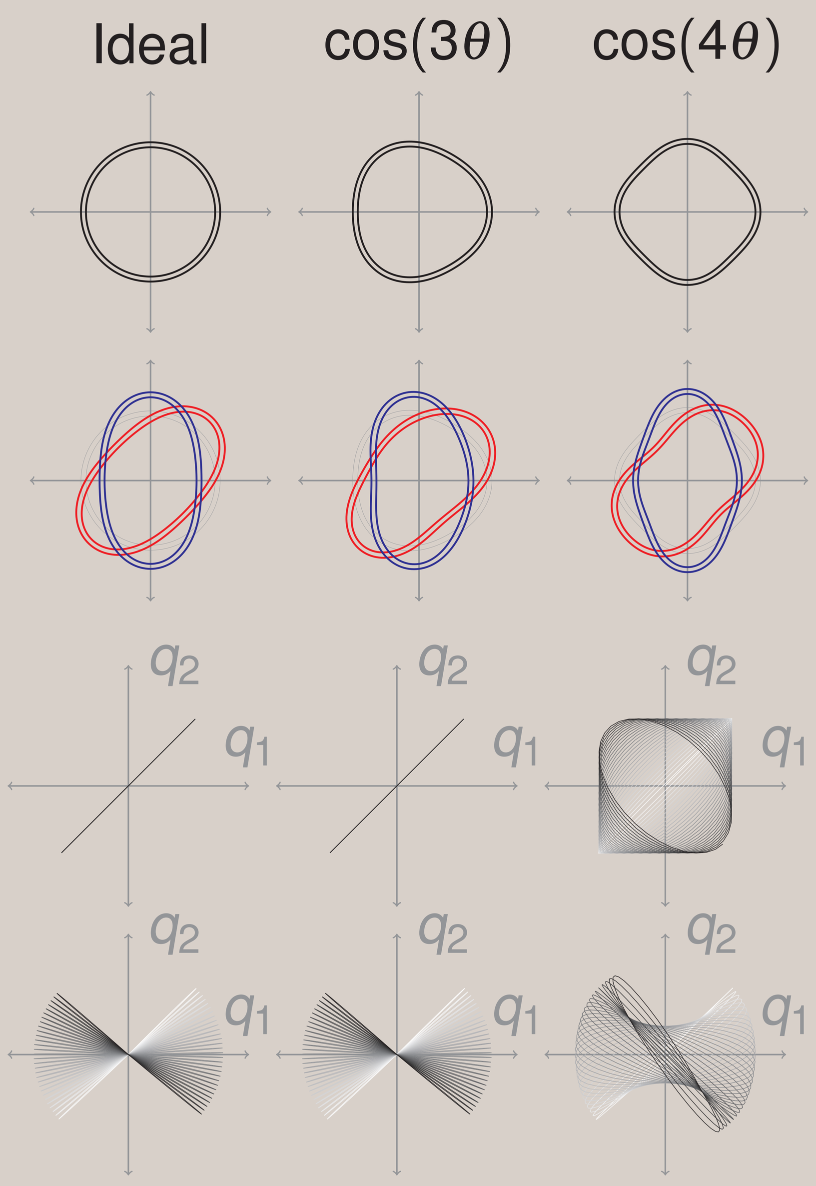
Goal: Fast, accurate simulation + qualitative understanding.

AxFEM: Finite Elements for Almost-Axisymmetric Structures



- Basic approach: Fourier analysis
  - Write motion *and* distortion in an axisymmetric reference system
  - Distortion involves few azimuthal numbers
- Use Fourier for simulation
  - Accurate: Discretization doesn't break symmetry
  - Fast: 3D problem → weakly coupled 2D problems
- Use *selection rules* for simulation and understanding
  - Modes with azimuthal numbers m, n, perturbation with p
  - Strongest coupling when $p = |m \pm n|$
  - Worst case: operate at m, distort at $p = 2m$

Analyzing Imperfect Rings



Simulate two distortions; compare $m = 2$ modes

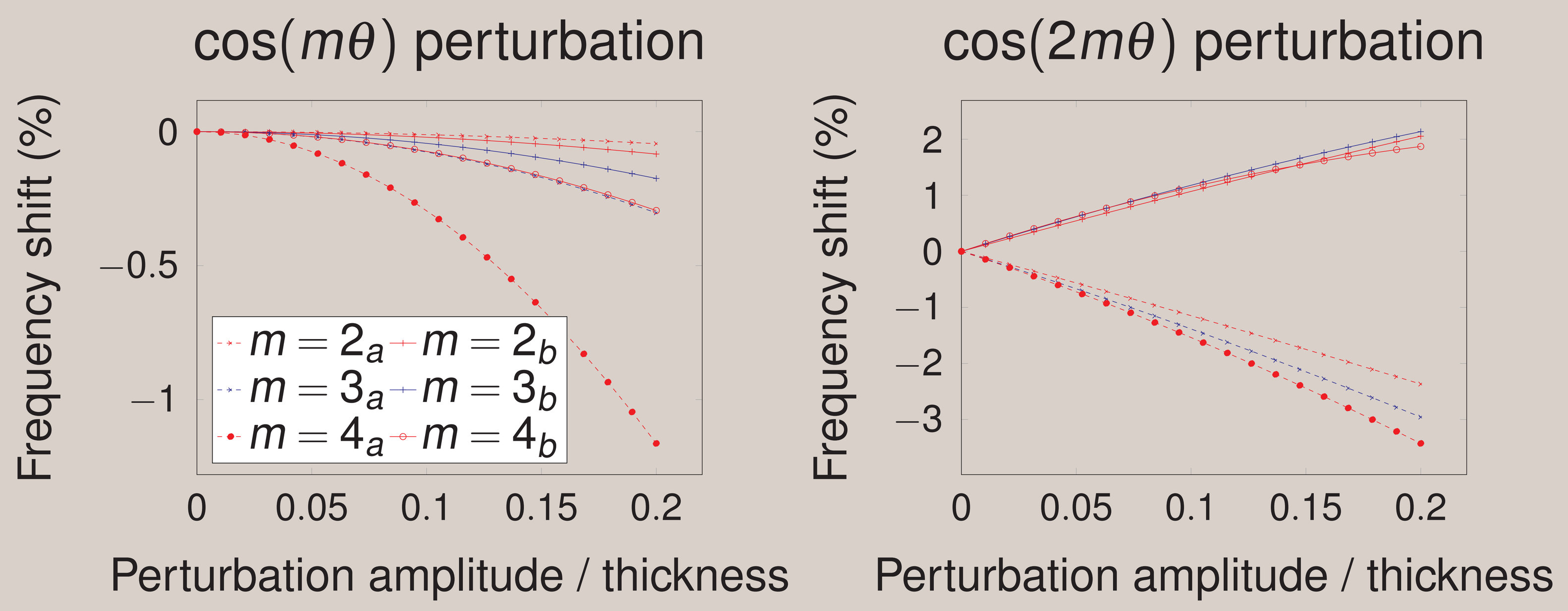
- Ideal: same
- $p = 3$: same
- $p = 4$: different!

Consider signal at rest (top) and rotating (bottom)

- $p = 3$: Ideal
- $p = 4$: Badly biased

As theory predicts!

Frequency Split for Rings



$p = 2m$ perturbations result in first-order frequency change; $p = m$ only changes frequency at second order.

For More

Erdal Yilmaz and David Bindel
"Effects of Imperfections on Solid-Wave Gyroscope Dynamics"
Proceedings of IEEE Sensors 2013, Nov 3–6.