Decoding the structure of the WWW: facts versus sampling biases

M. Ángeles Serrano\textsuperscript{1}  
mdserran@indiana.edu  
Ana Maguitman\textsuperscript{1}  
anmaguit@cs.indiana.edu  
Marián Boguñá\textsuperscript{2}  
marian.boguna@ub.edu  
Santo Fortunato\textsuperscript{1,3}  
santo@indiana.edu  
Alessandro Vespignani\textsuperscript{1}  
alexv@indiana.edu  

\textsuperscript{1} School of Informatics, Indiana University  
Bloomington, IN 47406, USA  
\textsuperscript{2} Departament de Física Fonamental, Universitat de Barcelona  
08028 Barcelona, Spain  
\textsuperscript{3} Fakultät für Physik, Universität Bielefeld  
D-33501 Bielefeld, Germany

ABSTRACT

The understanding of the immense and intricate topological structure of the World Wide Web (WWW) is a major scientific and technological challenge. This has been tackled recently by characterizing the properties of its representative graphs in which vertices and directed edges are identified with web-pages and hyperlinks, respectively. Data gathered in large scale crawls have been analyzed by several groups resulting in a general picture of the WWW that encompasses many of the complex properties typical of rapidly evolving networks \cite{4, 9, 19, 1, 12}. In this paper, we report a detailed statistical analysis of the topological properties of four different WWW graphs obtained with different crawlers. We find that, despite the very large size of the samples, the statistical measures characterizing these graphs differ quantitatively, and in some cases qualitatively, depending on the domain analyzed and the crawler used for gathering the data. This spurs the issue of the presence of sampling biases \cite{17} and structural differences of Web domains that might induce properties not representative of the actual global underlying graph. In order to provide a more accurate characterization of the Web graph and identify observables which are clearly discriminating with respect to the sampling process, we study the behavior of degree-degree correlation functions and the statistics of reciprocal connections. The latter appears to enclose the relevant correlations of the WWW graph and carry most of the topological information of the Web. The analysis of this quantity is also of major interest in relation to the navigability and searchability of the Web.

Categories and Subject Descriptors

H.4.m [Information Systems]: Miscellaneous; G.3 [Mathematics and Computing]: Probability and Statistics

Keywords

Web graph structure, Web measurement, crawler biases, statistical analysis

1. INTRODUCTION

The World Wide Web (WWW) has grown at an unprecedented pace. While it is not possible to provide a precise estimate of the WWW size in terms of pages, a recent study \cite{16}, which used Web searches in 75 different languages, determined that there were over 11.5 billion Web pages in the publicly indexable Web \cite{21, 22} at the end of January 2005. Furthermore, the Web growth lacks any regulation and physical constraint (contrary to what happens with the physical Internet infrastructure \cite{27}), with new documents being added or becoming obsolete very quickly.

A fundamental step in decoding and understanding the WWW organization consists in the experimental studies of the WWW graph structure in which vertices and directed edges are identified with Web pages and hyperlinks, respectively. These studies are based on crawlers that explore the WWW connectivity by following the links on each discovered page, thus reconstructing the topological properties of the representative graph. Several studies based on those graphs have been performed in order to reveal the large-scale topological properties of the WWW. Distributions of in-degrees and out-degrees have been found to exhibit heavy-tails and the macroscopic architecture of connected components has made evident a rich structural organization, i.e., the so-called bow-tie structure \cite{20, 4, 5, 9, 12}. Reciprocal links and transitive relations regarding thematic communities \cite{14} have attracted attention as well, giving rise to a generally accepted picture of the topological structure of the WWW.

While the importance of these studies is indisputable, the dynamical nature of the Web and its huge size make very difficult the process of compressing, ranking, indexing or mining the Web. Indeed, even the largest scale Web crawlers cover only a small portion of the publicly available information. In other words, it has been impossible so far to achieve any complete unbiased large-scale picture of the Web. On the other hand, the very large sizes of the gathered data sets have led to the general belief that the structural
and statistical properties observed in the WWW graphs were representative of the actual ones, thus leaving almost untouched the study of possible sampling biases [17]. In this respect, on the one hand it is crucial to understand clearly which is the exact information provided by crawl engines, and, on the other hand, to explore to which extent the Web properties we observe are not biased by the specific characteristics of the crawls.

In this paper, we study four different data sets obtained in different years with different crawls and for different domains of the WWW. We provide a careful analysis of the structural and statistical topological properties of the resulting Web graphs, making evident qualitative and quantitative differences across different samples. The presented analysis questions previously accepted results such as the stability of the bow-tie structure and the statistical properties of the out-degree distribution, providing evidence for sampling biases and domain’s dependent characteristics not addressed so far. We look at higher order statistical indicators characterizing single and two vertices degree-degree correlations in order to provide a full account of the connectivity pattern and structural ordering of the Web graph. We identify in reciprocal links [15], also referred in the literature as bidirectional links [7] or co-links [14], an important topological feature able to clearly discriminate among the statistical properties resulting from different crawls. Reciprocal links appear also to account for most of the statistical correlations present in Web graphs. The inspection of the subgraphs of vertices reciprocally connected provides interesting structural information. The structure and the correlations induced by reciprocal links are therefore important observables that might be crucial to assess how the underlying topology could affect processes running on the network as well as its functionality [7]. Indeed, navigability and searchability are intimately related to the functionality of the WWW, and those properties strongly depend on the communication patterns among the constituent sites of the network. In this sense, reciprocal edges deserve special attention since they arise as structural entities playing a key role in the organization of the WWW.

2. RELATED WORK

The first empirical topological studies of the Web as a directed graph focused on the measure of the directed degree distributions \( P(k_{in}) \) and \( P(k_{out}) \), where the in/out-degree, \( k_{in} \), or \( k_{out} \) respectively, is defined as the number of incoming/outgoing links connecting a page to its neighbors. The work by Kumar et al. [19] on a big crawl of about 40M nodes, and that by Barabási and Albert [4] on a smaller set of over 0.3M nodes restricted to the domain of the University of Notre Dame, suggested a scale-free nature for the WWW with power-law behaviors both for the in- and out-degree distributions.

Immediately after, a more complete investigation was published by Broder et al. [9]. There, two sets from AltaVista crawls were analyzed, corresponding to different months in the same year 1999, May and October. The sets were over 200 million pages and 1.5 billion links. The authors reported detailed measurements on local and global properties of the Web graph which covered, for instance, the degree distributions, corroborating earlier observations, and also the presence and organization of connected components, unfolding the so-called bow-tie structure of the Web. One of the most intriguing conclusions there was that, from the analysis of those two sets, the observed structure of the Web was relatively insensitive to the particular large crawl used. In addition, the connectivity structure of the Web was resilient to the removal of a significant number of nodes.

Successively, further work [12] along the same lines has been performed over a large 2001 data set of 200M pages and about 1.4 billion edges made available by the WebBase project at Stanford (See next section for references and a project description). In this work, new measures were introduced along with the standard statistical observables, and the obtained results were compared with the ones presented in the work by Broder et al.. One of the reported differences is the deviation from the power-law behavior of the out-degree distribution.

3. DATA SETS

To gain some insight about how the crawling strategy affects observations and on the existence of observable unbiased properties we have analyzed and compared four sets of data corresponding to different years, from 2001 to 2004, and different domains, general and .uk and .it domains. The sets have been gathered within two different projects: the WebBase project and the WebGraph project, each using its own Web crawler, WebVac and Ubicrawler respectively. The WebBase Project is a World Wide Web repository built as part of the Stanford Digital Libraries Project by the Stanford University InfoLab 1. The Stanford WebBase project [13] is investigating various issues in crawling, storage, indexing, and querying of large collections of Web pages. The project aims to build the necessary infrastructure to facilitate the development and testing of new algorithms for clustering, searching, mining, and classification of Web content. The Stanford WebBase has been collected by the spider WebVac and makes available a Web repository with access to general crawls, such as the ones used in this research, or specific domain crawls restricted, for instance, to universities or institutions. The incremental WebVac crawler [9,3] generally crawls to a depth of 10 levels fetching a maximum of 10-20k pages, with a pause of 2-8 seconds between pages depending on site ranking.

The WebGraph Project 3 is being developed by the Laboratory for Web Algorithmics 4 (LAW) at the University of Milano and analyzes data obtained by its own crawler, Ubicrawler [8], designed to achieve high scalability and to be tolerant to failures.

The above projects provide several data sets publicly available to researchers. We analyze four samples ranging from 2001 to 2004. The WebBase general crawl of 2001 (WBGC01)5 and the WebBase general crawl of 2003 (WBGC03)6 have been collected by the WebBase project in a general crawl using the WebVac spider. The remaining two sets collected by the Ubicrawler project, the WebGraph .uk domain of 2002 (WGBK02)7 and WebGraph .it domain of 2004 (WGIT04)8, are restricted to the domains .uk and .it, respectively. Note that the two domain crawls present a fundamental difference. While pages in the .uk domain have higher probability to point out to pages outside the domain, due to English being the official language in other influential countries, such as the USA, and to the widespread use of English, the links in the Italian .it domain may be much more endogenous, which could potentially have a high effect in the Web description derived from the data.

We have cleaned the four sets by disregarding multiple links and self-connections. In Table 1 we present a summary of the size in vertices and directed edges of the four sets analyzed in this paper.

1 http://www-db.stanford.edu/
2 http://dbpubs.stanford.edu:8091/testbed/doc2/WebBase/
3 http://webgraph.dsi.unimi.it/
4 http://law.dsi.unimi.it/
5 http://ubi.iit.cnr.it/projects/ubicrawler/
6 ftp://db.stanford.edu/pub/webbase/
7 ftp://db.stanford.edu/pub/webbase/
8 http://webdata.iit.cnr.it/united kingdom-2002/
9 http://webdata.iit.cnr.it/italy-2004/
4. STRUCTURAL PROPERTIES

Data gathered in large scale crawls [20, 4, 5, 9, 14, 12] have uncovered the presence of a complex architecture underlying the structure of the Web graph. A widespread feature is the small-world property. Despite its huge size, the average number of URL links that must be followed to navigate from one document to the other, technically the average shortest path length, seems to be very small as compared to the value for a regular lattice of comparable size, and it seems to grow with the system size very slowly at a logarithmic pace [2, 9]. Another important result is that the WWW exhibits a power-law relationship between the frequency of vertices and their degree, defined as the number of directed edges linking each vertex to its neighbors. This last feature is the signature of a very complex and heterogeneous topology with statistical fluctuations extending over many length scales [2, 4, 20]. Finally, a fascinating macroscopic description of the Web has been provided by the study of the connected components, taking into account the directed nature of the Web graph [2]. In the following, we perform a careful comparative analysis of the four Web crawls described in the previous section. This will allow us to critically examine the stability of the various results as a function of the crawl and discuss which properties appear to be genuine features of the global Web graph.

4.1 Sizes of connected components

The directed nature of the Web brings out a complex structure of connected components [27, 13] that has been captured in the famous bow-tie architecture highlighted in the study presented in [9]. If we disregard the directedness of links, the weakly connected component of the graph is made by all pages belonging to the giant component of the undirected corresponding graph. The undirected component becomes internally structured when the directed nature of the connections is considered. The most important of these new components is called the strongly connected component (SCC), which includes all pages mutually connected by a directed path. The other two relevant components are the in-component (IN) and the out-component (OUT). The first one is formed by the vertices from which it is possible to reach the SCC by means of a directed path. The second one refers to the set of vertices that can be reached from the SCC by means of a directed path. Finally, other secondary structures can also be present, such as tendrils, which contain pages that cannot reach the SCC and cannot be reached from it, or tubes which can directly connect the IN and OUT components without crossing the SCC. This complex composition is usually called the bow-tie structure because of the typical shape assumed by the figure sketching the relative size of each component (see Fig. 1). It is clear that such a component structure is extremely relevant in the discussion of the functionalities of the Web. For instance, the relative sizes of the SCC and the IN and OUT components give us information about the probabilities of returning to an original page after exploration, or the size of the accessible Web once a starting page has been selected. The size of the SCC is of particular importance, since it constitutes the subset of reversible and complete access navigability. When one starts to surf the Web from the IN component, it is very likely that after a while one ends up in the SCC, and maybe eventually in the OUT component, but can never go back to the original point. Once in the OUT component, one can never go back to the other main components. But within the SCC, all nodes are reachable and can be revisited.

The analysis of the four data sets considered in the present study shows a striking variability of the basic component structure of the resulting graph. We summarize the values for the sizes of the components in Table 2. In particular, the IN component is a very unstable feature that ranges from accounting for about 20% of the total structure to the case in which it is practically absent (WGIT04). This variability could be likely ascribed to the different crawling strategies and the fact that each of those may use different starting points. Moreover, crawlers perform a directed exploration in the sense that they follow outgoing hyperlinks to reach pointed pages, but cannot navigate backwards using incoming hyperlinks. This implies that the exploration of the IN component is strongly biased by the initial conditions used to start the crawl. Variations are however not limited to the IN component. Also the relative sizes of the SCC and the OUT component vary by a factor of two from sample to sample. Interestingly, the sum of the IN, OUT and SCC components amounts only to about 40% of the total size of the WGUK02, in contrast to the rest of the sets which show much higher values. This could be due to a large number of pages within the set pointing to other pages written in English but outside the.uk domain. In summary, it is evident from this analysis that the structure of Web graphs is strongly dependent on the crawler strategies as well as on...
the domain of the Web considered.

4.2 Degree distributions

A major interesting feature found in Web graphs is the presence of a highly heterogeneous topology, with degree distributions characterized by wide variability and heavy tails. The degree distribution $P(k)$ for undirected networks is defined as the probability that a node is connected to $k$ other nodes. For directed networks, this function splits in two separate functions, the in-degree distribution $P(k_{in})$ and the out-degree distribution $P(k_{out})$, which are measured separately as the probabilities of having $k_{in}$ incoming links and $k_{out}$ outgoing links, respectively. In Figs. 4 and 6 we report the behavior of the in-degree and out-degree distributions. These distributions, as for most real world networks, are found to be very different from the degree distribution of a random graph or an ordered lattice. They are both skewed and spanning several orders of magnitude in degree values. The in-degree distribution exhibits a heavy-tailed form approximated by a power-law behavior $P(k_{in}) \sim k_{in}^{-\gamma_{in}}$, generally over 3 to 4 orders of magnitude. In Figure 4 we show the region considered in the evaluation of the exponent obtained by a maximum likelihood algorithm for discrete distributions. The in-degree distributions also exhibit a noisy tail that cannot be well fitted with a specific analytic form. Yet it strengthens the evidence for the heavy-tailed character of $P(k_{in})$.

A different situation is faced in the case of the out-degree distribution $P(k_{out})$. In this case, a clear exponential cut-off is observed and the range of degree values is 2 to 4 orders of magnitude smaller than what found for the in-degree distribution. The origin of the cut-off can be explained by the different nature of the in-degree and out-degree evolution. The in-degree of a vertex is the sum of all the hyperlinks incoming from all the Web pages in the WWW. In principle, thus, there is no limit to the number of incoming hyperlinks, that is determined only by the popularity of the Web page itself. On the contrary, the out-degree is determined by the number of hyperlinks present in the page, which are controlled by Web administrators. For evident reasons (clarity, handling, data storage) it is very unlikely to find an excessively large number of hyperlinks in a given page. This represents a sort of finite capacity for the formation of outgoing hyperlinks that might naturally lead to a finite cut-off in the out-degree distribution.

The heavy-tailed behavior of the in-degree distribution implies that there is a statistically significant probability that a vertex has a very large number of connections compared to the average degree $\langle k_{in} \rangle$. In addition, the extremely large value of $\langle k_{max}^{in} \rangle$, and therefore of the variance $\sigma_{in}^2 = \langle k_{in}^2 \rangle - \langle k_{in} \rangle^2$ is signalling the extreme heterogeneity of the connectivity pattern, since it implies that statistical fluctuations are virtually unbounded, and tells us that the average degree is not the typical degree value in the system, i.e., we have scale-free distributions. The heavy-tailed nature of the degree distribution has also important consequences in the dynamics of processes taking place on top of these networks. Indeed, recent studies about network resilience to removal of vertices have shown that the relevant parameter for these phenomena is the ratio between the first two moments of the degree distribution $\kappa = \langle k^2 \rangle / \langle k \rangle$. If $\kappa \gg 1$ the network manifests some properties that are not observed for networks with exponentially decaying degree distributions. In the case of directed networks, this heterogeneity parameter has to be defined separately for in- and out-degrees as $\kappa_{in} = \langle k_{in}^2 \rangle / \langle k_{in} \rangle$ and $\kappa_{out} = \langle k_{out}^2 \rangle / \langle k_{out} \rangle$.

Table 3: Main statistical properties of the analyzed sets: average degree $\langle k \rangle$, maximum degree $k_{max}$, standard deviation $\sigma$, heterogeneity parameter $\kappa$, and maximum likelihood estimate of the exponent of the power-law in-degree distribution $\gamma$ (precision error $\pm 0.1$). All values are provided for in- and out-degrees and for the four data sets. The symbol $\infty$ means that the distribution decays faster than a power-law.

<table>
<thead>
<tr>
<th>Data set</th>
<th>WBGC01</th>
<th>WGU02</th>
<th>WBGC03</th>
<th>WGT04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle k_{in} \rangle$</td>
<td>9.3</td>
<td>15.8</td>
<td>24.1</td>
<td>27.5</td>
</tr>
<tr>
<td>$k_{max}^{in}$</td>
<td>788632</td>
<td>194942</td>
<td>378875</td>
<td>1326744</td>
</tr>
<tr>
<td>$\sigma_{in}$</td>
<td>200.2</td>
<td>143.3</td>
<td>421.6</td>
<td>881.4</td>
</tr>
<tr>
<td>$\kappa_{in}$</td>
<td>4298.6</td>
<td>1317.5</td>
<td>7414.9</td>
<td>28269.9</td>
</tr>
<tr>
<td>$\gamma_{in}$</td>
<td>1.9</td>
<td>1.7</td>
<td>2.2</td>
<td>1.6</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\langle k_{out} \rangle$</td>
<td>9.3</td>
<td>15.8</td>
<td>24.1</td>
<td>27.5</td>
</tr>
<tr>
<td>$k_{max}^{out}$</td>
<td>552</td>
<td>2449</td>
<td>629</td>
<td>9964</td>
</tr>
<tr>
<td>$\sigma_{out}$</td>
<td>13.1</td>
<td>27.4</td>
<td>29.5</td>
<td>67.1</td>
</tr>
<tr>
<td>$\kappa_{out}$</td>
<td>27.7</td>
<td>63.4</td>
<td>60.3</td>
<td>191.0</td>
</tr>
<tr>
<td>$\gamma_{out}$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
Web samples restricted to specific thematic groups [23]. Another oddity that has to be signalled is the fact that the general crawls WBGC01 and WBGC03 exhibit a much smaller cut-off of the out-degree distribution than observed in the two domain crawls. This is somehow counterintuitive given the larger sizes of the general crawls that should include pages of the domain crawls in their exploration. This might hint to the presence of a bias in the way hyperlinks are explored by different crawlers, again purporting evidence for the presence of sampling biases that affect the observed statistical properties of Web graphs.

5. DEGREE-DEGREE CORRELATIONS

As an initial discriminant of structural ordering, the attention has been focused on the networks’ degree distribution. This function is, however, only one of the many statistics characterizing the structural and hierarchical ordering of a network; a full account of the connectivity pattern calls for the detailed study of the multipoint degree correlation functions and/or opportune combinations of those. Along these lines, for instance, it is possible to provide a quantitative study of the mixing properties of networks through opportune projection of the degree-degree joint probability distribution. This allows the large scale distinction between assortative networks, in which large degree nodes preferentially attach to large degree nodes, and disassortative networks, showing the opposite tendency [24]. These structural properties are the signature of specific ordering principles and provide a wealth of useful information on the topology and physical properties of complex networks.

5.1 Single vertex degree correlations

First, we examine local one-point degree correlations for individual nodes, in order to understand if there is a relation between the number of incoming and outgoing links in single pages. Since most of the analyzed degree distributions are heavy-tailed, fluctuations are extremely large so that the linear correlation coefficient is not well defined for those cases. Instead, we provide the crossed one-point correlations, \( \langle k_{in}, k_{out} \rangle \), normalized by the corresponding uncorrelated value, \( \langle k_{in} \rangle \langle k_{out} \rangle \). We also report the function

\[
\langle k_{out}(k_{in}) \rangle = \frac{1}{N_{k_{in}}} \sum_{i \in k_{in}} k_{out,i}, \tag{1}
\]

which measures the average out-degree of nodes as a function of their in-degree. The notation \( i \in k_{in} \) indicates that the summation has to be performed over the set of nodes of degree \( k_{in} \). The results can be found in Table 4 and in Fig. 4.

A significant positive correlation between the in-degrees and the out-degrees of single nodes is found for all the sets. That means that more popular pages tend to point to a higher number of other pages. This positive correlation is found to be true for a range of in-degrees that spans from \( k_{in} = 1 \) to \( k_{in} = 10^2 \rightarrow 10^3 \), depending on the specific set. Beyond this point no noticeable correlation is observed, see Fig. 4. The set for the Italian domain is more noisy, but this pattern appears to be independent of the crawl used to gather the data and, thus, it seems to be a genuine feature of the Web.

5.2 Two vertices degree correlations

Another important source of information about the network structural organization lies in the correlations of the degrees of neighboring vertices. These correlations can be probed in undirected networks by inspecting the average degree of nearest neighbors of a vertex \( i \)

\[
\overline{k}_{nn,i} = \frac{1}{k_i} \sum_{j \sim i} k_j, \tag{2}
\]

where the sum runs on the nearest neighbor vertices of each vertex \( i \). From this quantity, a convenient measure is obtained by averaging over degree classes to obtain the average degree of the nearest neighbors for vertices of degree \( k \), defined as

\[
\overline{k}_{nn}(k) = \frac{1}{N_k} \sum_{i \in k} k_{nn,i} = \sum_{k'} k' P(k' | k), \tag{3}
\]

where \( P(k' | k) \) quantifies the conditional probability that a vertex with degree \( k \) is connected to a vertex with degree \( k' \). This measure provides a sharp proof of the presence or absence of correlations. In the case of uncorrelated networks, each edge has an end vertex of a given degree with a probability which is independent of the degree of the vertex at the other end of the edge, so that \( P(k' | k) \) is only a function of \( k' \). In this case, \( \overline{k}_{nn}(k) \) does not depend on \( k \) and equals \( \langle k^2 \rangle / \langle k \rangle \). Therefore, a function \( k_{nn}(k) \) showing any explicit dependence on \( k \) signals the presence of degree correlations in the system. Real networks usually tend to display one of two different patterns [24]. Assortative networks exhibit \( k_{nn}(k) \) functions increasing with \( k \), which denotes that vertices are preferentially connected to other vertices with similar degree. Examples...
of assortative behavior are typically found in many social structures. On the other hand, disassortative networks exhibit $f_{in}(k)$ functions decreasing with $k$, which denotes that vertices are preferentially connected to other vertices with very different degree. Examples of disassortative behavior are typically found in several technological networks, as well as in communication and biological networks.

In the case of the WWW, the study of the degree-degree correlation functions is naturally affected by the directed nature of the graph. In [6], a set of directed degree-degree correlation functions was defined considering that, in this case, the neighbors can be restricted to those connected by a certain type of directed link, either incoming or outgoing. For the WWW, we study the most significant distributions, taking into account that we can partition the neighborhood of each single node $i$ into neighboring nodes connected to it by incoming links and neighboring nodes connected to it by outgoing links. A first correlation indicator, $f_{in,nn}(k_{in})$, is defined as the normalized average in-degree of the neighbors of in-degree $k_{in}$, when those neighboring nodes are found following incoming links of the original node, see Fig. 5(a). If we measure the Web pages popularity in terms of the number of pages pointing to them, this function quantifies the average popularity of pages pointing to pages with a certain popularity. The exact definition is given in Appendix A along with the expression for the normalization factor. The rest of the correlation functions can be defined in an analogous manner. $f_{out,nn}(k_{out})$ quantifies the average number of references in pages pointing to pages with a certain popularity, and technically it is the normalized average out-degree of the neighbors of nodes of in-degree $k_{in}$, when the neighbors are found following incoming links as for the previous correlation measure, see Fig. 5(c). Analogous quantities can be defined following outgoing connections. $f_{out,nn}(k_{out})$ is the normalized average out-degree of neighbors of nodes of out-degree $k_{out}$, when the neighbors are found following outgoing links, and gives information about the average number of references in pages pointed by pages with a certain number of references, see Fig. 5(b). Finally, we consider $f_{in,nn}(k_{out})$, the normalized average in-degree of neighbors of nodes of out-degree $k_{out}$, when the neighbors are found following outgoing links, and it quantifies the average popularity of pages pointed by pages with a certain number of references, see Fig. 5(d). Each plot in Fig. 5 shows the above correlation functions for the four data sets analyzed in this paper. Remarkably, only one of the functions shows an increasing pattern denoting the presence of assortative correlations for the four data sets. The average out-degree of neighbors of nodes of high out-degree is also high, so that the average number of references is high in pages pointed by pages with a high number of references. In all other cases, very mild or a complete lack of correlation is observed. This is somehow surprising since the different structural properties observed in Sec. 4.1 for the different Web graphs do not find a clear qualitative signature in any correlation function.

6. THE ROLE OF RECIPROCAL LINKS

While a directed network, the Web has many pages pointing to each other. A couple of pages pointing to each other corresponds to an effective presence of a reciprocal link that can be considered as undirected. These reciprocal connections play an important role and in this section we introduce and investigate reciprocal links as crucial elements in the understanding of the WWW. To this end, we will differentiate into incoming, outgoing, and reciprocal links, where incoming and outgoing links do not include the ones taking part in reciprocal connections and are referred to as non-reciprocal. This allows us to consider reciprocal and non-reciprocal connections as separate and well-defined independent entities and provides a statistical analysis able to capture additional information.
of the Web structure and the sampling biases eventually observed in different data sets.

6.1 Degree distributions

For the sake of notation, in the following we will identify the non-reciprocal in-degree and out-degree of a given vertex \( i \) with \( q_{in,i} \) and \( q_{out,i} \), respectively. Analogously, the reciprocal degree (r-degree) \( q_{r,i} \) indicates the number of reciprocal connections to neighboring vertices. While the degree distributions of non-reciprocal links are extremely similar to those obtained for the global in and out-degree, the reciprocal degree distribution appears to exhibit a striking different behavior depending on the crawl examined. In particular, general crawls show a distribution \( P(q_r) \) with an exponentially fast decaying behavior, while the domain crawls have a heavy-tailed distribution varying over four orders of magnitude (see Fig. 7). In Table 5, we summarize the main properties of \( P(q_r) \) for the various data sets. Also from the values shown there one can easily see the mild fluctuations and heterogeneity expressed by the general crawl data sets. The evident differences in the reciprocal degree distributions match the dissimilar component structure observed in general and domain crawls. On the other hand, the origin of the two different statistical behaviors does not find a clear explanation and deserves further investigation. In particular, it is not possible to find an easy explanation either in the crawling strategies or in the eventual features of Web specific domains. Finally, once again we have to emphasize the odd finding of general crawls showing reciprocal degree distribution cut-offs much smaller than those observed for domain crawls. This is at odds with the fact that general crawls should contain part of the domain Web graphs, thus recovering some of the high values for the reciprocal degree of vertices observed in those Web graphs. This again indicates the presence of sampling biases due to the different crawling strategies used in the experiments.

6.2 One-point degree correlations

The distinction between reciprocal and non-reciprocal links induces a higher complexity even at the most local level. In this case, each node is characterized by three different quantities. Consequently, we need to introduce three correlation measures, i.e., the average non-reciprocal out-degree as a function of the non-reciprocal in-degree, \( < q_{out}(q_{in}) > \), and the average r-degree as a function of the number of non-reciprocal incoming and outgoing links, \( < q_r(q_{in}) > \) and \( < q_r(q_{out}) > \), respectively (see Fig. 8). A surprising result is that, in this case, there is no clear correlation between non-reciprocal in- and out-degrees but there is a positive correlation between reciprocal and non-reciprocal in-degrees. So, the positive correlation previously observed between in- and out-degrees is just a consequence of this new correlation. This is also observed in the positive correlation between r-degree and non-reciprocal out-degree, although in this case the correlation is weaker.

6.3 Degree-degree correlations

The two vertices correlation analysis presented in section 5.2 can be repeated for the non-reciprocal and reciprocal decomposition of the network. Now, we have to differentiate reciprocal links and segregate the neighborhood of each single node \( i \) into neighboring nodes connected to it by non-reciprocal incoming links, neighboring nodes connected to it by non-reciprocal outgoing links, and neighboring nodes connected to it by reciprocal links. The degree-degree correlation functions corresponding to the first two cases give similar results to the ones presented in the previous section and do not signal the presence of any relevant correlation pattern (not plotted).

A very different picture is obtained when we measure correlations following reciprocal connections. A strong positive correlation is observed between the in-degrees of nodes connected by reciprocal links. This is clearly visible in the upper left plot of Fig. 9 which shows the normalized average non-reciprocal in-degree of the neighbors of nodes of non-reciprocal in-degree \( q_{in} \), when the
neighbors are found following reciprocal links, \( \langle q_{\text{rec},\text{nn}}(q_{\text{in}}) | r \rangle \). This function shows a clear increase of two orders of magnitude as a function of \( q_{\text{in}} \), indicating an assortative correlation. The same behavior is found between non-reciprocal out-degrees (lower right figure of Fig. 8). Concerning the crossed correlations, we observe again a positive correlation between the neighboring non-reciprocal in-degree and the non-reciprocal out-degree but no noticeable correlation in the opposite one, that is, the average non-reciprocal out-degree of the reciprocal neighbors of a node is independent of the non-reciprocal in-degree of that node (see lower left figure in Fig. 8). In summary, the analysis of the two vertices degree correlation behavior indicates that most of the structural correlations of Web graphs are found in vertices connected by reciprocal links. This type of links therefore represents an element of particular interest in that they express the ordering principles (beyond simple randomness) at the basis of the Web structure.

6.4 The reciprocal subgraph

Very interesting information is provided by the study of how reciprocal links are structurally organized among them. If we look at the subgraph formed by the vertices and the reciprocal links we can use the tools adopted for undirected graphs. A measure of the two vertices correlation function is therefore expressed by

Figure 8: One node correlations for the four different data sets. The functions shown are the normalized average non-reciprocal out-degree as a function of the non-reciprocal in-degree, and the normalized average r-degree as a function of the non-reciprocal in- and out-degrees.

\[
\langle q_{\text{rec},\text{out}}(q_{\text{in}}) | q_{\text{out}} \rangle = \frac{1}{\langle q_{\text{out}} \rangle} \langle q_{\text{rec},\text{out}}(q_{\text{in}}) \rangle
\]

Figure 9: Non reciprocal degree-degree correlations for the four different data sets. Each graph shows the same correlation function for all the sets.

\[
\langle q_{\text{rec},\text{out}}(q_{\text{in}}) | q_{\text{out}} \rangle (\text{see Sec. 2}), \text{i.e., the standard measure of an undirected network if we identify reciprocal links as undirected. This function shows a first decrease, for } q_{\text{in}} < 10, \text{ followed by a linear increase up to a critical value depending on the crawler. At high reciprocal degrees, a cloud of points is populating the low r-degree region of the average nearest neighbor reciprocal degree, see Fig. 10. This defines a bi-modal pattern which indicates two different behaviors. The low values cloud can be interpreted as a collection of star-like structures, with central hubs connected to low degree nodes. This effect is probably due to the "home" button in many Web pages that belong to a bigger site. The linear behavior may have two different interpretations. The first one is that the network is a tree in which high degree nodes are connected to other high degree nodes. The second one is that the network forms clique-like structures, that is, groups of pages pointing simultaneously to each other. To discern which scenario is more appropriate we inspect the local connectivity properties of reciprocally linked vertices. Since we can treat the reciprocal subgraph as an undirected one, we can probe the local interconnectedness by analyzing the clustering coefficient defined as the fraction of inter-connected neighbors of \( j \): 
\[
c_j = 2 \cdot \frac{n_{1\text{nnk}}}{\langle q_{\text{rec},j} \rangle (q_{\text{rec},j} - 1)} ,
\]
where \( n_{1\text{nnk}} \) is the number of reciprocal links between the \( q_{\text{rec},j} \) reciprocal neighbors of \( j \). This quantity measures the density of interconnected vertex triplets and it is therefore close to one in the case of a fully interconnected neighborhood and zero in the case of a tree structure. Global statistical information can be gathered by inspecting the average clustering coefficient \( \bar{c}(q_{\text{rec}}) \) restricted to classes of vertices with reciprocal degree \( q_{\text{rec}} \). In the first scenario, \( \bar{c}(q_{\text{rec}}) \) should be very small and decreasing with the degree because of the tree-like structure. In the second one \( \bar{c}(q_{\text{rec}}) \) should be significant and independent of the degree. In Fig 10 we show the function \( \bar{c}(q_{\text{rec}}) \) which exhibits a high and constant value followed by a cloud of points with very low clustering coefficient at the same point where the function \( \bar{q}_{\text{rec},j}(q_{\text{rec}}) \) also splits. This indicates that the organization of the reciprocal subgraph is a set of star-like structures combined with cliques, or communities, of highly interconnected pages. Very interestingly, this pictorial characterization appears to be the same in all Web graphs considered, pointing out to a genuine feature of the Web graph. The present analysis identifies in the reciprocal subgraph an
be a starting point to approach this problem and have a preliminary assessment of the intrinsic biases. In this sense, numerical important element that might help in decoding the structure of the WWW. Finally we have to stress that the reciprocal component is surely extremely important for the analysis and understanding of navigation patterns and the network resilience to link removal.

7. OUTLOOK

Contrary to what happened with the scrutiny of Internet maps, the issue of sampling biases in the structure of the WWW has been left almost untouched. The large size of the data sets has led to the belief that the global properties were well defined in view of the abundant statistics available. Noticeably, from the present analysis, it appears that the resulting picture of the WWW structure and its statistical characterization can be considerably affected by the design of the tools we use to observe it. While some of the basic properties are qualitatively preserved across different data sets, other features and quantities are highly variable. This results in a fuzzy picture of the WWW structure, where sampling biases still play a major role. In other words, we are still in a position where it is impossible to have a definite conceptual framework to decode the structure of the global Web and how effectively we can navigate, search, index, or mine the Web. The present work thus highlights the need for a theoretical framework able to approach a detailed analysis and understanding of the sampling biases implicit in the most widely used crawling strategies. In this sense, numerical studies of simulated exploration of directed network models could be a starting point to approach this problem and have a preliminary assessment of the intrinsic biases.

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9. REFERENCES


**APPENDIX**

**A. DEGREE-DEGREE CORRELATIONS: QUANTITATIVE DEFINITIONS**

We study the most significant two-point correlation functions, taking into account that we can segregate the neighborhood of each single node \(i\) into neighboring nodes connected to it by incoming links, the set \(\nu_{in}(i)\), and neighboring nodes connected to it by outgoing links, the set \(\nu_{out}(i)\). Following Eq. (4), we can write

\[
\begin{align*}
\langle k_{in,nn}(k_{in}) \rangle &= \frac{1}{N_{in}} \frac{1}{N_{in}} \sum_{i \in k_{in}} \sum_{j \in k_{in}} \frac{1}{k_{in,j}} \kappa_{in,i,j} \\
\langle k_{out,nn}(k_{in}) \rangle &= \frac{1}{N_{k_{out}}} \sum_{i \in k_{in}} \sum_{j \in k_{in}} \frac{1}{k_{out,i}} \kappa_{out,i,j} \\
\langle k_{in,nn}(k_{out}) \rangle &= \frac{1}{N_{k_{in}}} \sum_{i \in k_{out}} \sum_{j \in k_{out}} \frac{1}{k_{out,i}} \kappa_{in,i,j} \\
\langle k_{out,nn}(k_{out}) \rangle &= \frac{1}{N_{k_{out}}} \sum_{i \in k_{out}} \sum_{j \in k_{out}} \frac{1}{k_{out,i}} \kappa_{out,i,j}.
\end{align*}
\]

The same quantities can be calculated when non-reciprocal and reciprocal links are differentiated. Now, the neighborhood of each single node \(i\) is segregated into neighbors connected to it by non-reciprocal incoming links, the set \(\nu_{non}(i)\), neighbors connected to it by non-reciprocal outgoing links, the set \(\nu_{non}(i)\), and neighbors connected to it by reciprocal links, the set \(\nu_{r}(i)\). The functions given in Eq. (4) are valid whenever the in and out subscripts are restricted to non-reciprocal links. When following only reciprocal links, one can redefine them in a similar way:

\[
\begin{align*}
\langle q_{in,nn}(q_{in}) \rangle &= \frac{1}{N_{k_{in}}} \frac{1}{N_{k_{in}}} \sum_{i \in k_{in}} \sum_{j \in k_{in}} \frac{1}{q_{r,i}} \kappa_{in,i,j} \\
\langle q_{out,nn}(q_{in}) \rangle &= \frac{1}{N_{k_{out}}} \sum_{i \in k_{in}} \sum_{j \in k_{in}} \frac{1}{q_{r,i}} \kappa_{out,i,j} \\
\langle q_{in,nn}(q_{out}) \rangle &= \frac{1}{N_{k_{in}}} \sum_{i \in k_{out}} \sum_{j \in k_{out}} \frac{1}{q_{r,i}} \kappa_{in,i,j} \\
\langle q_{out,nn}(q_{out}) \rangle &= \frac{1}{N_{k_{out}}} \sum_{i \in k_{out}} \sum_{j \in k_{out}} \frac{1}{q_{r,i}} \kappa_{out,i,j}.
\end{align*}
\]

and the normalization terms in this case are

\[
\begin{align*}
\kappa_{r,in} &= \langle q_{r,nn} \rangle \\
\kappa_{r,out} &= \langle q_{r,nn} \rangle.
\end{align*}
\]