- $x \in \{0,1\}^k$

1. $x$ of length $k$ - information word

2. encoder: $E - E : \{0,1\}^k \rightarrow \{0,1\}^n$

   - obtains a code word of $n$ bits

   - with $n$ = the code length $n$

   - block length $n$

3. Transmit code word $y = E(x)$ over
   noisy channel

   - Rate of the code $r = \frac{k}{n}$

   - Assume binary symmetric channel, BSC.

   - Each bit flipped independently with probability $p$ (error probability)
4) Reset receiver, put \( y \) as

concatenated \((z_0, h)\) and \( y \) of length \( n \).

5) decode: \( D: \{0, 1\}^n \to \{0, 1\}^k \)

and even probability \( P_0 \) of \( D \)

decoder into probability \( 2^n \)

\[ D(z) \neq x. \]

6) The maximum-likelihood (ML)

information word \( x \) is the one that

maximize \( P(x|E_0) \) for \( x \) (\( y \) was received)
\[
\max_{\tilde{v}, x} \left( \alpha_x \left( E(\tilde{v}) \mid \text{obs. } \tilde{v}, \text{recvd. } x \right) \right)
\]

\[
= \max_{\tilde{v}} \left( \sum_x \alpha_x \left( E(\tilde{v}) \mid \text{obs. } x, \text{recvd. } \tilde{v} \right) \right) \frac{P(\tilde{v} \text{ received})}{P(\tilde{v} \text{ received})}
\]

\[
= \max_{\tilde{v}} \left( \alpha_{\tilde{v}} \left( E(\tilde{v}) \mid \text{obs. } \tilde{v} \right) \right) \frac{P(\tilde{v} \text{ received})}{P(\tilde{v} \text{ received})}
\]

\[
= \max_{\tilde{v}, x} \left( \alpha_x \left( E(\tilde{v}) \mid \text{obs. } \tilde{v}, \text{recvd. } x \right) \right)
\]

\[
\text{assumed all } P_x \left( E(\tilde{v}) \mid \text{obs. } \tilde{v} \right) \\
\text{are equal.}
\]

\[
\text{code } \tilde{v} \text{ given } \text{fixed.}
\]

\[
\max_{\tilde{v}} \left( \alpha_{\tilde{v}} \left( E(\tilde{v}) \mid \text{obs. } \tilde{v} \right) \right)
\]

\[
\text{known.}
\]

Thus, under DSC, we can look for

\[
E(\tilde{v}) \text{ that is closest to } \tilde{v} \text{.}
\]
ON BBC channel

Min Hamming dist = Max. Codew. length

t code used: decode

Word with error

Hamming distance between received word

\[ d(x,v) \triangleq 5 \]

2 codes received

\[ x = 1011 \]

\[ E(11) = 101110 \]

\[ v = 100110 \]

\[ E(101110) \text{ not a valid code word} \]

\[ E(100110) \text{ valid code word} \]

\[ \text{decoded: } v_{\text{decoded}} = \begin{cases} E(101110) & \text{if } \exists \hat{v} \\
E(100110) & \text{otherwise} \end{cases} \]

2 valid code words

Hamming dist. > 1

\[ d(v_{\text{decoded}}, E(\hat{v})) = (t-p)^2 \cdot p^4 \]
Minimum distance of a code.

= minimum Hamming distance between all pairs of code words.

The decoder always return (correctly)

\[ d \geq \frac{d}{2} + 1 \text{ enw.} \]
But Mr. Dim. is a man our women—

E.g. Kumbu cute very much but not with
gov't min. dir. property