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Coding & Information Theory

BASIC CODING THEOY -
SYNPEHIC CHANNEL

Source
\[ p(X=0) = 0.95 \]
\[ p(X=1) = 0.05 \]

Symmetry: Defined by set of conditional probabilities

\[ P(Y|X) \]

So, 1 in 10 'ones' are turned to a '0':
\[ P(Y=0|X=1) = 0.9 \]
\[ P(Y=1|X=1) = 0.1 - 'te note' \]

So, 1 in 10 'zeros' are turned to a '1':
\[ P(Y=1|X=0) = 0.9 \]
\[ P(Y=0|X=0) = 0.1 - 'te nine' \]

So, without observing anything, output:
\[ P(X=1) = 0.05 \]
\[ P(X=0) = 0.95 \]

Now, after you observe the output: 25% of
\[ P(X|Y=0) \]
\[ P(X|Y=1) \]
\[ P(X = 0 \mid Y = 0) = \frac{P(X = 0 \land Y = 0)}{P(Y = 0)} \]

\[ = \frac{P(Y = 0 \mid X = 0)}{P(Y = 0)} \cdot P(X = 0) \]

\[ = P(Y = 0 \mid X = 0) \cdot P(X = 0) \]

\[ P(X = 0 \land Y = 0) \]

\[ = P(X = 0) \cdot P(Y = 0 \mid X = 0) \]

\[ + P(X = 1) \cdot P(Y = 0 \mid X = 1) \]

\[ \text{by definition of probability,} \]

\[ P(A \cup B) = \frac{P(A \cup B)}{P(C \cup B)} \]

\[ P(Y = 0) = P(X = 0 \land Y = 0) + P(X = 1 \land Y = 0) \]

\[ \text{by definition of probability,} \]

\[ \text{two mutually exclusive and exhaustive events} \]

\[ P_X(x) = \sum_y P_{X,Y}(x,y) \]

\[ = P(X = x) \]

\[ = \sum_y P(Y = y \mid X = x) \]

\[ = \sum_y P_{Y \mid X = x}(y) \]

\[ = \text{inclusion-exclusion of } \]

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \text{ disjoint events} \]

\[ x = (0.95 \cdot 0.9) + (0.05 \cdot 0.1) \]

\[ = (0.95 \cdot 0.9) + (0.05 \cdot 0.1) \]

\[ = 0.9419 \]

\[ T = 0.0056 \]

\[ 0.0056 + \frac{0.05 \cdot 0.1}{0.95 \cdot 0.9} = 0.9419 \]
So, a priori

\[ P(X=1) = 0.05 \Rightarrow P(X=1|Y=1) = 0.006 \]
\[ P(X=0) = 0.95 \Rightarrow P(X=0|Y=1) = 0.994 \]

Second scenario - observe a "1".

\[ X \quad \text{observation} \quad Y \]

\[
\begin{align*}
& 0.000001 \quad 1 \quad 0.0001 \\
& 100 \Rightarrow & 95.05 & 0.1 \text{ noise} & (95.0.1 = 9.5) \\
& & 5.1 & 0.9 & 5.0.9 = 4.5 \\
\end{align*}
\]

\[ 95.0.9 = 85.5 \\
5.0.1 = 0.5 \\
86 "0" \]

Observe "1" \[ \Rightarrow \frac{4.5}{14} \text{ pm } "1" \Rightarrow 0.322 = P(X=1|Y=1) \]

"Inside" & \[ 9.5 \text{ pm } "0" \Rightarrow 0.678 = P(X=0|Y=1) \]

So, "Confirmed", overall.
\[ P(X=1 \mid Y=0) = \frac{P(X=1 \land Y=0)}{P(Y=0)} \]

\[ = \frac{P(Y=0 \mid X=1) \cdot P(X=1)}{P(Y=0)} \]

\[ = \frac{0.95 \cdot 0.1 \cdot 0.05}{(0.95 \cdot 0.9) + (0.1 \cdot 0.05)} \]

\[ = \frac{0.004775}{1} \]

\[ = 0.004775 \]

\[ = 0.00481 \]

\[ \approx 0.0048 \]
How observe a "0" -

\[ \frac{0.55}{0^0} \text{ for real "0" } = 0.994 \]

\[ \frac{0.5}{0^0} \text{ for complex } "0" = 0.006 \]

(US. 0.05 Apriori)
Open curve, will be observed
\[ \Rightarrow \text{ no role } \]
Fold randomly

Special case now = 0

\[ \begin{align*}
P(X = 0) &= 0.95 \quad \text{(US. 1)} \\
P(X = 1) &= 0.05
\end{align*} \]

But, \[ P(X = 0 \mid Y = 0) = 1.0 \] \[ P(X = 1 \mid Y = 0) = 0 \]
\[ P(X = 0 \mid Y = 1) = 1.0 \] \[ P(X = 0 \mid Y = 1) = 0.0 \]
Will 1/3 noise:

\[
\begin{array}{c}
0001101000 \\
\downarrow \\
0001100000
\end{array}
\]

Find

\[
\begin{array}{c}
0001101000 \\
\downarrow \\
01101100 \\
\downarrow \\
00101100
\end{array}
\]

two whole 0's remain? 2 real 1's?

0 way none which it.

0 way we already the current 0's.

0 "together" 0 "also one.

So, when you see a 1 , it may very well be a corrupted 0 .
Especially when a mini 1 's are very rare & there is reasonable noise.

Also, when you see a 0 , quite likely a real Zero - rare a mini 1 = very real corrupted 0.
With \( \frac{1}{2} \) noise:

\[ P(\text{out} \text{ will be random bits}) = 0.5 \]

\[ P(X = 0 | Y = 0) = 0.5 \quad \text{and} \quad P(X = 1 | Y = 0) = 0.5 \]

\[ P(X = 0 | Y = 1) = 0.5 \quad \text{and} \quad P(X = 1 | Y = 1) = 0.5 \]

So, the sequence

\[ 0 0 0 1 0 1 0 0 0 \]

equals a priori

\[ 1 0 1 1 0 0 0 1 0 0 \]

E.g., \( P(X = 0 | Y = 0) = 1 \)

5 cm & 5 cm
3.5 cm & 1 cm
3.5 cm & 1 cm
3.5 cm & 1 cm

0.75 \& 0.75
0.75 \& 0.75
0.75 \& 0.75
0.75 \& 0.75

\[ 5 \text{ total } 3 \text{ cm} \text{ down.} \]

\[ \frac{3.5}{5} = \frac{7}{10} \quad \text{and} \quad \frac{3}{5} = \frac{3}{5} \]

\[ 0.75 \]

\[ 0.75 \]

\[ 0.75 \]