Using LP to decode linear codes

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Linear code $C$ w. parity check

Matrix $A$

represented by a Tanner or factor graph $G$

$n \times m$ parity check matrix of the code $C$:

$$A = \begin{pmatrix} m \\ m \end{pmatrix}$$

$$N(v_i)$$

$G$ - bipartite

Clock nodes

Example assignment:

$$N(c_j)$$
Example:

Variable nodes:

Code words:

Code words include:

(7, 4, 3) Hamming Code.

Note: all parity checks \( c_1, c_2, c_3 \) are even.
code word $\gamma$

$$\gamma \rightarrow \bigg[ \text{noisy channel} \bigg] \rightarrow \hat{\gamma}$$

ML decoder's problem:
given $\hat{\gamma}$ find the $\gamma$ that maximizes the likelihood of receiving $\hat{\gamma}$.

Cost function:
$$\sum_{i=1}^{n} f_i \cdot v_i$$

where $f_i = -\log \left( \frac{p_i|I_1 \hat{y}_i |}{p_i|0 \hat{y}_i |} \right)$

In the regular log likelihood ratio (LLR) at each unit's code.
For example, given a symmetric binary binary channel (BSC) with error prob. \( p \), we set

\[
Y_i = -\log \left[ \frac{(1-p)}{p} \right] \quad \text{if} \quad \text{received bit} \; \hat{Y}_i = 1
\]

\[
H \left[ 1 \mid Y_i = 1 \right] = (1-p)
\]

\[
H \left[ 0 \mid Y_i = 0 \right] = p
\]

&

\[
Y_i = -\log \left[ \frac{p}{(1-p)} \right] \quad \text{if} \quad \text{received bit} \; \hat{Y}_i = 0
\]

\[
H \left[ 1 \mid Y_i = 0 \right] = (1-p)
\]

\[
H \left[ 0 \mid Y_i = 0 \right] = (1-p)
\]