Cornell University

A radiative transfer framework for rendering materials with anisotropic structure

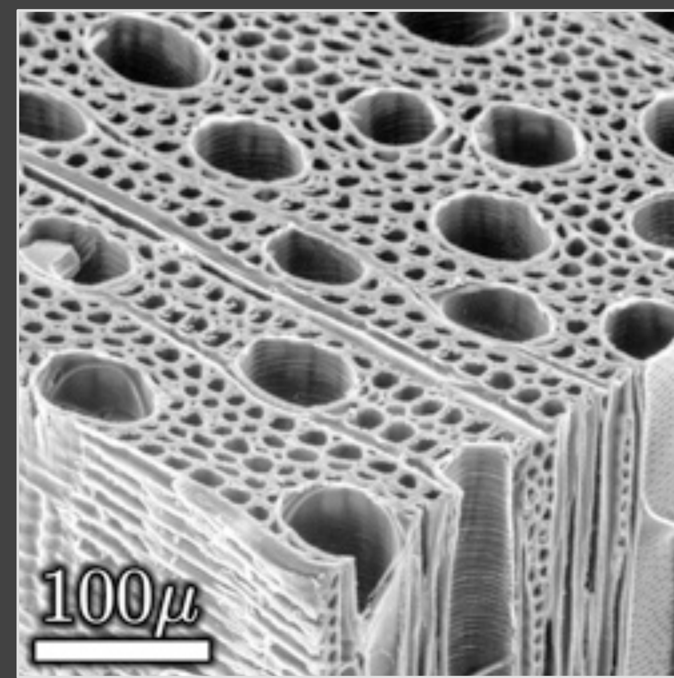
Wenzel Jakob[*] Adam Arbree[§] Jonathan T. Moon[†]
Kavita Bala[*] Steve Marschner[*]

[*] Cornell University
[§] Autodesk
[†] Google

Wood photograph

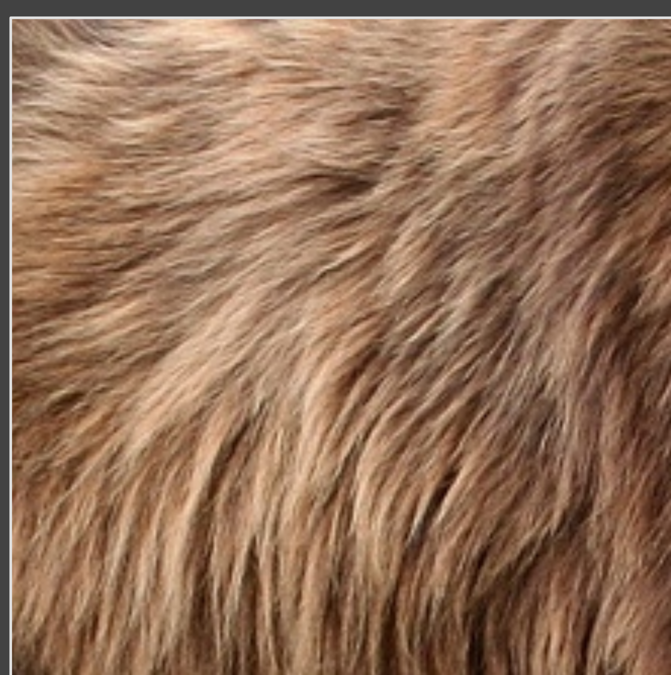
[Marschner et al.]

Wood micrograph



[NC Brown Center for Ultrastructure Studies]

Animal fur



[Wikimedia commons]

Moss

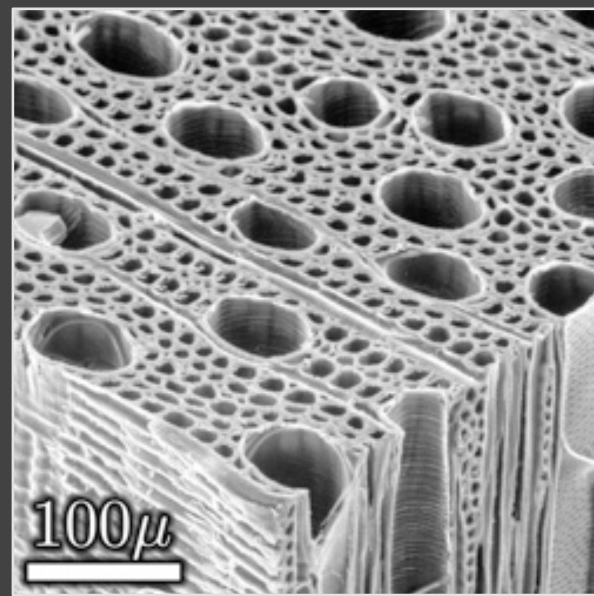
[Wikimedia commons]

Here's a sample of several interesting types of materials. Common to all of them is their complex internal structure, which has a profound influence on their appearance. In this talk, I'm going to present a principled way to render such materials using a unified volumetric formulation.

Wood photograph

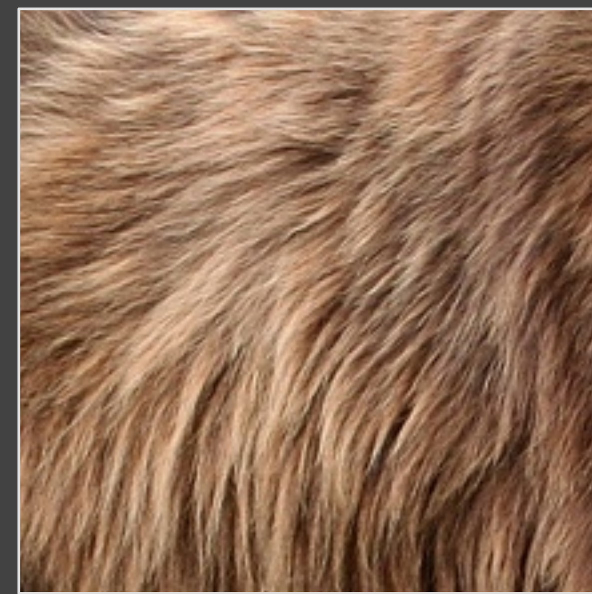
[Marschner et al.]

Wood micrograph



[NC Brown Center for Ultrastructure Studies]

Animal fur



[Wikimedia commons]

Moss

[Wikimedia commons]

Anisotropic volumes

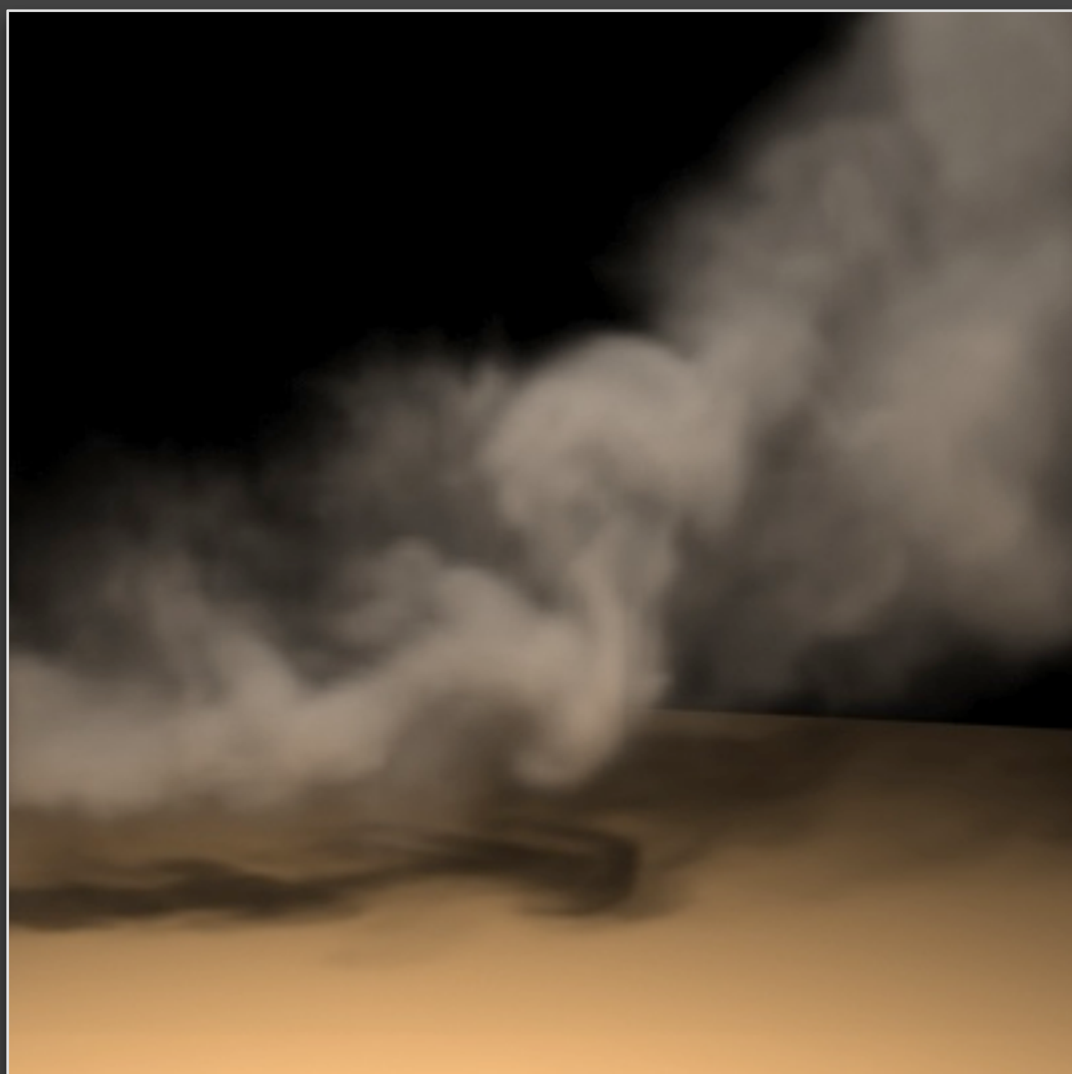
- Objects with suitable volumetric representations
- Anisotropy caused by internal structure
- Current theory doesn’t handle this!

For objects of such vast geometric detail, its preferable to consider them as volumes. We’re interested in anisotropic materials whose internal structure causes them to reflect light differently depending on the directions, from which they are illuminated and observed. Currently however, when you mix radiative transfer and anisotropy, things begin to break. So we need a way to combine these two.

Two disjoint approaches for rendering volumes :

There are two groups that make up most of the current volume rendering techniques:

Two disjoint approaches for rendering volumes :



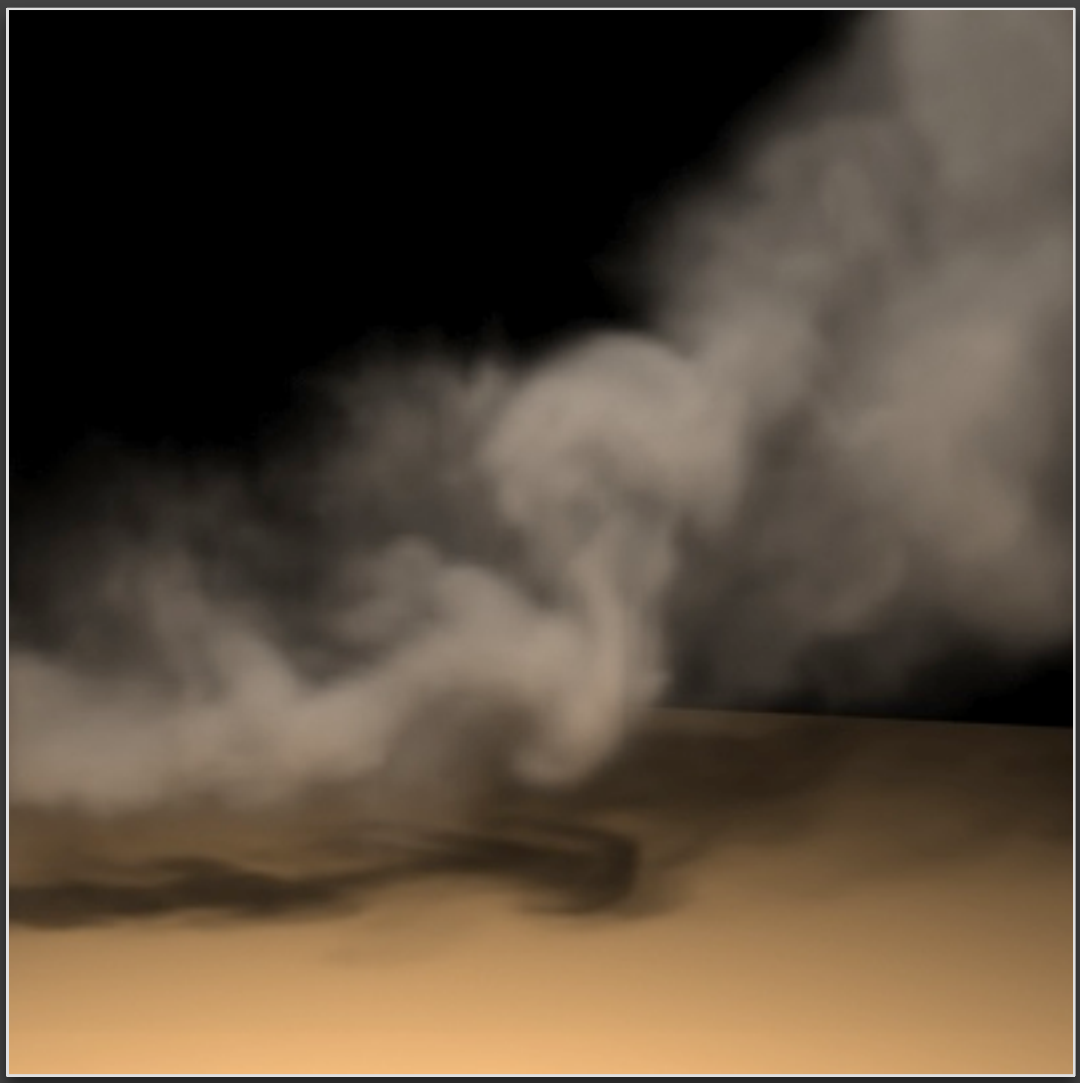
[Fedkiw et al.]

Physically based radiative transfer

- Sound foundation
- Inherently isotropic

First, there are physically based methods built on radiative transfer formulations from the hydrologic or atmospheric optics communities. The problem with such participating media methods is that the underlying equations that date back to the 40s are fundamentally built on the assumption of an isotropic medium. So to salvage this approach, we really have to go to the ground floor and start fixing all the way up.

Two disjoint approaches for rendering volumes :



[Fedkiw et al.]

[Wikipedia commons.]

Physically based radiative transfer

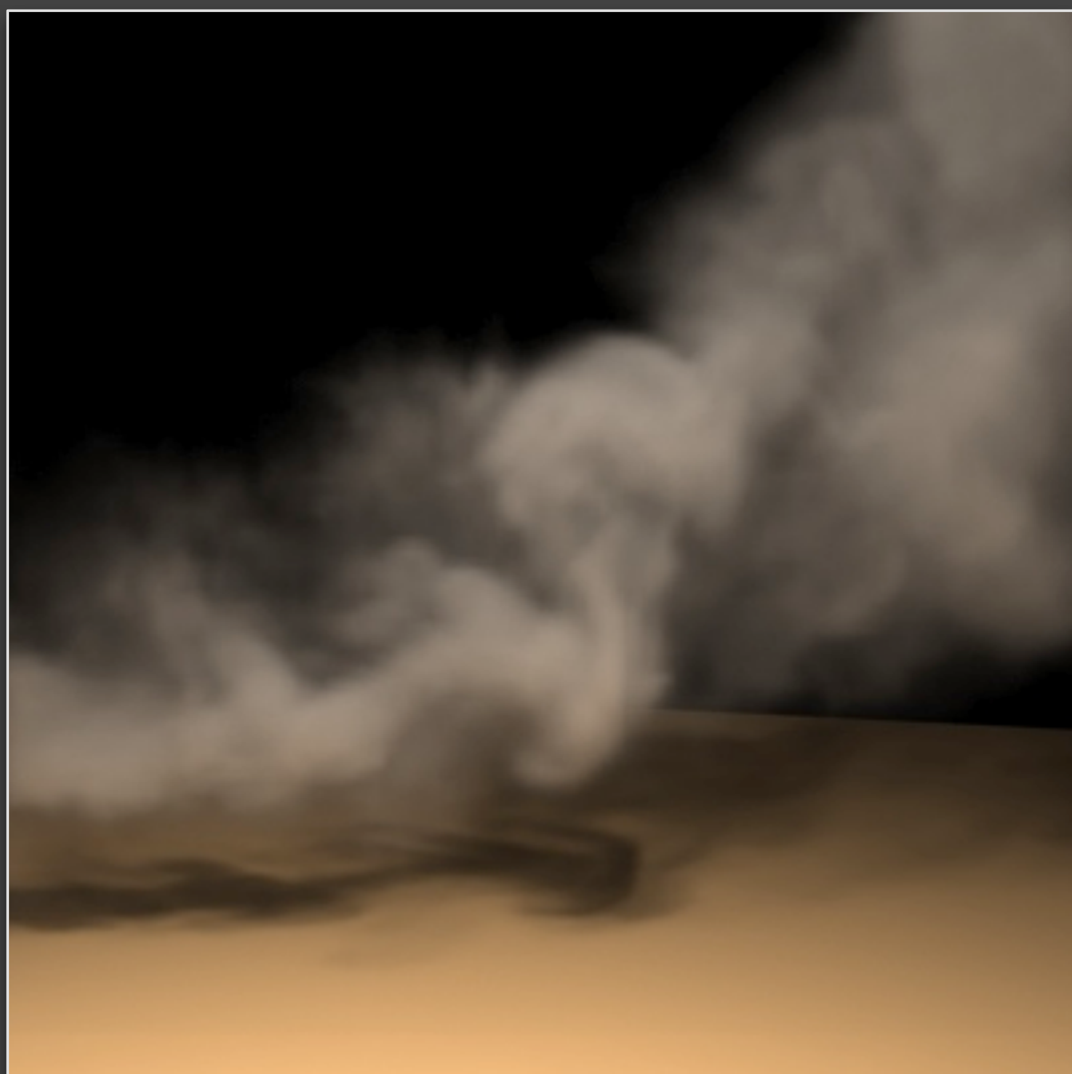
- Sound foundation
- Inherently isotropic

Heuristic models volume visualization

- Support anisotropy
- Not suitable for multiple scattering

And then there is what could be described as volume visualization with heuristic scattering models. For example, medical renderings of volumes often use surface shading models that, in the context of volume scattering, are actually anisotropic. But their heuristic nature prevents them from being usable in full volumetric light transport simulations.

Two disjoint approaches for rendering volumes :



[Fedkiw et al.]

[Wikipedia commons.]

Goal: bridge this gap.

The goal of this paper is to bridge this gap by upgrading the radiative transfer framework to handle such anisotropic effects, but to do so a physically meaningful way.

Limitations of previous systems

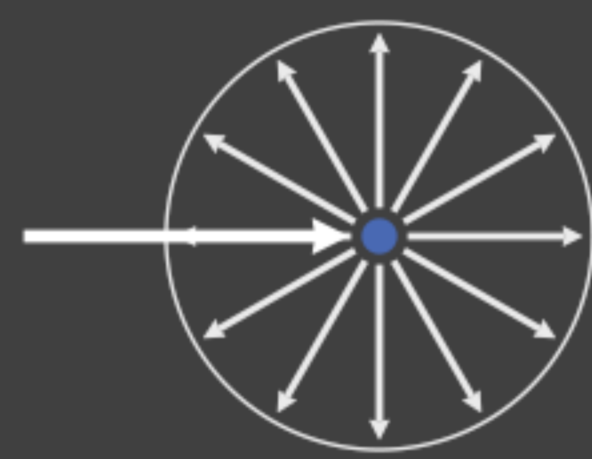
Isotropic scattering

Let’s now clarify some terminology and limitations of previous systems.

Currently, the common convention is to call completely uniform scattering “isotropic”. The blue point here indicates a scattering interaction, and the large arrow is the incident direction In the uniform case, nothing changes when the incident direction moves.

Limitations of previous systems

Isotropic scattering

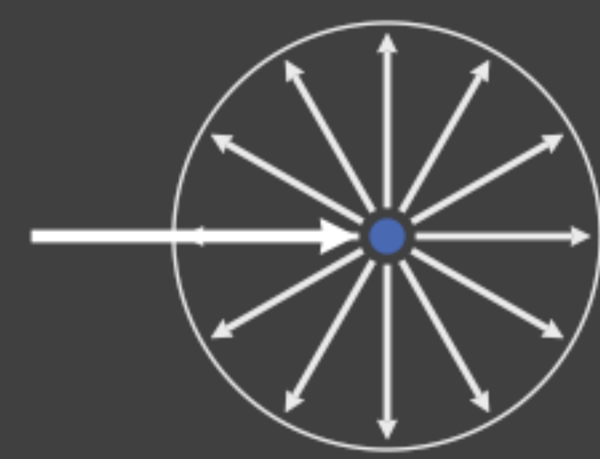


Let’s now clarify some terminology and limitations of previous systems.

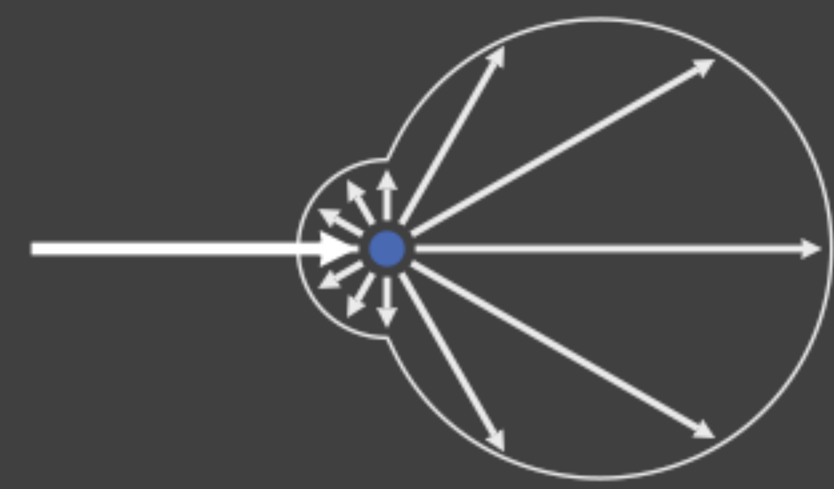
Currently, the common convention is to call completely uniform scattering “isotropic”. The blue point here indicates a scattering interaction, and the large arrow is the incident direction In the uniform case, nothing changes when the incident direction moves.

Limitations of previous systems

Isotropic scattering



“Anisotropic” scattering



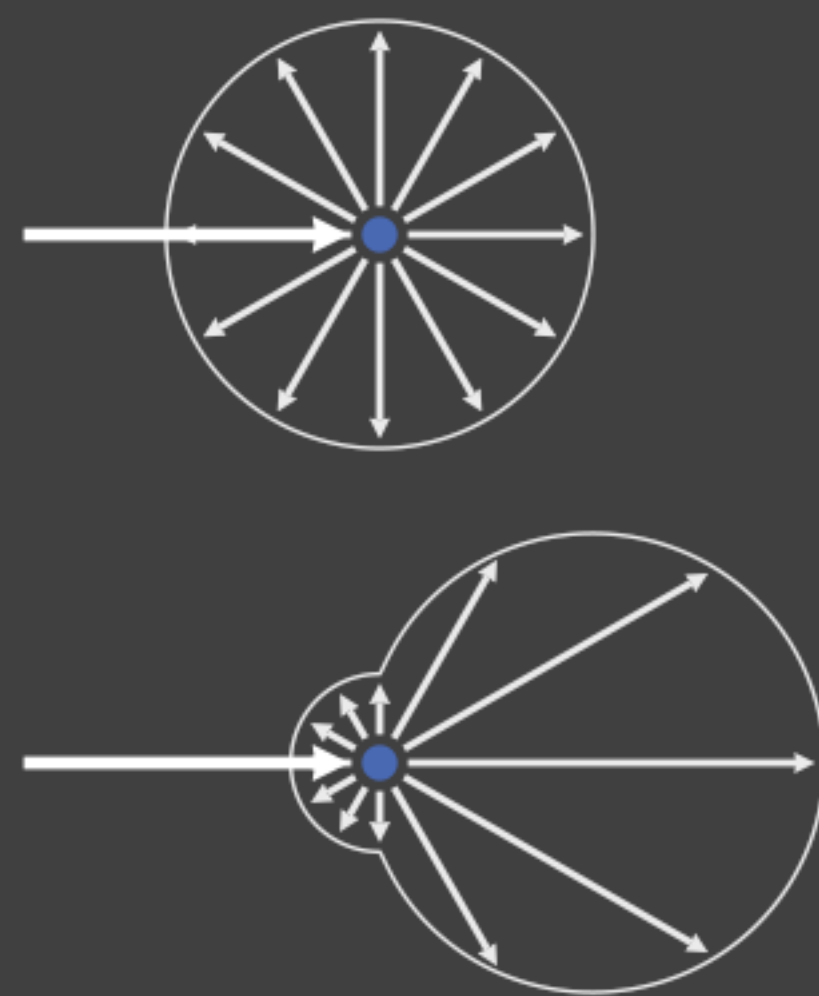
More general forms of scattering where the scattered energy depends on the angle between the incident and outgoing directions have traditionally been referred to as “anisotropic”.

This is good enough to handle things like a cloud of steam filled with spherical water droplets. And it even extends to things like a cloud filled with non-spherical ice crystals, assuming that they are all randomly oriented. But the main limitation common to both of these cases is that the medium must always behave the same way independently of the direction of propagation, which if you think about it is really the definition of the term isotropic. So in our paper, we actually refer to both of them as isotropic.

Limitations of previous systems

Isotropic scattering

“Anisotropic” scattering



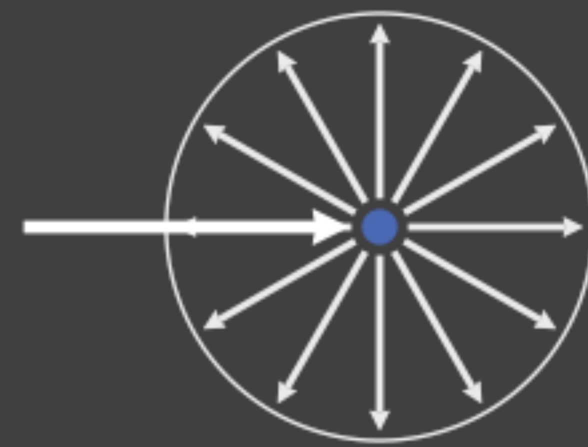
Good enough for:

- Smoke, steam
- Cloud of randomly oriented ice crystals

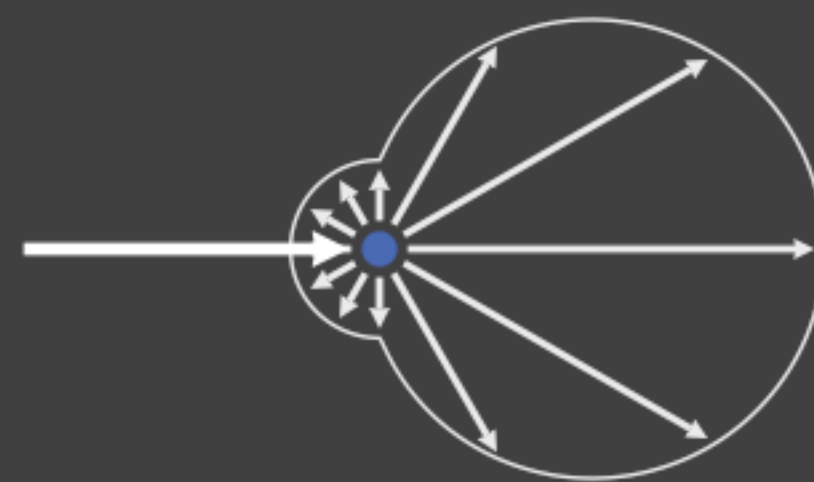
More general forms of scattering where the scattered energy depends on the angle between the incident and outgoing directions have traditionally been referred to as “anisotropic”.
This is good enough to handle things like a cloud of steam filled with spherical water droplets. And it even extends to things like a cloud filled with non-spherical ice crystals, assuming that they are all randomly oriented. But the main limitation common to both of these cases is that the medium must always behave the same way independently of the direction of propagation, which if you think about it is really the definition of the term isotropic. So in our paper, we actually refer to both of them as isotropic.

Limitations of previous systems

Isotropic scattering



Isotropic
"~~Anisotropic~~" scattering



Built-in assumption:

The medium behaves the same independently of the direction of propagation.

More general forms of scattering where the scattered energy depends on the angle between the incident and outgoing directions have traditionally been referred to as "anisotropic".
This is good enough to handle things like a cloud of steam filled with spherical water droplets. And it even extends to things like a cloud filled with non-spherical ice crystals, assuming that they are all randomly oriented. But the main limitation common to both of these cases is that the medium must always behave the same way independently of the direction of propagation, which if you think about it is really the definition of the term isotropic. So in our paper, we actually refer to both of them as isotropic.

Anisotropic scattering

- Cloud of aligned ice crystals
- Cloth fibers, wood, ..

In comparison, we want to be able to handle materials like cloth fibers or wood, where the scattering behavior does depend on the direction of propagation.

It's a good question to ask if we can use scattering models like these together with the existing equations. Hopefully, by the end of this talk, you will agree with me that if you do this, then things will break in subtle and unanticipated ways.

Anisotropic scattering

- Cloud of aligned ice crystals
- Cloth fibers, wood, ..

In comparison, we want to be able to handle materials like cloth fibers or wood, where the scattering behavior does depend on the direction of propagation.

It's a good question to ask if we can use scattering models like these together with the existing equations. Hopefully, by the end of this talk, you will agree with me that if you do this, then things will break in subtle and unanticipated ways.

Prior work

Anisotropic volume models

[Kajiya and Kay 1989], [Neyret 1998], ...

[Kajiya and Kay 1989]

[Neyret 1998]

Let's take a look at some related work: In the past, anisotropic volume models have been used to render materials with complex surfaces and volumetric structure.

Prior work

Anisotropic volume models

[Kajiya and Kay 1989], [Neyret 1998], ...

Anisotropic diffusion

[Heiskala et al. 2005], [Heino et al. 2003],

[Johnson and Lagendijk 2009], ...

[Kajiya and Kay 1989]

[Neyret 1998]

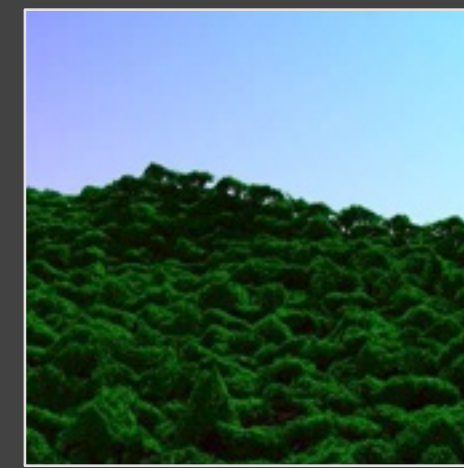
In optical tomography, anisotropic diffusion has been used to recover the contents of a volume using only external observations followed by the solution of a complicated inverse problem. We also use anisotropic diffusion, but we're mainly interested in the opposite direction.

Prior work

Anisotropic volume models

[Kajiya and Kay 1989], [Neyret 1998], ...

[Kajiya and Kay 1989]



[Neyret 1998]

Anisotropic diffusion

[Heiskala et al. 2005], [Heino et al. 2003],

[Johnson and Lagendijk 2009], ...

Dipole and multipole solutions

[Jensen et al. 2001], [Donner and Jensen 2005],

[Dudko and Weiss 2005], ...

[Jensen et al. 2001]

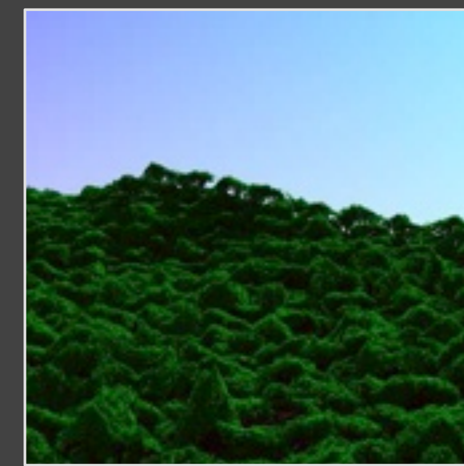
Both dipole and multipole solutions to the diffusion equation have been proposed for the isotropic and anisotropic case. The existing anisotropic solutions from biophysics are very complicated though and are not well-suited for graphics, because they were developed specifically with this inverse problem in mind. In this paper, we show how to construct an anisotropic dipole that leads to a much simpler solution <u>and</u> can be used for rendering.

Prior work

Anisotropic volume models

[Kajiya and Kay 1989], [Neyret 1998], ...

[Kajiya and Kay 1989]



[Neyret 1998]

Anisotropic diffusion

[Heiskala et al. 2005], [Heino et al. 2003],

[Johnson and Lagendijk 2009], ...

[Jensen et al. 2001]

Dipole and multipole solutions

[Jensen et al. 2001], [Donner and Jensen 2005],

[Dudko and Weiss 2005], ...

Microfacet models

[Cook and Torrance 1982],

[Ashikhmin and Shirley 2002], ...

[Cook and Torrance 1982]

[Ashikhmin and Shirley 2002]

And finally: microfacet models introduced to graphics by Cook and Torrance in 1982, have been hugely successful in representing scattering from a wide range of materials with rough surfaces. Motivated by this success, we generalize them to the volume setting and we call the resulting model a “micro-flake model”. As we will see later, this turns out to be a nice way of describing the most important types of anisotropic scattering in volumes.

Contributions

Models	Equations	Solution Techniques
	Radiative transfer equation	

Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.

Contributions

Models | Equations | Solution Techniques

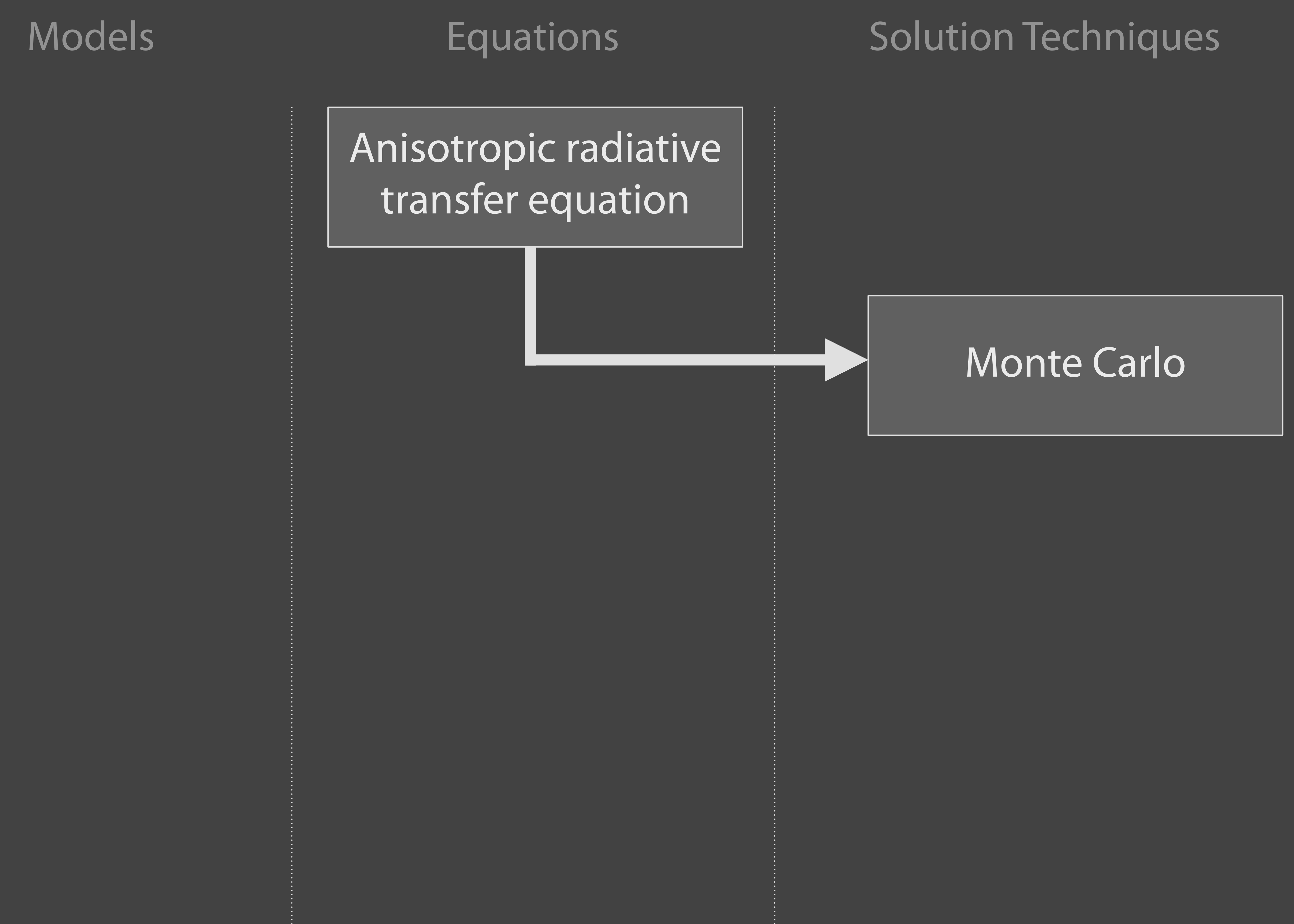
Anisotropic radiative transfer equation

Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.

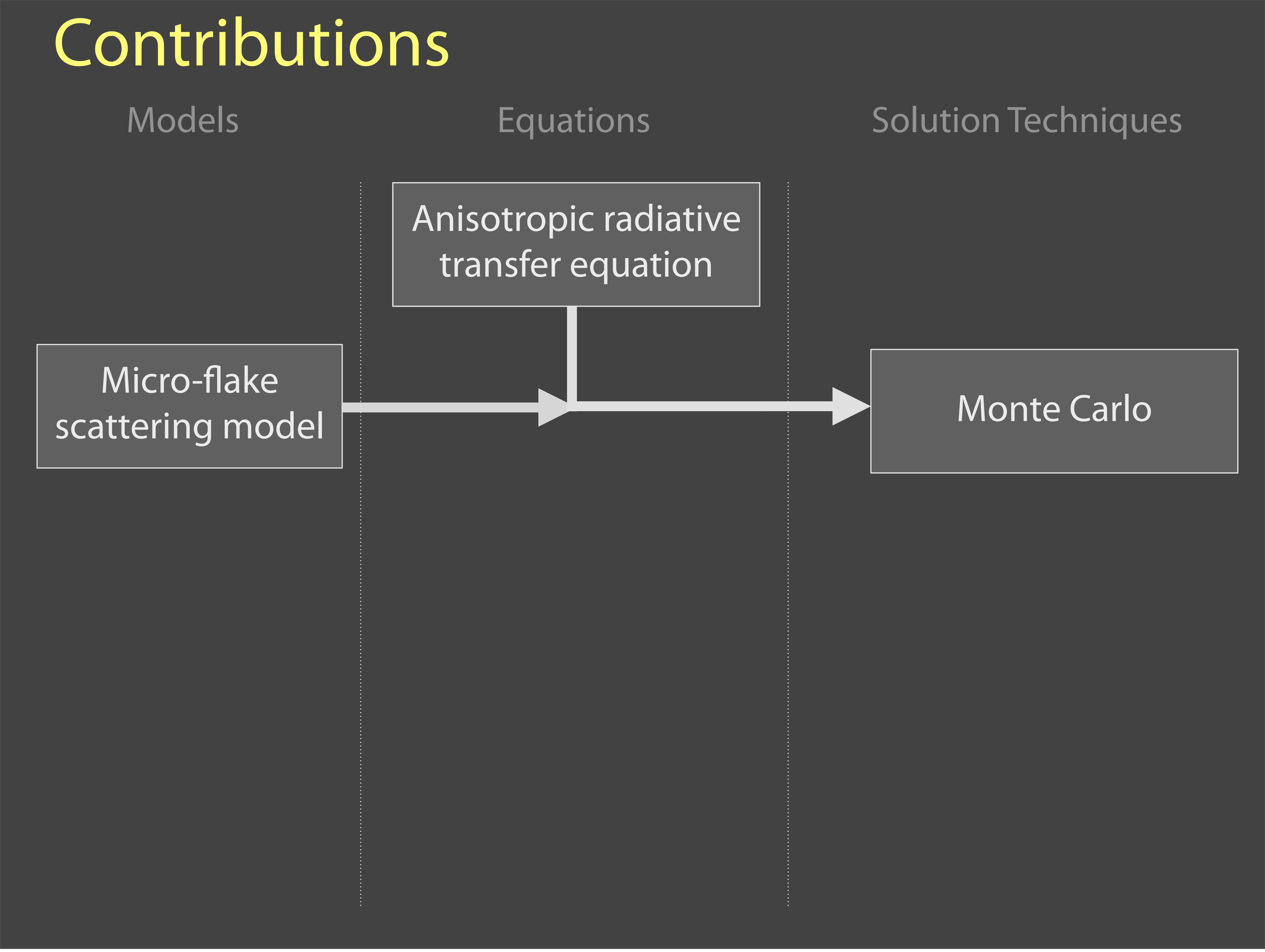
Contributions



Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.

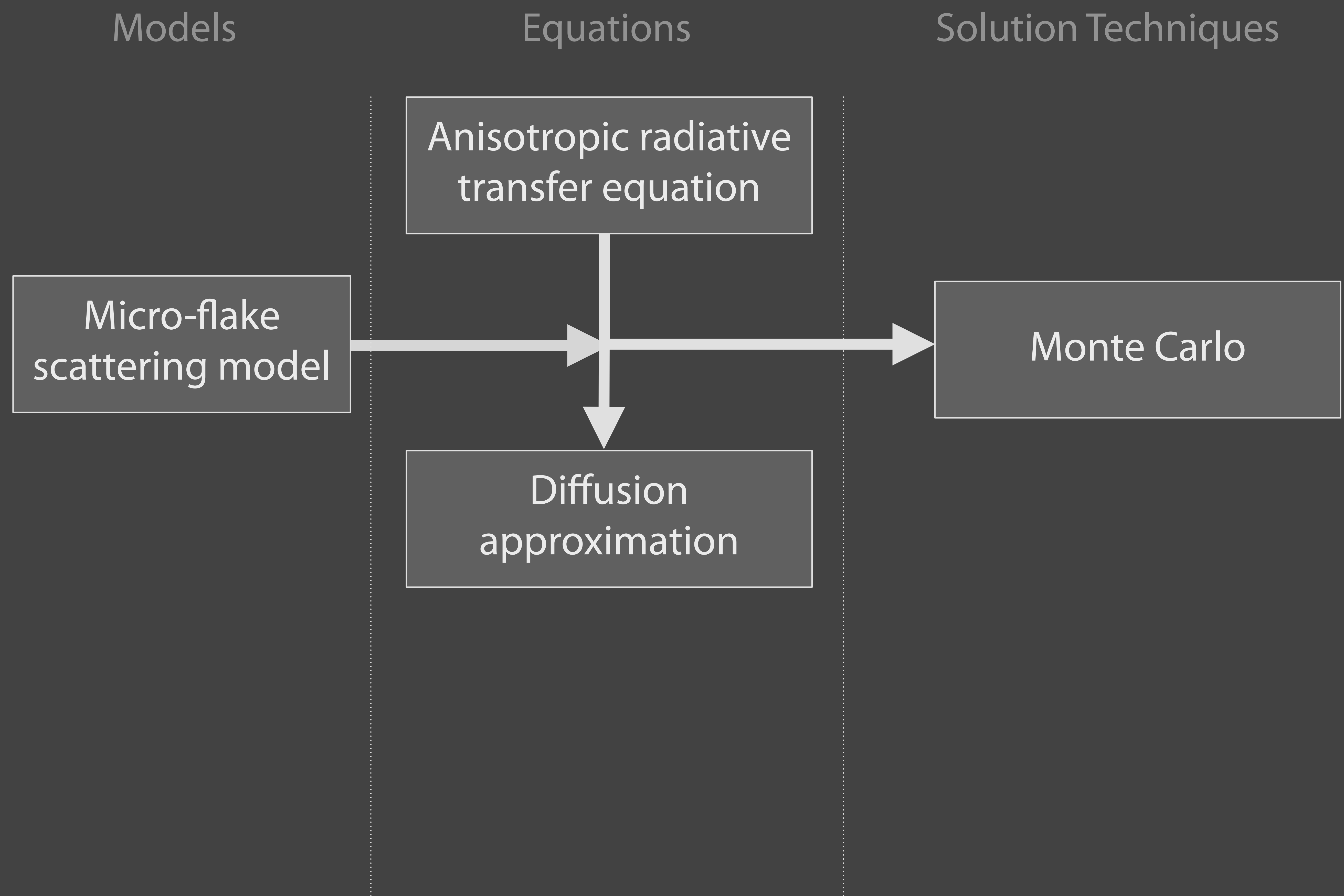


Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.

Contributions

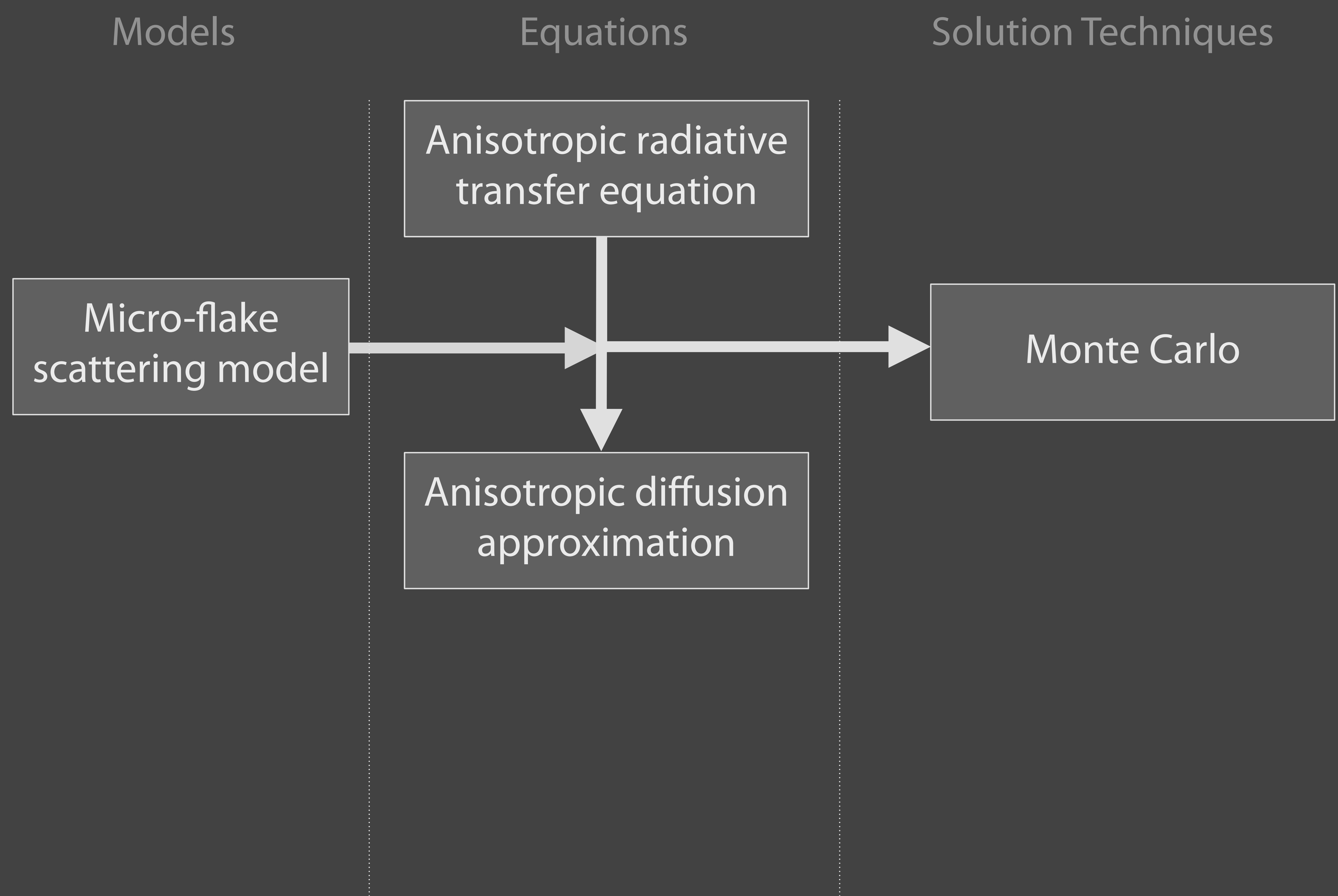


Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.

Contributions

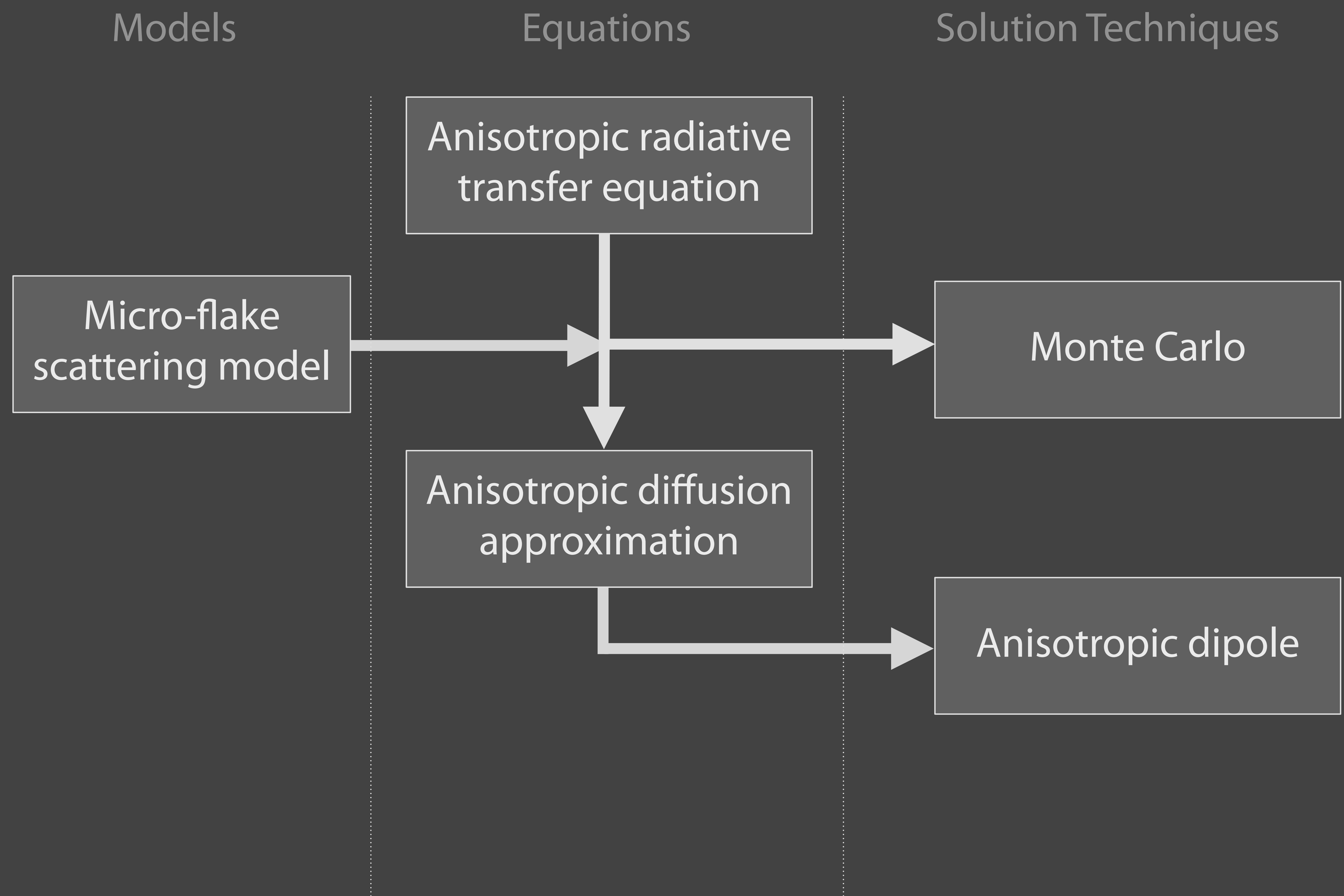


Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.

Contributions

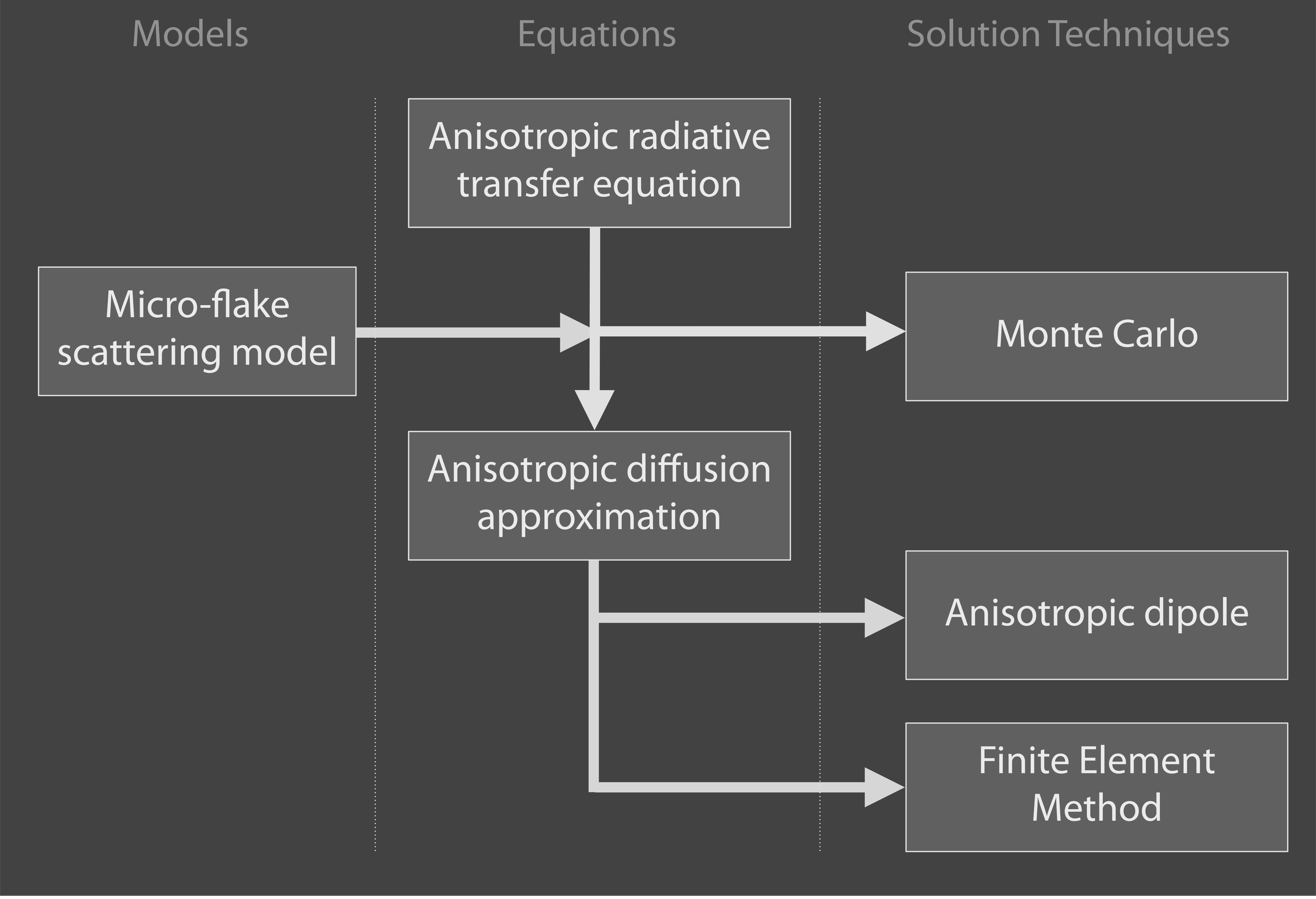


Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.

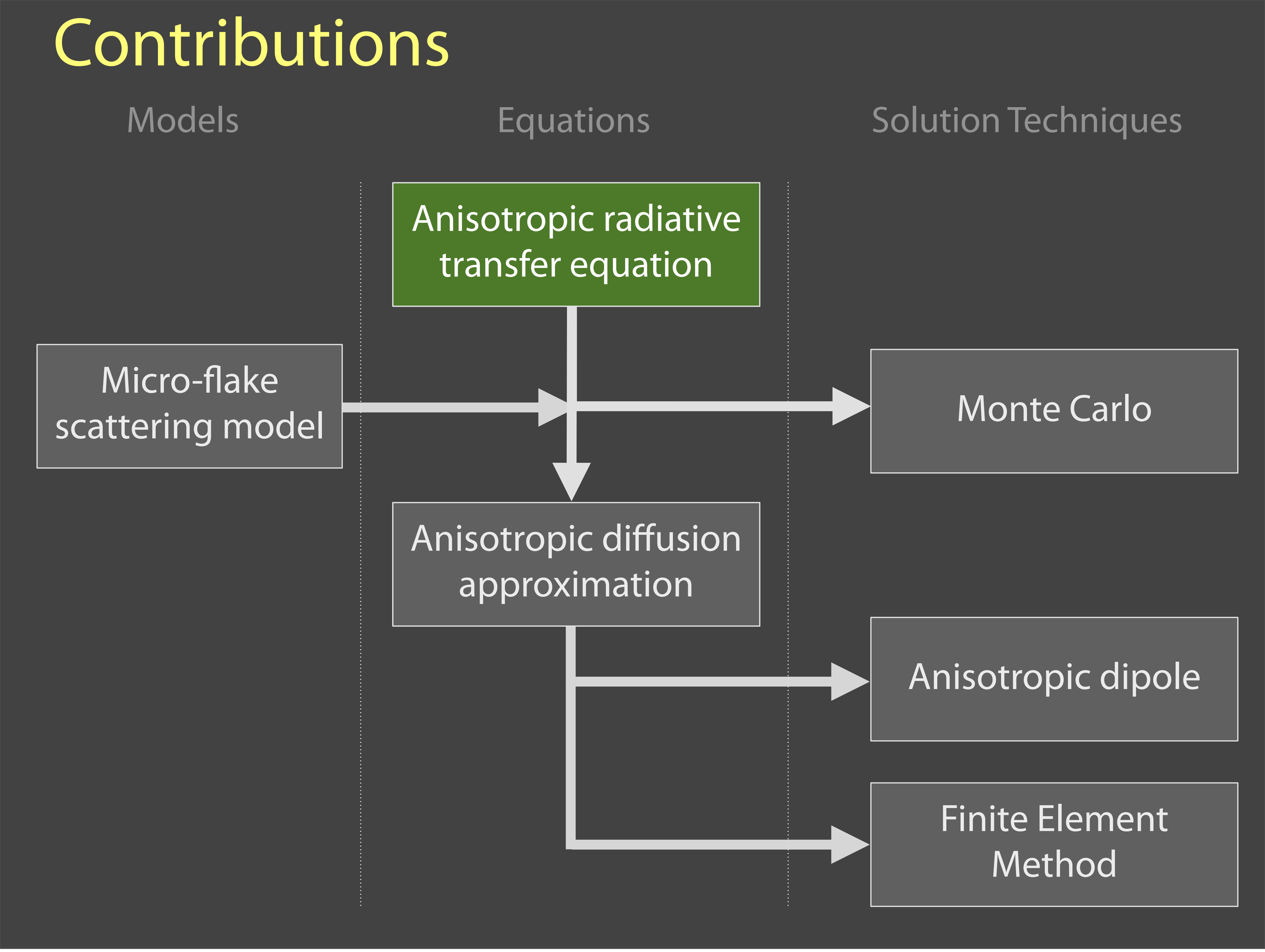
Contributions



Here is an outline of the talk: We'll start by making a modification to the radiative transfer equation, which leads to an anisotropic form. Before we're able to start rendering using Monte Carlo techniques, we still need a suitable scattering model, and here we propose one that is based on specularly reflecting flakes.

One common approximation that can be derived now is the diffusion approx. But because our earlier changes propagate, it takes on a new anisotropic form. We also adapt two associated diffusion-based solution techniques to suit this new equation.

Due to the time limitations, we can only give a very high-level overview of these steps, and we refer you to the paper and technical report for details.



Before continuing, let’s do a quick review of the radiative transfer equation.

Radiative transfer equation

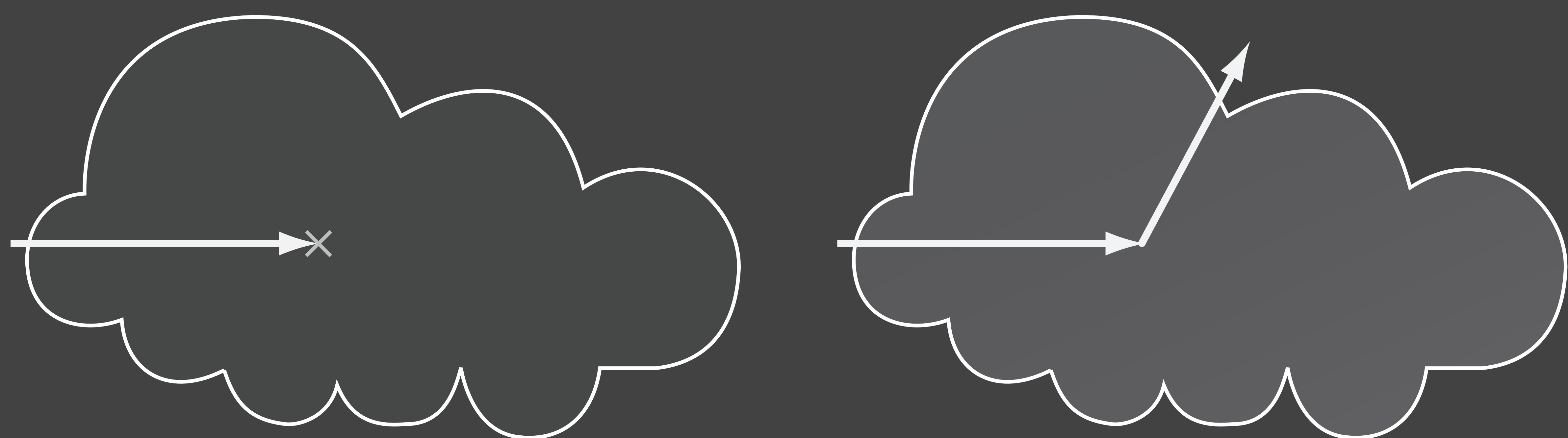
$$(\omega \cdot \nabla) L(\omega) = -\sigma_t \, L(\omega) + \sigma_s \int_{S^2} f_p(\omega' \cdot \omega) L(\omega') \, \mathrm{d}\omega' + Q(\omega)$$

(spatial dependence dropped for readability)

This is the form in which it is usually written down in computer graphics.

Radiative transfer equation

$$\underbrace{(\omega \cdot \nabla) L(\omega)}_{\text{local change}} = \underbrace{-\sigma_t \, L(\omega)}_{\text{extinction}} + \sigma_s \int_{S^2} f_p(\omega' \cdot \omega) L(\omega') \, \mathrm{d}\omega' + Q(\omega)$$

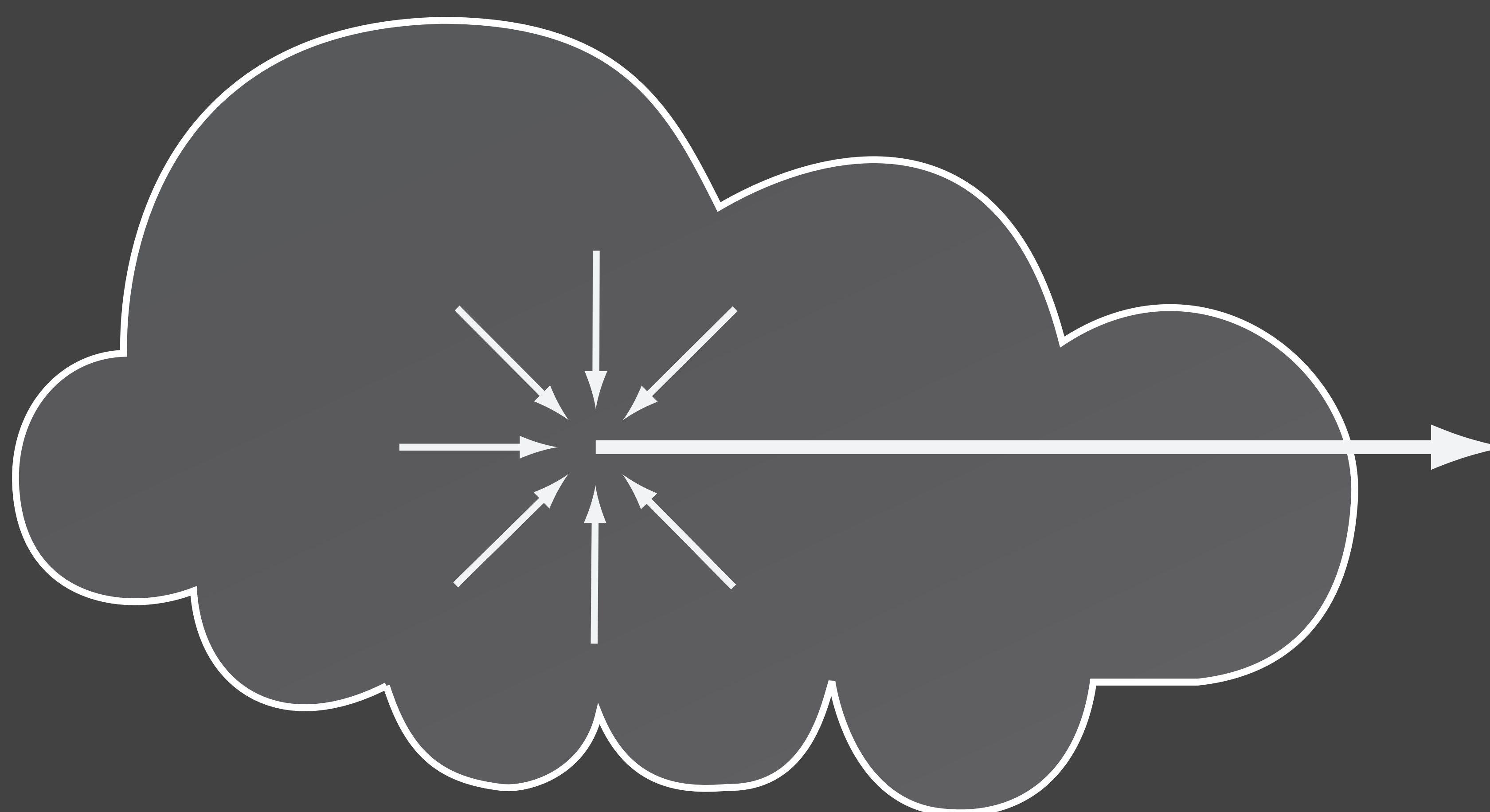


(spatial dependence dropped for readability)

Essentially, this equation describes the local change of radiance in a certain direction as a sum of terms, which nicely map to physical interpretations. The first one accounts for the decrease in radiance through extinction, which corresponds to light that is either absorbed or scattered elsewhere.

Radiative transfer equation

$$\underbrace{(\omega \cdot \nabla) L(\omega)}_{\text{local change}} = \underbrace{-\sigma_t \, L(\omega)}_{\text{extinction}} \underbrace{+ \sigma_s \int_{S^2} f_p(\omega' \cdot \omega) L(\omega') \, \mathrm{d}\omega'}_{\text{in-scattering}} + Q(\omega)$$

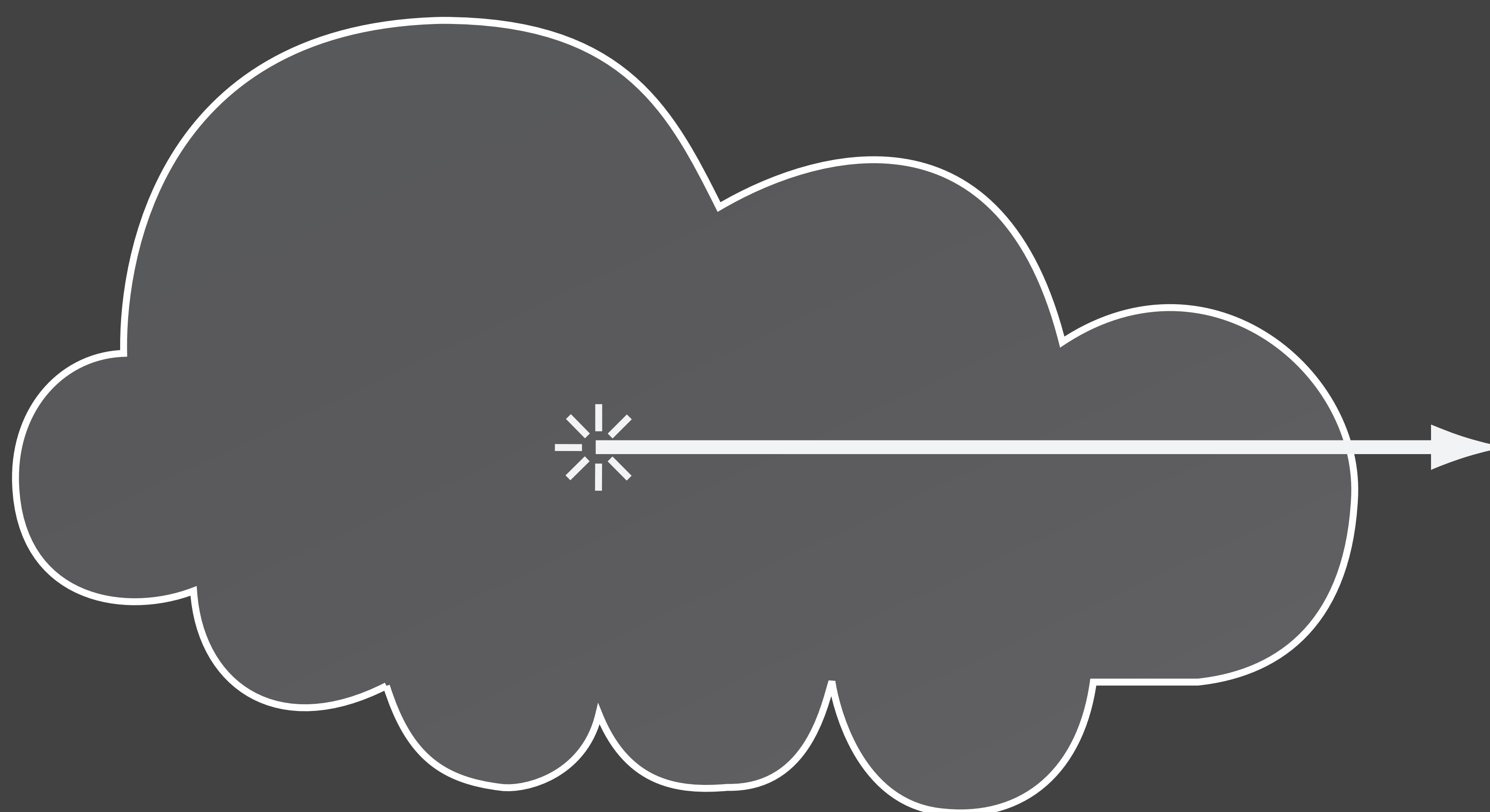


(spatial dependence dropped for readability)

The second term describes the in-scattering from other directions, which turns into an integral over the sphere containing the the so-called phase function f sub p.

Radiative transfer equation

$$\underbrace{(\omega \cdot \nabla) L(\omega)}_{\text{local change}} = \underbrace{-\sigma_t L(\omega)}_{\text{extinction}} \underbrace{+ \sigma_s \int_{S^2} f_p(\omega' \cdot \omega) L(\omega') \, \mathrm{d}\omega'}_{\text{in-scattering}} \underbrace{+ Q(\omega)}_{\text{source}}$$



(spatial dependence dropped for readability)

And finally, the radiance might also increase because the medium itself acts as an emitter -- this behavior is captured by the source term Q.

Radiative transfer equation

Isotropic form:

$$\underbrace{(\omega\cdot\nabla)L(\omega)}_{\text{local change}} = \underbrace{-\sigma_t\, L(\omega)}_{\text{extinction}} \underbrace{+\,\sigma_s \int_{S^2} f_p(\omega'\cdot\omega)L(\omega')\,\mathrm{d}\omega'}_{\text{in-scattering}} \underbrace{+\,Q(\omega)}_{\text{source}}$$

Anisotropic form:

$$\underbrace{(\omega\cdot\nabla)L(\omega)}_{\text{local change}} = \underbrace{-\sigma_t(\omega)L(\omega)}_{\text{extinction}} \underbrace{+\,\sigma_s(\omega)\int_{S^2} f_p(\omega'\to\omega)L(\omega')\,\mathrm{d}\omega'}_{\text{in-scattering}} \underbrace{+\,Q(\omega)}_{\text{source}}$$

(spatial dependence dropped for readability)

Let's take a look how this equation changes in the anisotropic case, which is shown at the bottom here. You can see that the extinction and scattering coefficients now have a directional dependence, and that the phase function is a proper function of two directions, as opposed to just the angle between them.

It's a really bad idea to just stick some arbitrary functions sigma_s, sigma_t and f sub p into this equation because, they are in fact all related to each other. To find out what these relations are, it necessary to take a step back and derive this equation from first principles. For radiative transfer, this means that we need to reason about the particles that make up the volume. We won't have time to see how this derivation works in detail, but we can take a look at the ingredients. One general thing to note here is that the material might actually not be made of particles. Despite that, the particle abstraction has proven itself in the past.

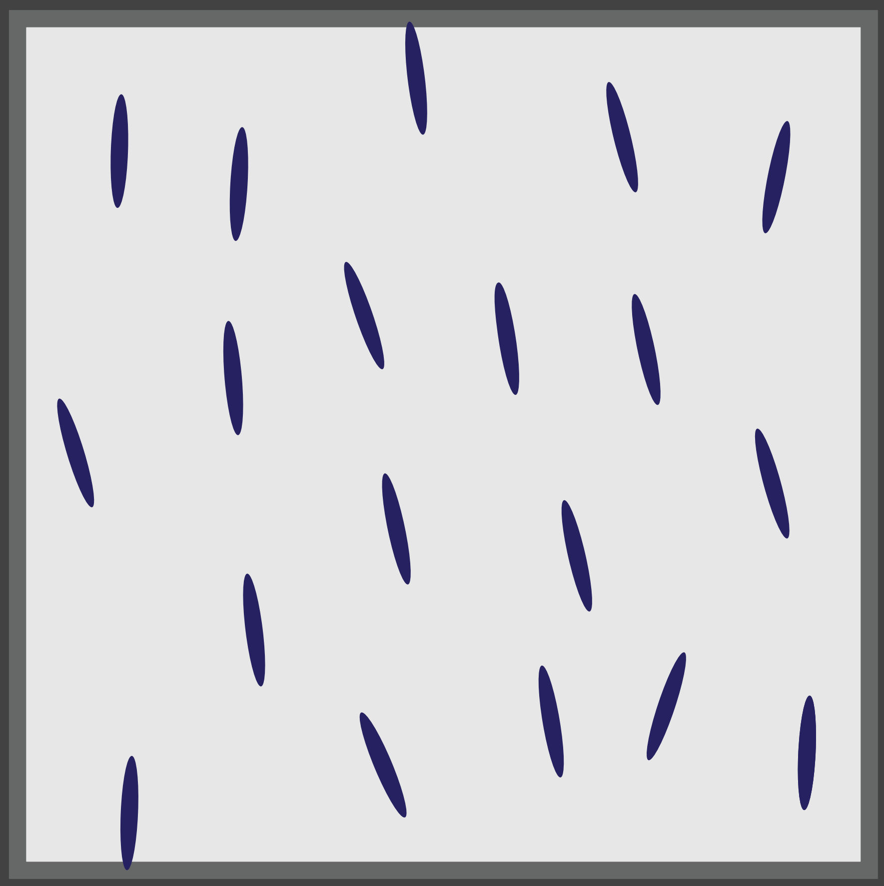
Particle description

Need several pieces of information:

The goal here is to find a compact way of fully characterizing the underlying particles. We do this using several pieces of information:

Particle description

Need several pieces of information:

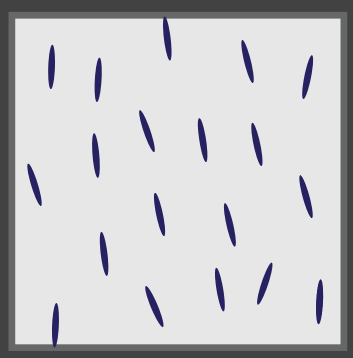


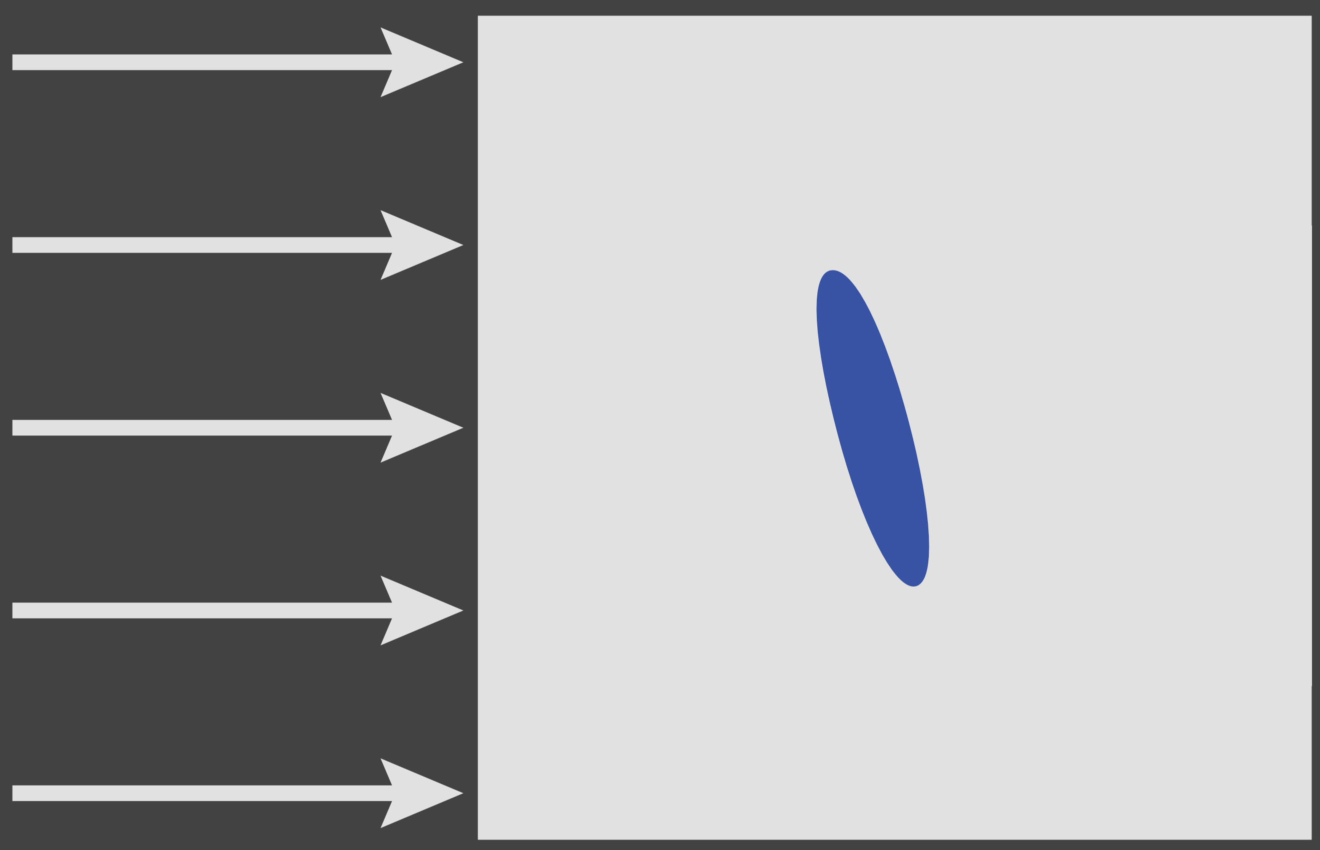
First, we need a density function that tells us how the particles are distributed both spatially and directionally.

Particle description

Need several pieces of information:

Particle distribution



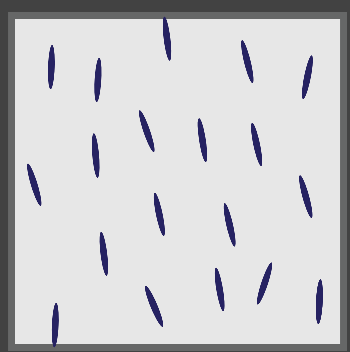


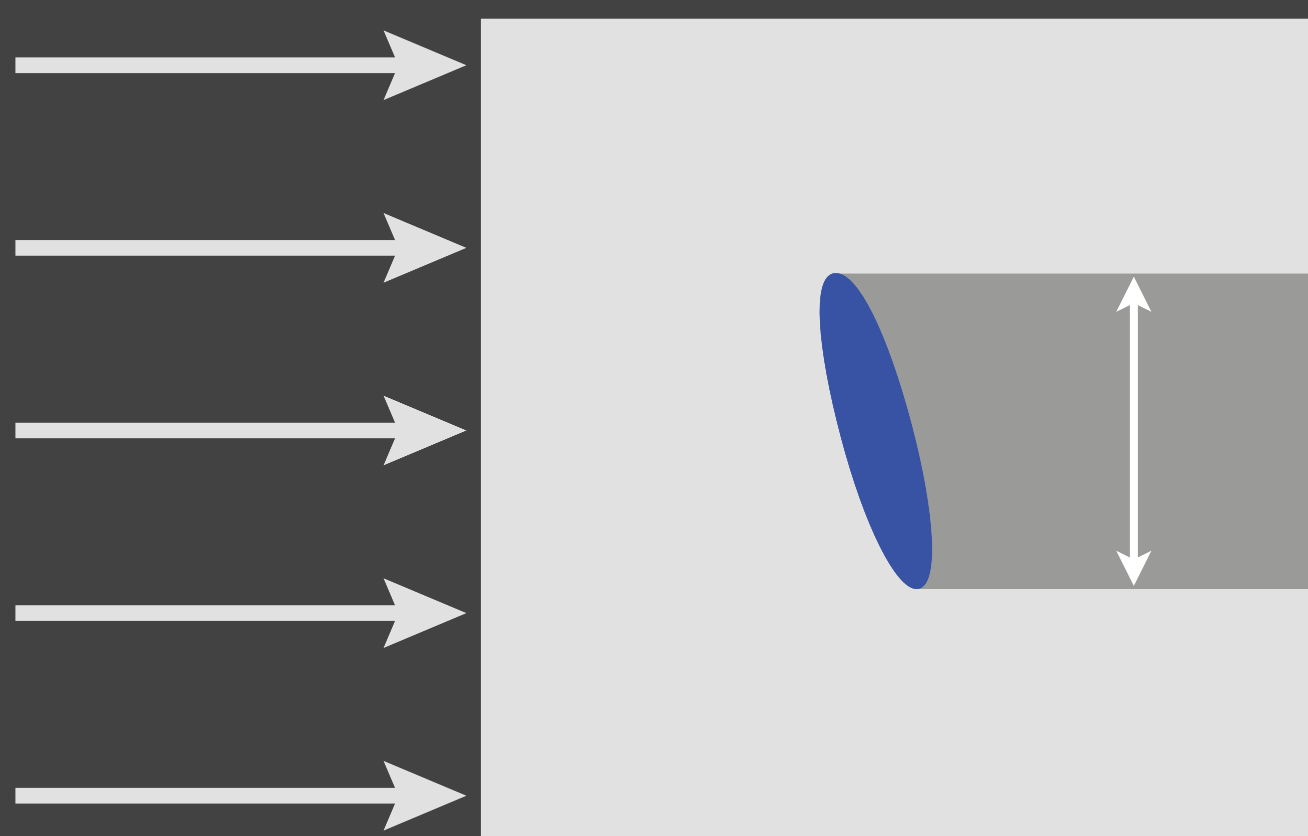
Secondly, we need to know how much light a particle intercepts --

Particle description

Need several pieces of information:

Particle distribution

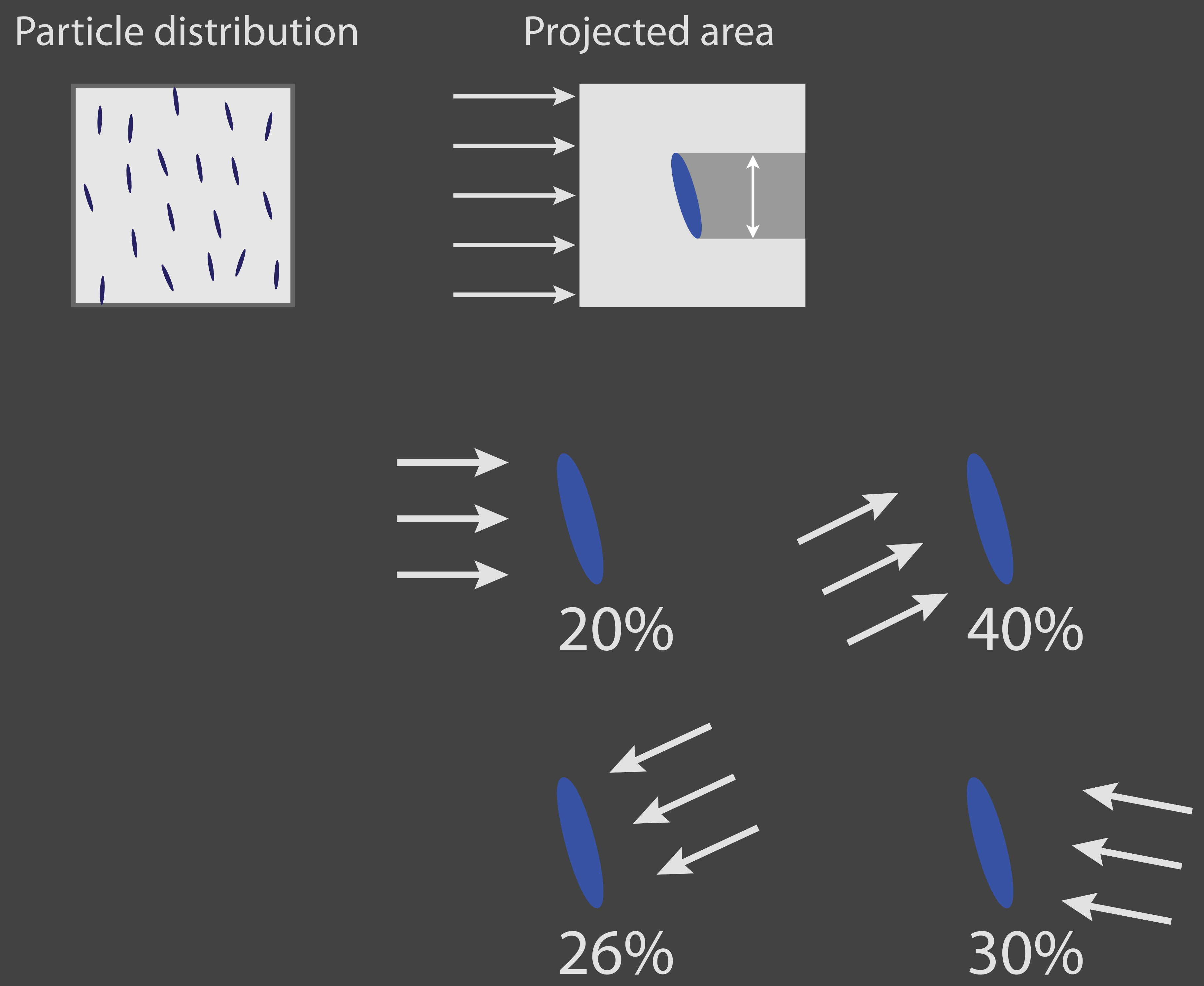




so we need a function that tells us the projected area from different directions.

Particle description

Need several pieces of information:

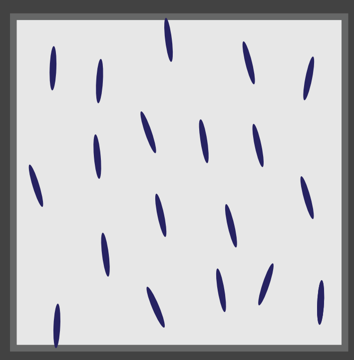


The particle might reflect different amounts of light depending on the direction from which it is illuminated, so we need to provide a directionally varying albedo function.

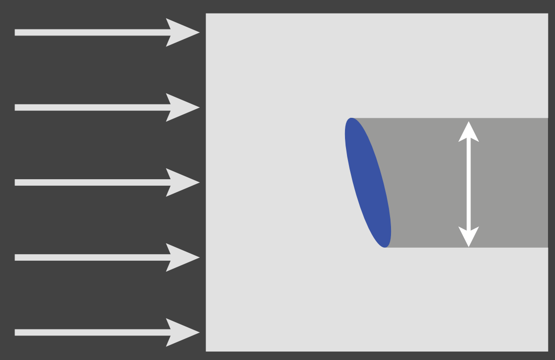
Particle description

Need several pieces of information:

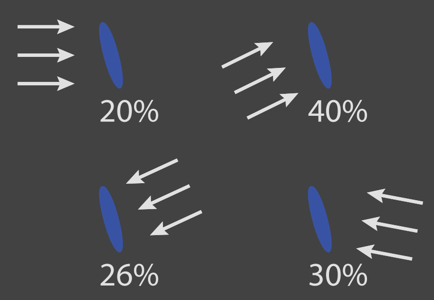
Particle distribution

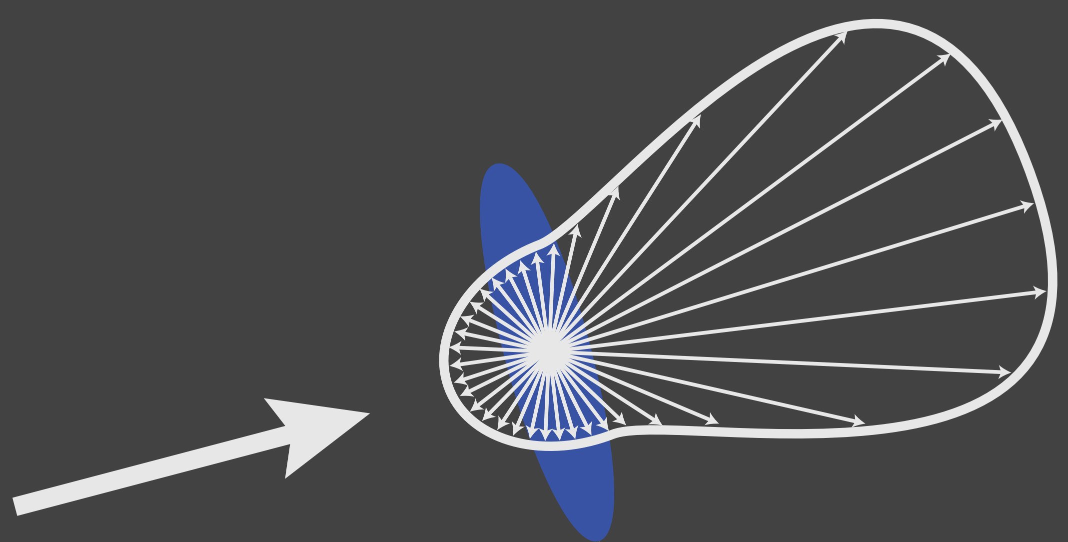


Projected area



Albedo

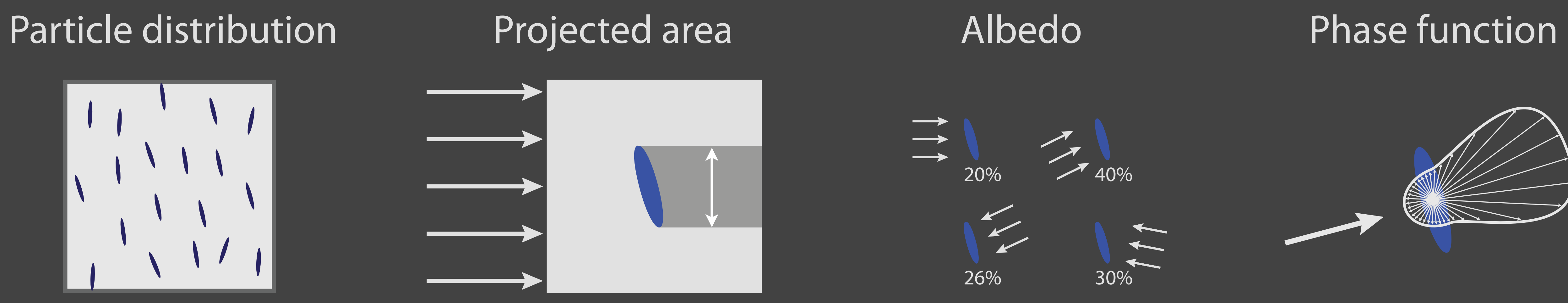




And each particle itself also has a phase function that determines the scattered direction after an interaction takes place.

Particle description

Need several pieces of information:

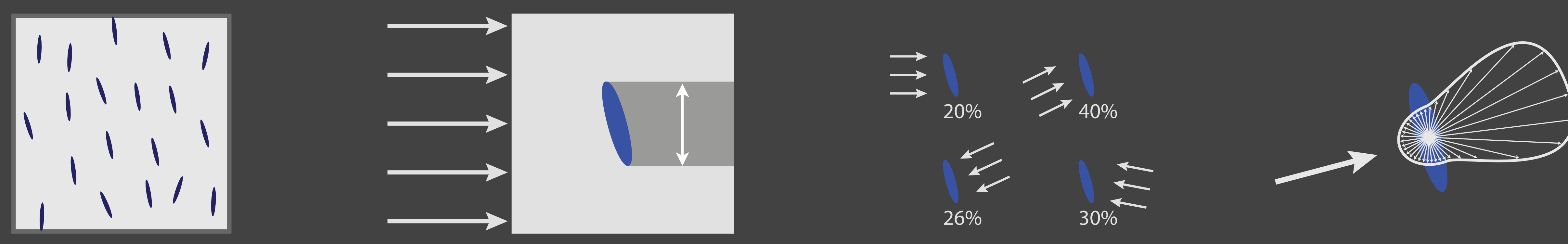


Now, given these ingredients --

Particle description

Need several pieces of information:

Particle distribution | Projected area | Albedo | Phase function

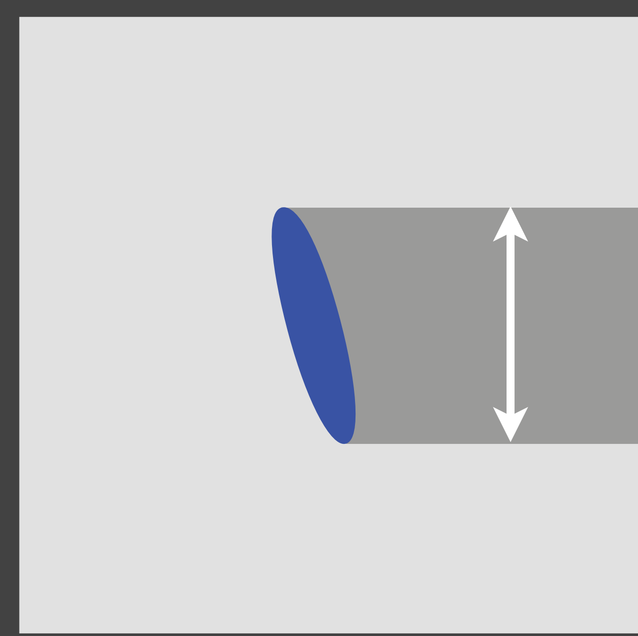


$$\sigma_s, \sigma_t, f_p$$

the paper provides a way of determining what the volume's scattering coefficients and the phase function should be.

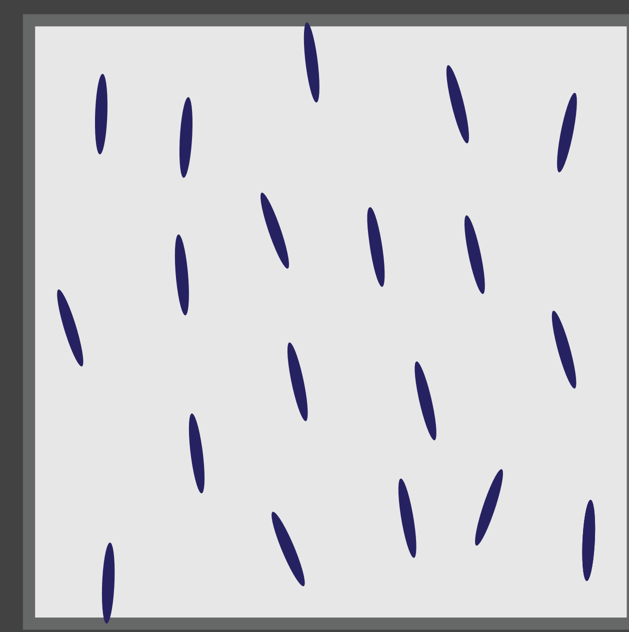
Example: extinction coefficient

$$\sigma_t(\omega) =$$



projected area function

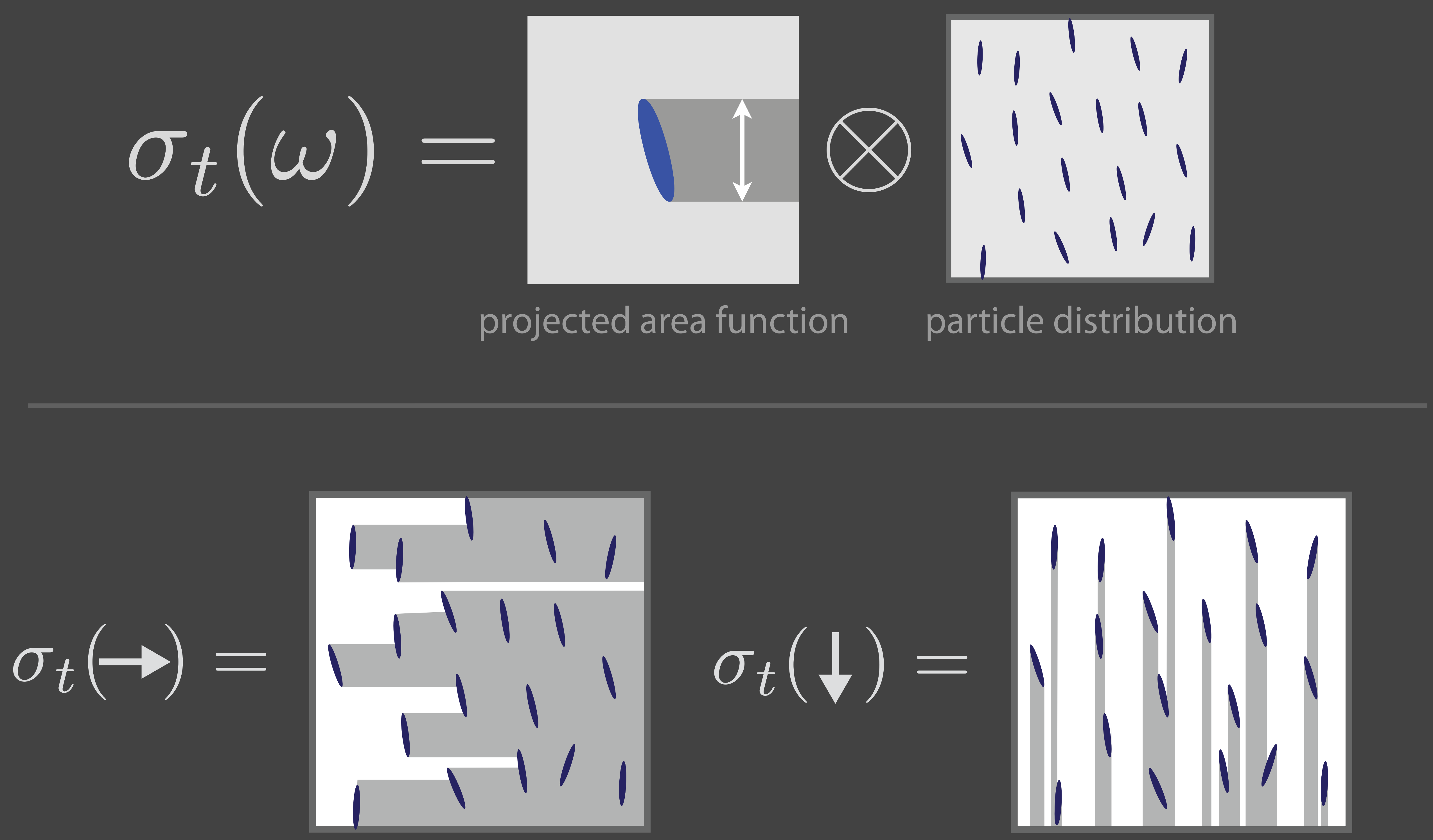
$$\otimes$$



particle distribution

All of them turn into integrals over the sphere. The easiest one is the extinction coefficient, which is simply the convolution of the projected area function of a single particle with the particle distribution.

Example: extinction coefficient



This means that the amount of extinction can potentially vary quite strongly with the direction of propagation, and here is a just picture to illustrate that.

Interesting properties

Normalization

- f_p cannot be normalized in both arguments
  — *one must be chosen*!

Some things are very different compared to the isotropic case. For example, the phase function can't be normalized in both of its arguments anymore -- it is necessary to pick one of them. The most intuitive choice is the direction in which it gets sampled in the Monte Carlo rendering context, but one could just as well choose the other argument.

Interesting properties

Normalization

- f_p cannot be normalized in both arguments

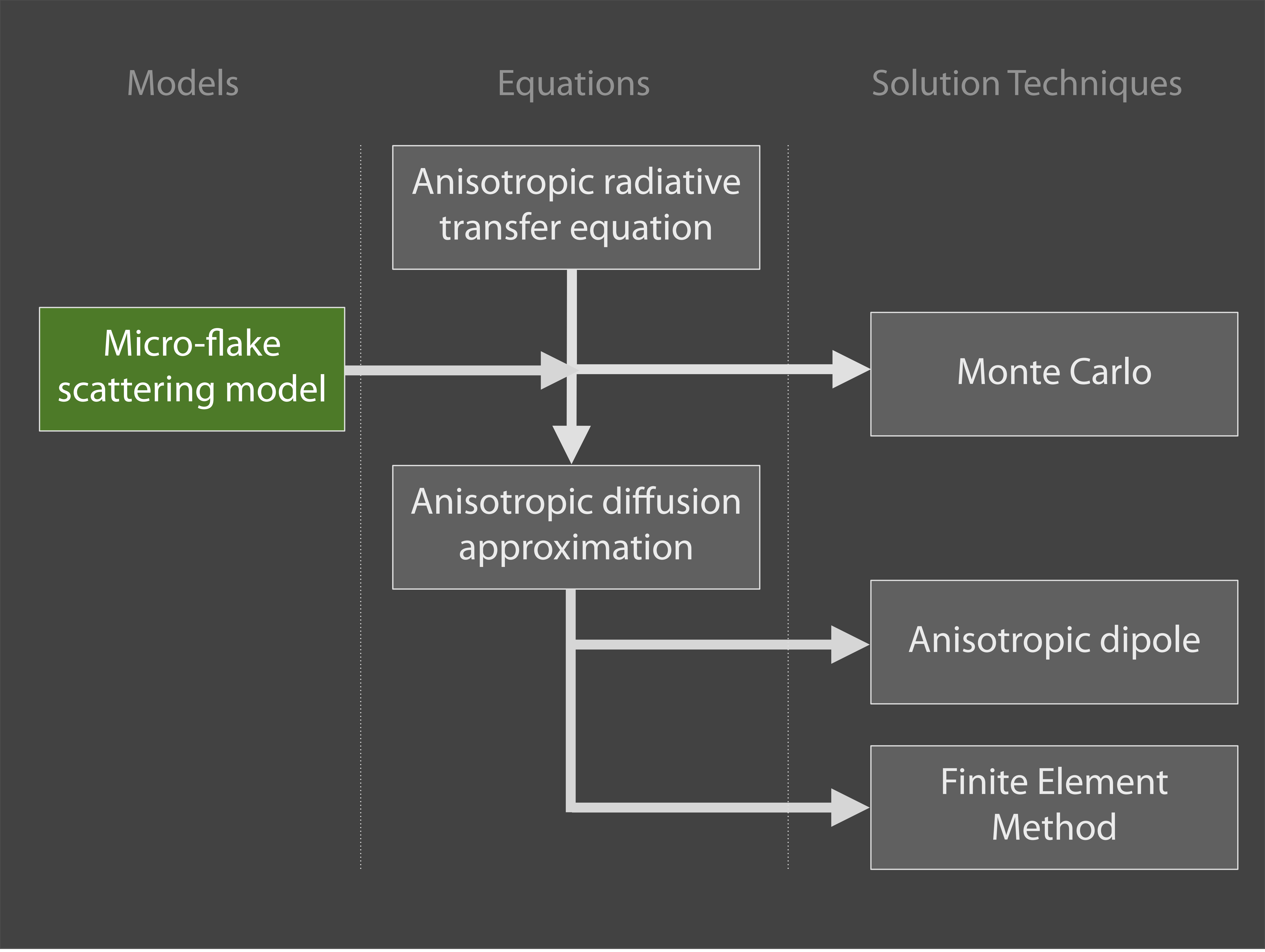
  — *one must be chosen*!

Reciprocity

- f_p is usually not symmetric! Instead, it satisfies

$$\boxed{\sigma_s(\omega) f_p(\omega' \to \omega) = \sigma_s(\omega') f_p(\omega \to \omega')}$$

And secondly, the phase function is not a symmetric function anymore, and it instead satisfies a slightly more complicated relation that links it to the scattering coefficient. Of course, the underlying physical system is still reciprocal, but the equations encapsulate this in a different way.



So now we have this equation to work with — but by itself we can't use it yet. The reason for this is simply that nobody has ever used this anisotropic RTE before, so there are no scattering models for it. To fix this, we propose a new model called "micro-flakes". One thing to note here is that this is not a restriction -- most of the paper is equally applicable if you'd rather come up with your own model.

Micro-flakes

Approach

- Plugs into the discussed particle abstraction
- Simple ideal mirror-like reflector on both sides

We present a simple model inspired by microfacet models, which turns out to get you pretty far. It's based on little flakes with ideal mirror-like reflection on both sides. These plug directly into the earlier particle abstraction, which gives you scattering coefficients and a phase function.

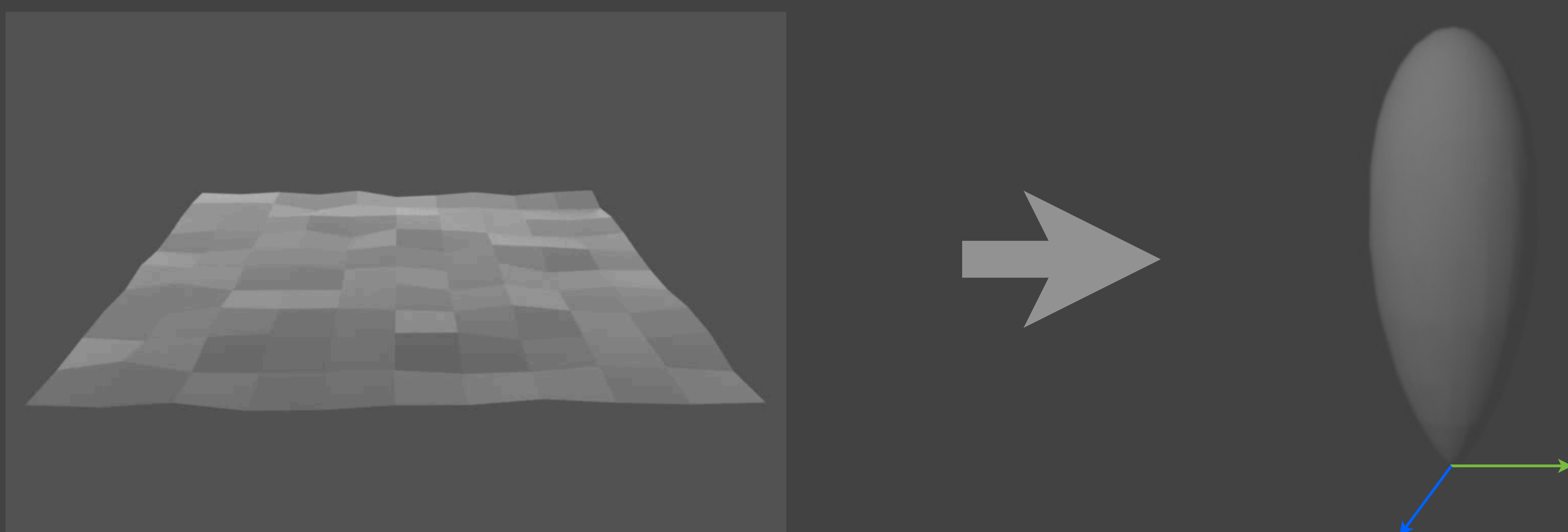
We use flakes to simulate various materials, even if they aren't actually made out of flakes in real-life. We mainly consider them to be a flexible tool for expressing different types of scattering, but without necessarily implying a specific internal makeup of your material.

The next question is: how do we choose the particle distributions. This decision is guided by the type of reflection we want our volume to represent.

Micro-flakes

Approach

- Plugs into the discussed particle abstraction
- Simple ideal mirror-like reflector on both sides



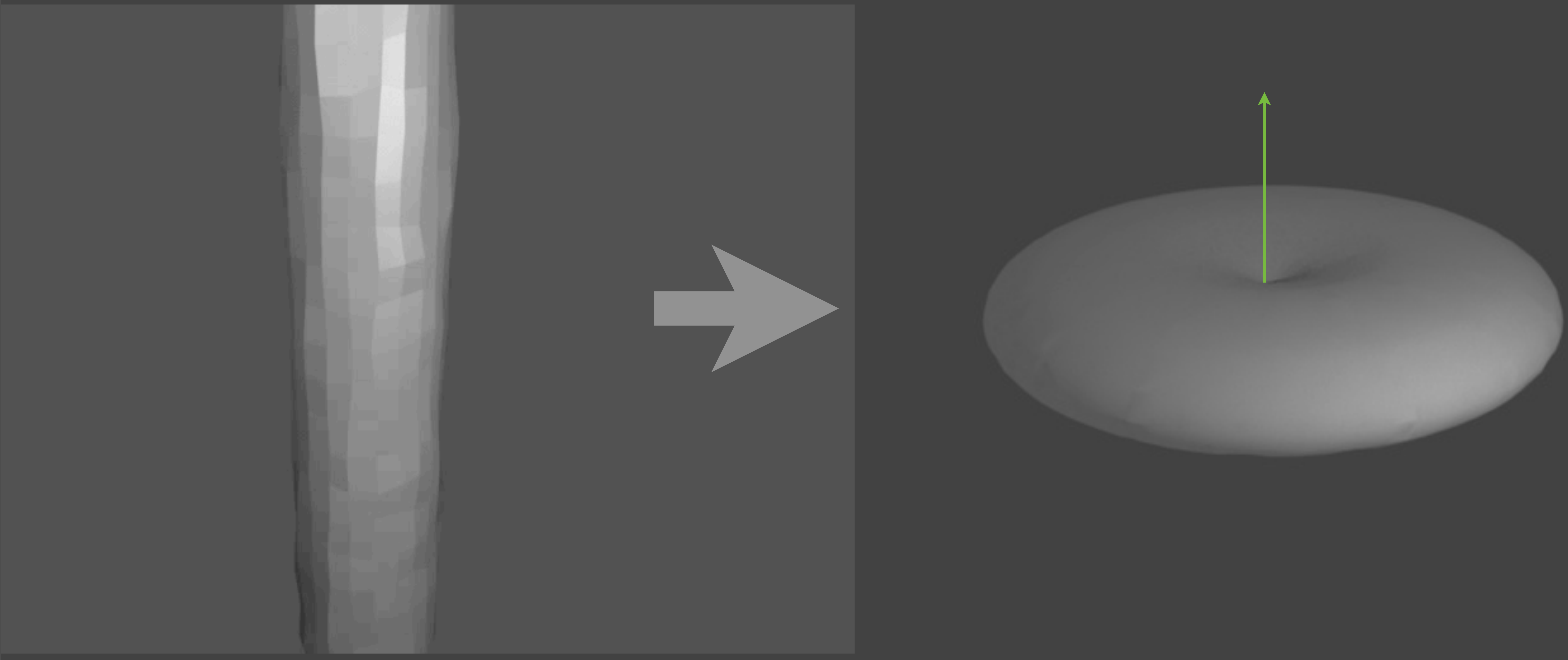
surface structure   flake distribution

For instance, to make a volume behave similarly to a rough surface, we choose the flake distribution on the right side here shown as a polar plot over the flake normals. Because most point upwards, the volume behaves like a translucent rough surface, which is oriented in that direction. Another way to think about this is as chopping up a facet representation of a surface and then building a histogram over the observed normals.

Micro-flakes

Approach

- Plugs into the discussed particle abstraction
- Simple ideal mirror-like reflector on both sides



surface structure flake distribution

And to make volume region behave like a rough fiber, we choose a “fibrous” equatorial flake distribution using the same principle.

Micro-flakes

Approach

- Plugs into the discussed particle abstraction
- Simple ideal mirror-like reflector on both sides

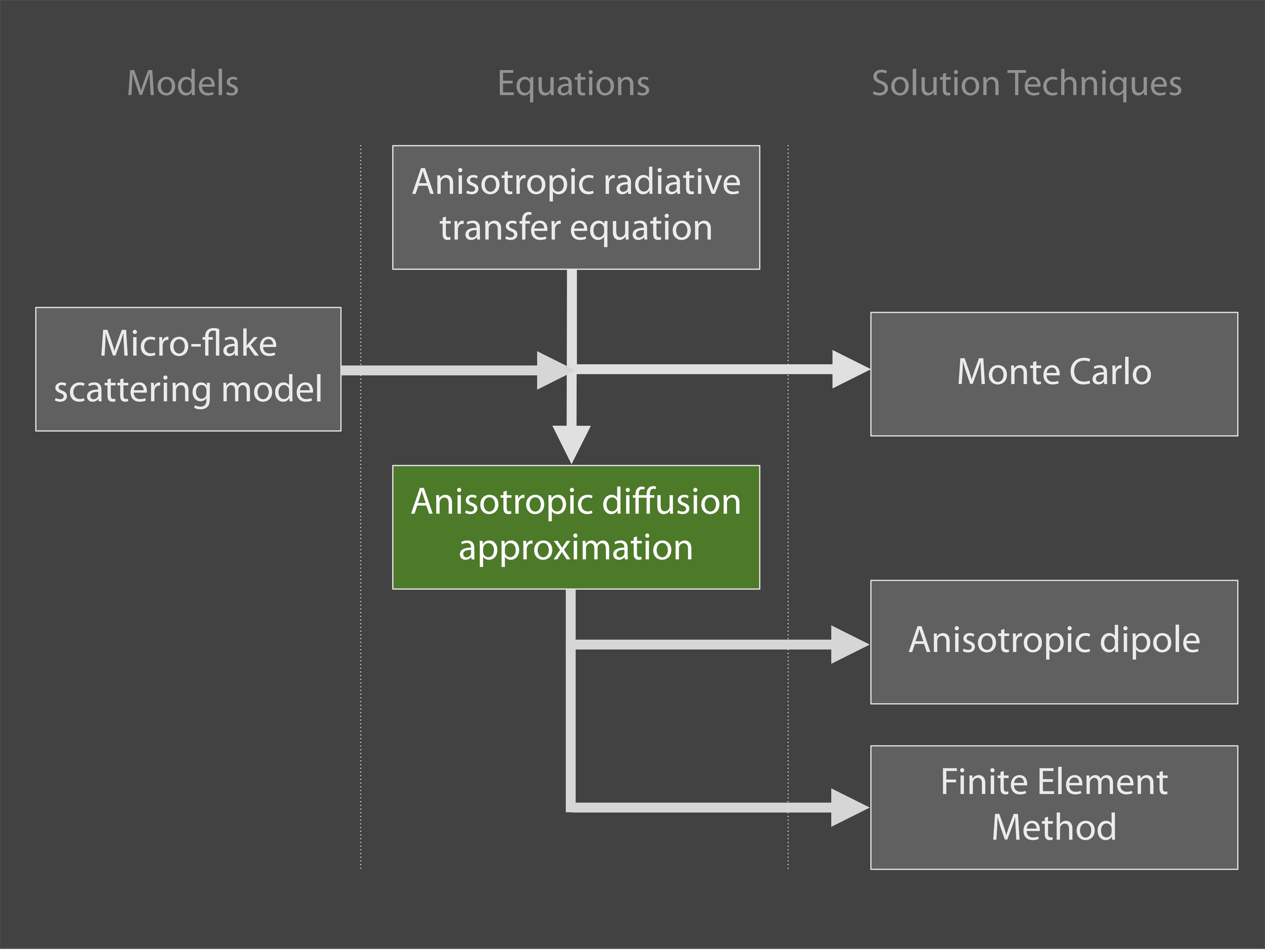
Properties

- Model subsumes traditional "anisotropic" scattering
- Leads to analytic results later on
- Half-angle formulation

Some more useful facts: First, this model is general enough subsume all traditional volume scattering models. So if you wanted to imitate Mie scattering using flakes, then you can find a specific type flake that will behave exactly the same way.

Another motivation for this model is that it leads to analytic solutions later on, when passing from the radiative transfer interpretation to that of anisotropic diffusion.

And finally, it results in a half-angle formulation, which is often a desirable property.



The next step is to find the anisotropic form of the diffusion equation.

Diffusion approximation

Preliminaries

- Key assumption: radiance is sufficiently smooth so that it can be approximated using a 2-term expansion:

$$L(x, \omega) \approx \frac{1}{4\pi}\phi(x) + \frac{3}{4\pi}\omega \cdot E(x)$$

Step 1

- Substitute this approximation into the RTE

We will start with a review of how this works in general.

The initial assumption of the diffusion approximation is that the radiance is sufficiently smooth so that the it can be written as a simple two-term expansion. That approximate radiance fn. is then substituted into the radiative transfer equation.

Diffusion approximation

Preliminaries

- Key assumption: radiance is sufficiently smooth so that it can be approximated using a 2-term expansion:

$$L(x,\omega) \approx \frac{1}{4\pi}\phi(x) + \frac{3}{4\pi}\omega \cdot E(x)$$

Step 1

- Substitute this approximation into the RTE

Step 2

- Require equality amongst constant & linear terms

Afterwards, we can't really solve that equation exactly anymore, and so a relaxed version of equality is used instead, which only requires the constant and linear components to agree.

The consequence of this step is that we'll actually get two equations.

Diffusion approximation – isotropic

0th order equation:

$$\nabla \cdot E(x) = -\sigma_t \phi(x) + \sigma_s \phi(x) + Q_0(x)$$

1st order equation:

$$\frac{1}{3}\nabla\phi(x) = -\sigma_t E(x) + g\sigma_s E(x) + Q_1(x)$$

local change | extinction | in-scattering | source

In the isotropic case, that looks something like this, and you can see that every term still nicely maps to a corresponding part in the original radiative transfer equation. A common step which follows now is to substitute one equation in the other, and then you'll get the form in which it is usually written down.

The important thing to note here is that all of these steps can be applied to the anisotropic radiative transfer equation in pretty much the same way.

And if we do that, then this is what we get

Diffusion approximation – anisotropic

0th order equation:

$$\nabla \cdot E(x) = C_0\phi(x) - V(x) \cdot E(x) + Q_0(x)$$

1st order equation:

$$\underbrace{\frac{1}{3}\nabla\phi(x)}_{\text{local change}} = \underbrace{C_1\phi(x) - M(x)E(x)}_{\text{extinction \& in-scattering}} \underbrace{+\, Q_1(x)}_{\text{source}}$$

The main, interesting change are the indicated “V” and “M” terms. “V” is a 3-vector and “M” a symmetric 3x3 matrix, and both capture information about the low-frequency anisotropy of the medium.
You can find expression for them in the paper, and much more detail our 50-page supplement. -- I’ll just say that each consists of three nested integrals over the sphere, which makes them daunting to compute.

One nice thing about the micro-flake model is it leads to significant simplifications at this point. Just by assuming that the medium is filled with flakes, then amongst others,

Diffusion approximation – anisotropic

0th order equation:

$$\nabla \cdot E(x) = C_0\phi(x) - V(x)\cdot E(x) + Q_0(x)$$

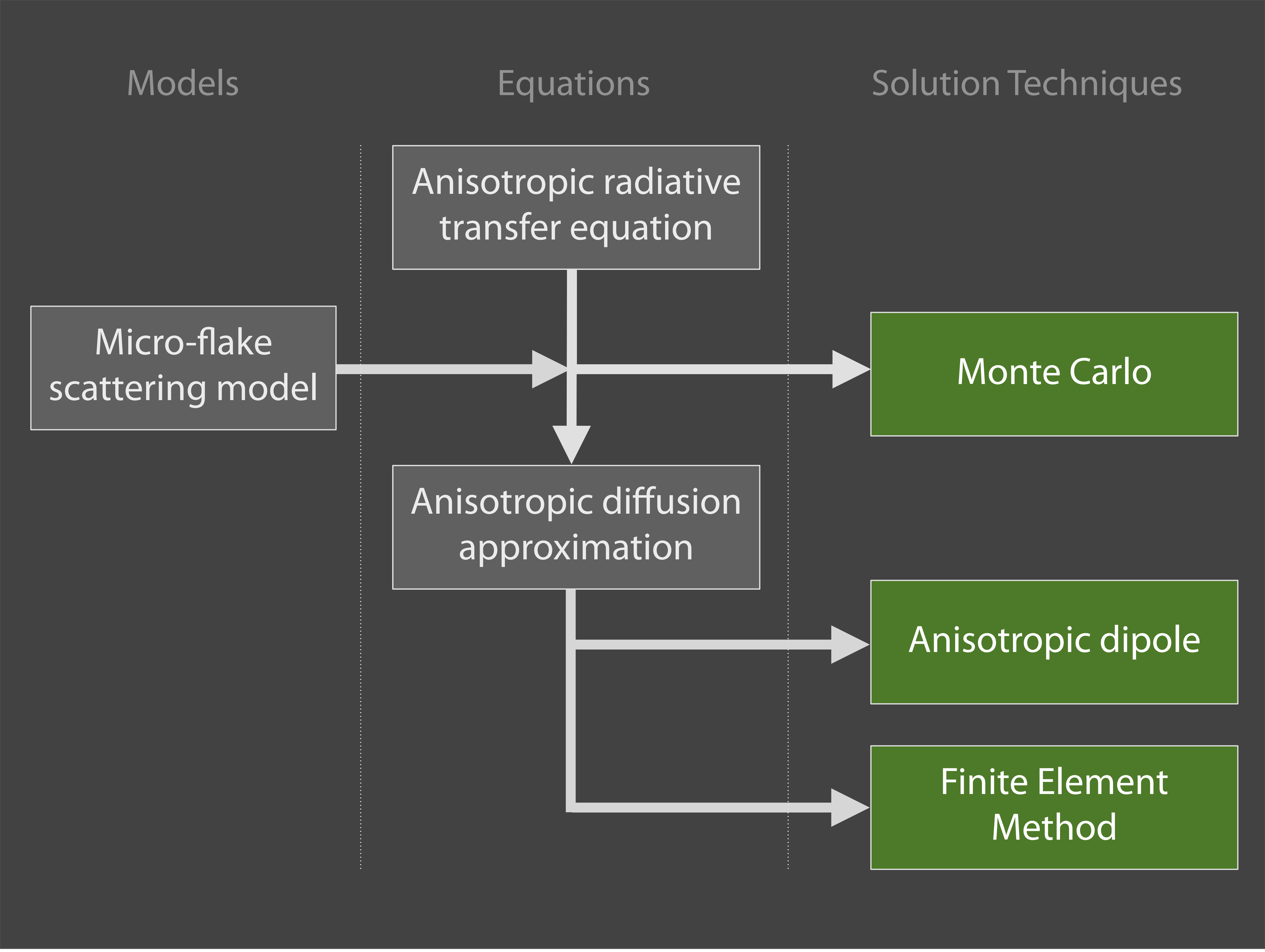
1st order equation:

$$\frac{1}{3}\nabla\phi(x) = C_1\phi(x) - M(x)E(x) + Q_1(x)$$

local change | extinction & in-scattering | source

the "V" term drops out and "M" is found to have a very simple closed form, which was a surprise to us.

The practical consequence is that if you know the spherical harmonics coefficients of the flake distribution, then you can compute "M" essentially for free by looking up the lower-order coefficients. That in turn now makes it possible to build fast diffusion-based rendering algorithms that are based on this equation.



Having having seen the equations and the model we use, we'll now take a look at the three solution techniques that can be used to render images of these materials.

Solution technique 1: Monte Carlo

Changes

- Must account for the directional dependence of the scattering coefficients
- Need good importance sampling support for the anisotropic phase function

The most accurate, but the slowest is Monte Carlo, and just few changes are required for this method. What does need to be addressed is that the scattering coefficients are not constants anymore, so they often need to be evaluated with respect to direction. Also, for highly scattering anisotropic media, it is important to have a good way of importance sampling the phase function, otherwise there will just be too much noise. The paper contains some information on how to solve these problems.

Here are some renderings made using Monte Carlo:

This is a high resolution scarf model represented as a 415 megavoxel volume. At every point in the medium, it contains both a density value and a local fiber orientation. The blue glow is completely due to multiple scattering. We would expect small highlights to run along the plies that make up the yarn, but because the rendering seen here is isotropic, the image looks a bit dull, and the illumination is relatively washed out.

But if we use the stored fiber orientations to define micro-flake distributions of the equatorial type at every point in the medium, we can switch to the anisotropic form of the radiative transfer equation and create this image:

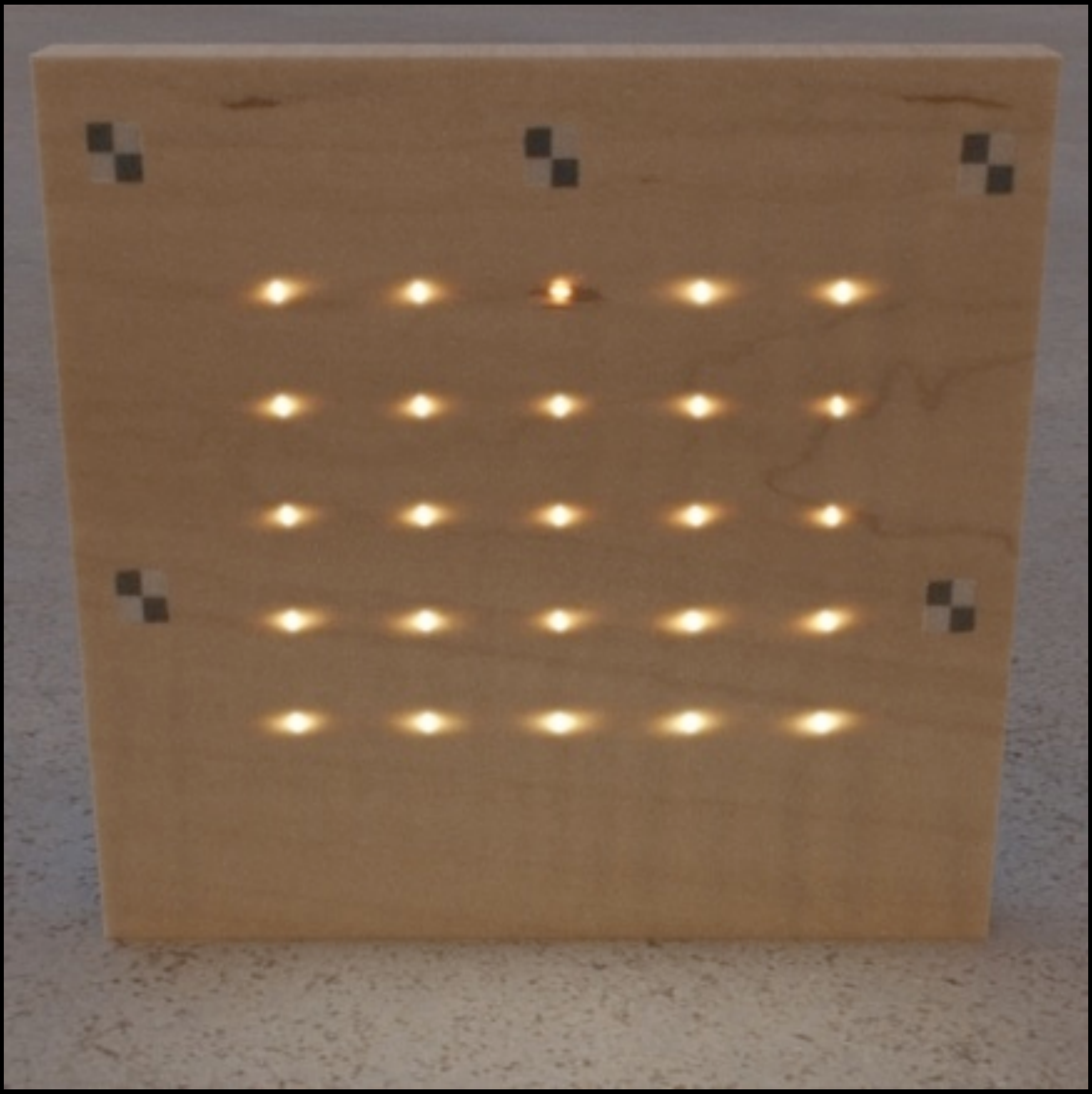
Here is an animated comparison

As you can see, accounting for the anisotropic structure leads to a significantly changed appearance, including much more realistic highlights. Up to now, physically based renderings of this kind have not been possible.

We have also made some experiments with the captured wood fiber orientations from the 2005 SIGGRAPH paper on finished wood.

That paper contained a *completely* specialized model that was only suitable for wood, and it also contained an ad-hoc diffuse component. We tried to reproduce the shifting highlights observed in that work, but now using the much more general micro-flakes and multiple scattering.

And even though the flakes really weren't made with wood in mind, this turns out to work, and we automatically get things like get energy conservation and reciprocity for free.



We also projected some very bright parallel beams onto the same slab, and we see these interesting spatially varying diffusion effects along the grain direction of the wood. This is something that the original model wouldn’t have been able to do.

One downside to the Monte Carlo approach is that all of these images take a really long time to render, from a few hours to almost a day. To improve on that, we’ll take a look at the FEM solver:

Solution technique 2: FEM

Finite elements approach

- Based on anisotropic diffusion equation
- Implementation built on [Arbree et al. 09]
- Much faster; rendering times in the minute range

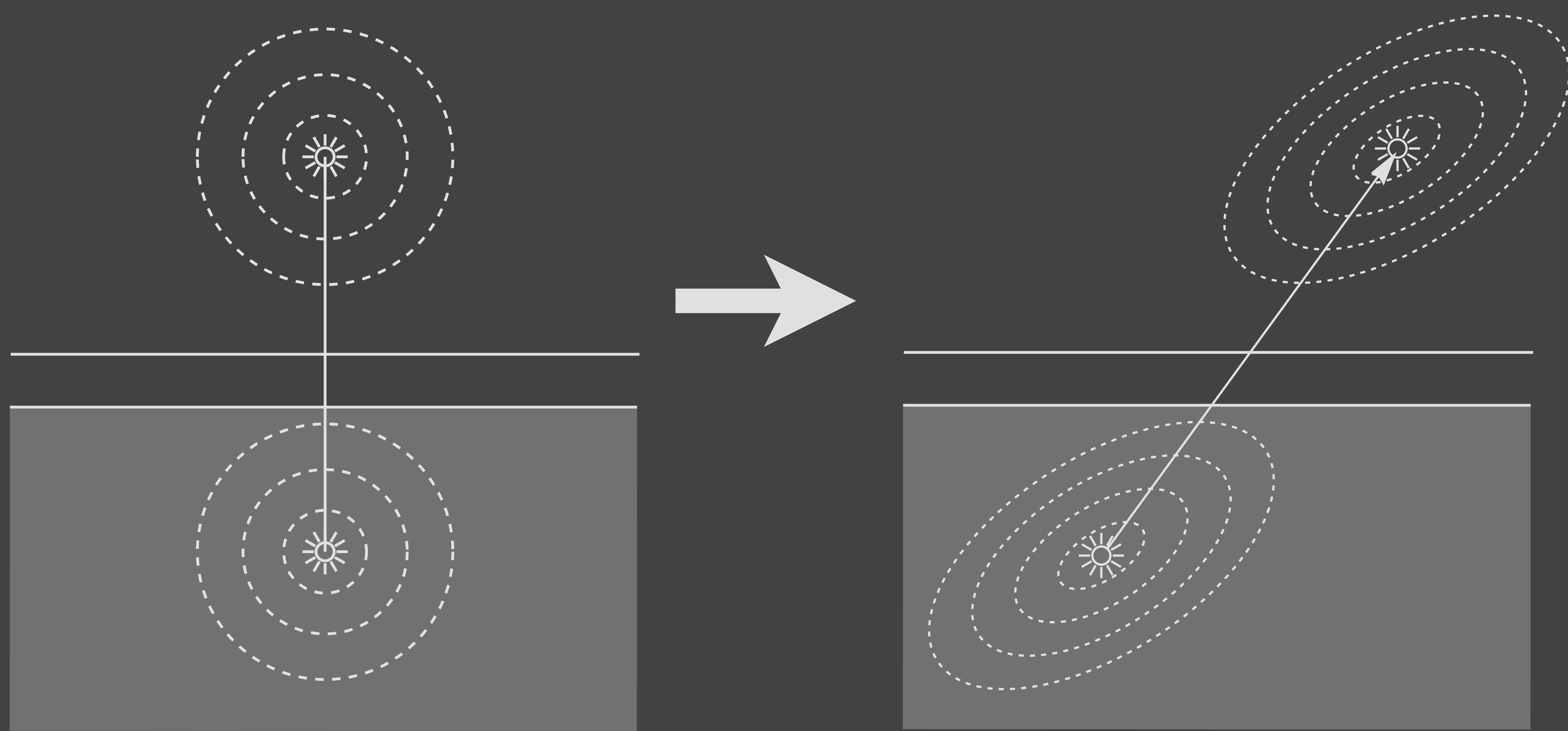
Changes

- Straightforward — turning some scalar multiplications into matrix-vector products

If already happen to have an isotropic FEM solver sitting around, as we did, then all you need to do is to turn some scalar multiplications into matrix-vector products. Otherwise, you've got some work to do.

Here is a simple test we made with a medium whose diffusion matrix is a function of Perlin noise. For that reason, the parallel beams projected onto the object diffuse along curved paths. Rendering an image using finite elements is quite fast and takes on the order of minutes.

Solution technique 3: Dipole



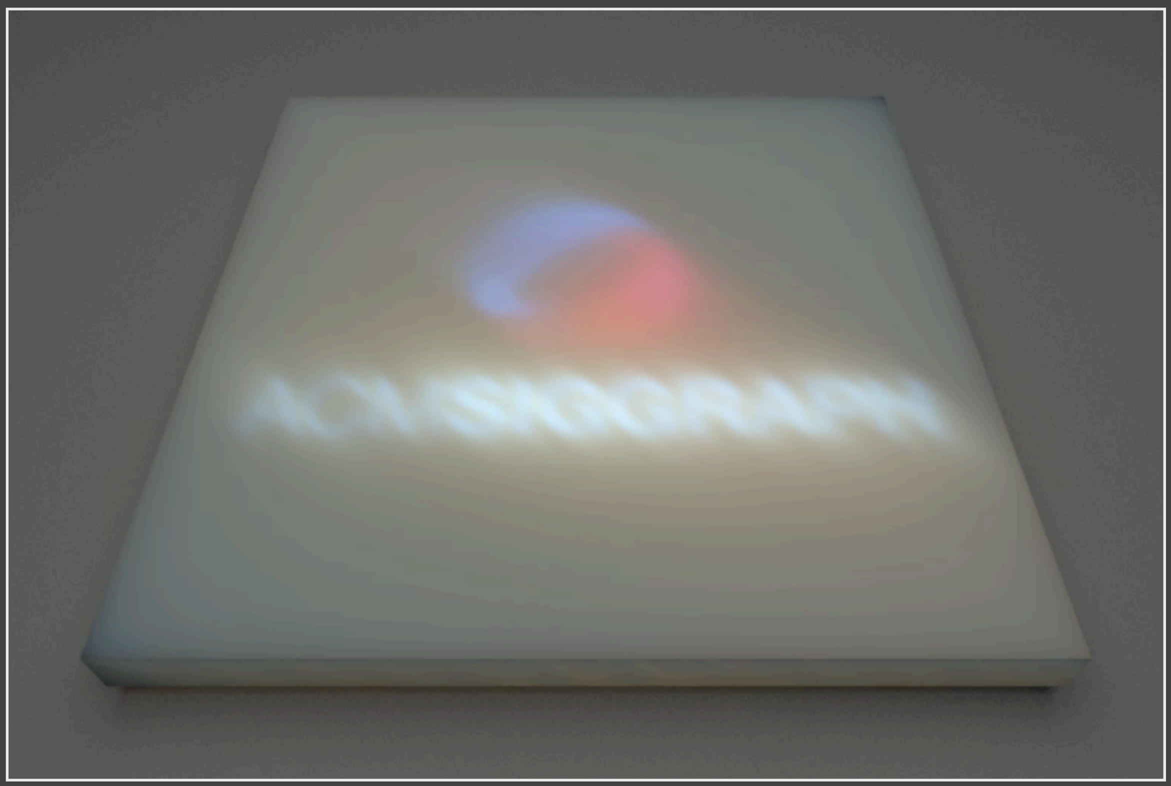
It's also possible to generalize the dipole solution approaches to anisotropic media.

The main change here is that the positions of the sources now have to be computed differently. Whereas in the isotropic case, the two sources line up perpendicularly to the surfaces, they are arranged in an increasingly tilted configuration as the volume becomes more and more anisotropic.

Render time (1 sample/pixel): 3.4 sec

isotropic

9.8 sec



anisotropic

Next, you’ll see a video, which shows a rotating colored pattern being projected onto two slabs, one isotropic and one anisotropic, where diffusion happens mainly along diagonal.

Render time (1 sample/pixel): 3.4 sec 9.8 sec

isotropic anisotropic

Of all 3 solution techniques, this by far the fastest one, and rendering an image takes just a few seconds.

Implications and future work

Contributions

- Principled foundation for work on complex materials
- **Theory**: Anisotropic RTE, Anisotropic diffusion equation
- **Model**: Micro-flake scattering model
- **Solution techniques**: Monte Carlo, FEM, Dipole

In conclusion, this paper provides end-to-end derivation of the changes required to support anisotropy in current volume rendering systems. First and foremost, we believe that this framework can provide a solid foundation for a principled and powerful new way of thinking about complex materials that couldn't be handled in the past.

The derivations led to two new equations, a modified radiative transfer equation and a generalized anisotropic form of the diffusion equation. To use these equations in practice, we proposed a new scattering model, and we showed how to then solve them using three different solution techniques.

Implications and future work

Contributions

- Principled foundation for work on complex materials
- **Theory**: Anisotropic RTE, Anisotropic diffusion equation
- **Model**: Micro-flake scattering model
- **Solution techniques**: Monte Carlo, FEM, Dipole

Future work

- Level of detail (LoD)
- Bidirectional rendering schemes
- Fitting of micro-flake distributions

There are several directions for future work we have in mind:

One is to use this framework to do volume level of detail correctly. The challenge is that volumes generally start to become increasingly anisotropic as you look at larger regions, even if you initially started out with something isotropic.

We would also like to investigate integration with bidirectional rendering schemes like Metropolis Light Transport.

And finally, another question is just how expressive the flake model is. As with microfacet models, it certainly cannot represent any kind of scattering, so it'll be interesting to explore the underlying limitations a bit more, and to use it to fit existing data.

Acknowledgements

Scarf dataset

- Jonathan Kaldor
- Manuel Vargas
- Manolis Savva

Funding

- NSF grants CCF-0347303 and CCF-0541105
- Unilever Corporation

A radiative transfer framework for rendering materials with anisotropic structure

http://www.cs.cornell.edu/projects/diffusion-sg10

http://www.mitsuba-renderer.org

We'd like to thank Jon, Manuel and Manolis for providing the scarf dataset
and also our funding agencies.

If you're interested in using the framework explained in this paper, we recommend taking a look at the supplementary technical report, which contains a wealth of additional material.

We've also released Mitsuba, the renderer used in this paper, which we invite you to download and play with. It can work with micro-flake volumes and it contains an implementation of the anisotropic dipole.