Seeing is Believing: A Client-Centric Specification of Database Isolation

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ABSTRACT
This paper introduces the first state-based formalization of isolation guarantees. Our approach is premised on a simple observation: applications view storage systems as black-boxes that transition through a series of states, a subset of which are observed by applications. Defining isolation guarantees in terms of these states frees definitions from implementation-specific assumptions. It makes immediately clear what anomalies, if any, applications can expect to observe, thus bridging the gap that exists today between how isolation guarantees are defined and how they are perceived. The clarity that results from definitions based on client-observable states brings forth several benefits. First, it allows us to easily compare the guarantees of distinct, but semantically close, isolation guarantees. We find that several well-known guarantees, previously thought to be distinct, are in fact equivalent, and that many previously incomparable flavors of snapshot isolation can be organized in a clean hierarchy. Second, freeing definitions from implementation-specific artefacts can suggest more efficient implementations of the same isolation guarantee. We show how a client-centric implementation of parallel snapshot isolation can be more resilient to slowdown cascades, a common phenomenon in large-scale datacenters.

ACM Reference format:

1 INTRODUCTION
Large-scale applications such as Facebook [1], or Twitter [56] offload the managing of data at scale to replicated and/or distributed systems. These systems, which often span multiple regions or continents, must sustain high-throughput, guarantee low-latency, and remain available across failures. To meet these demands, commercial databases or distributed storage systems like MySQL [45], Oracle [46], or SQL Server [42] often give up serializability [47]—(the gold-standard correctness criterion: an interleaved execution of transactions must be equivalent to a serial schedule) and instead privilege weaker but more scalable correctness criteria [2, 15, 34, 45, 49, 50, 53, 61] (referred to as weak isolation) such as snapshot isolation [15] or read committed [15]. In fact, to the best of our knowledge, almost all SQL databases use read committed as their default isolation level [39, 42, 45, 46, 50, 51], with some only supporting read-committed or snapshot isolation [45, 51].

This trend poses an additional burden on the application programmer, as these weaker isolation guarantees allow for counter-intuitive application behaviors: relaxing the ordering of operations yields better performance, but introduces schedules and anomalies that could not arise if transactions executed atomically and sequentially. These anomalies may affect application logic: consider a bank account with a $50 balance and no overdraft allowed. If the application runs under read-committed [15], the underlying database may allow two transactions to concurrently withdraw $45, incorrectly leaving the account with a negative balance [15].

To mitigate this increased programming complexity, many commercial databases and distributed storage systems [14, 28, 29, 39–41, 45, 46, 50] interact with applications through a front-end that gives applications the illusion of querying or writing to a logically centralized, failure-free node that will scale as much as one’s wallet will allow [28, 29, 39, 40, 46]. In practice, however, this abstraction is leaky: a careful understanding of the system that implements a given isolation level is oftentimes necessary to determine which anomalies the system will admit and how these will affect application correctness.

Indeed, the guarantees provided by isolation levels are often dependent on specific and occasionally implicit system properties—be it properties of storage (e.g., whether it is single or multiversioned [16]); of the chosen concurrency control (e.g., whether it is based on locking or timestamps [15]); or other system features (e.g., the existence of a centralized timestamp [27]).

Consider for example serializability [47]: the original ANSI SQL specification states that guaranteeing serializability is equivalent to preventing four phenomena [15]. This equivalence, however, only holds for lock-based, single version databases. Such implicit dependencies continue to have practical consequences: current multiversioned commercial databases that prevent these four phenomena, such as Oracle 12c, claim to implement serializability, when they in fact implement the weaker notion of snapshot isolation [11, 27, 46]. As such, they accept non-serializable schedules akin to the schedule in Figure 1(r), which exhibits an anomaly commonly referred to as write skew [15]. In contrast, a
majority reject the (serializable) schedule of Figure 1(l) because, for performance reasons, these systems choose not reorder writes.

We submit that the root of this complexity is a fundamental semantic gap between how application programmers experience isolation guarantees and how they are currently formally defined. From a programmer’s perspective, isolation guarantees are contracts between the storage systems and its clients, specifying the set of behaviors that clients can expect to observe—i.e., the set of admissible values that each read is allowed to return. When it comes to formally defining these guarantees, however, the current practice is to focus, rather than on the values that the clients can observe, on the mechanisms that can produce those values—i.e., histories capturing the relative ordering of low-level read and write operations.

Expressing isolation at such a low level of abstraction has significant drawbacks. First, it requires application programmers to reason about the ordering of operations that they cannot directly observe. Second, it makes it easy, as we have seen, to inadvertently contaminate what should be system-independent guarantees with system-specific assumptions. Third, by relying on operations that are only meaningful within one of the layers in the system’s stack, it makes it hard to reason end-to-end about the system’s guarantees.

To address these issues, we propose a new model that, for the first time, expresses isolation guarantees exclusively as properties of states that applications can observe, without relying on traditional notions—such as dependency graphs, histories, or version orders—that are instead invisible to applications. This new foundation comes at no cost in terms of generality or expressiveness: we offer below state-based and client-centric definitions of most modern isolation definitions, and prove that they are equivalent to their existing counterparts. It does, however, result in greater clarity, which yields significant benefits.

First, this model makes clear to developers what anomalies, if any, their applications can expect to observe, thus bridging the semantic gap between how isolation is experienced and how it is formalized. For example, we show (§5.1) how a state-based and client-centric definition brings immediately into focus the root cause of the write-skew anomaly, which distinguishes snapshot isolation from serializability.

Second, by removing the distorting effects of implementation artefacts, our approach makes it easy to compare the guarantees of distinct, but semantically close, isolation guarantees. The results are sometimes surprising. We prove (§5.2) that several well-known flavors of isolation in fact provide the same guarantees: parallel snapshot isolation (PSI) [20, 53] is equivalent to lazy consistency (PL-2+) [2, 3]; similarly, generalized snapshot isolation (GSI) [48] is actually equivalent to ANSI snapshot isolation (ANSI SI) [15], though GSI was proposed as a more scalable alternative to ANSI SI. Likewise, we also show that the lesser known strong session SI [24] and prefix-consistent SI [48] are also equivalent. Ultimately, the insights offered by state-based definitions enable us to organize in a clean hierarchy (§5.2) what used to be incomparable flavors of snapshot isolation [2, 9, 15, 24, 37, 48, 53].

Finally, by focusing on how clients perceive a given isolation guarantee, rather than on the mechanisms currently used to implement it, a state-based formalization can lead to a fresh, end-to-end perspective on how that guarantee should be implemented. Specifically, a state-based definition of parallel snapshot isolation (PSI) makes clear that the requirement of totally ordering transactions at each datacenter, which is baked into its current definition [53], is only an implementation artefact. Removing it offers the opportunity of an alternative implementation of PSI that makes it resilient to slowdown cascades [37], a common failure scenario in large-scale datacenters that has inhibited the adoption of stronger isolation models in industry [5].

After quickly reviewing in Section 2 the current approach to formalizing isolation guarantees, we introduce our state-based model in Section 3, and use it in Section 4 to define several isolation guarantees. We highlight the benefits of our approach in Section 5 and summarize related work in Section 6, before outlining our work’s limitations and concluding in Section 7.

## 2 BACKGROUND

Isolation guarantees have been formalized in many different ways: initial specifications of serializability [47] define correctness as a function of schedule equivalence, while weaker isolation guarantees have been defined using implementation-oriented operational specifications [13, 53] or by relating the order in which transactions commit with the values that they observe [20, 21, 52]. The most prevalent approach, however, has been to formulate isolation guarantees as dependency graphs, with edges denoting conflicts between transactions: this method was introduced by Bernstein et al. to formalize serializability for both single-version databases [16] and multiversioned databases [17], and subsequently refined by Adya to specify weak isolation guarantees [2]. Adya’s specification has since been adopted as the de-facto language for specifying isolation [25, 37, 54, 58]. We select Adya’s model as a baseline and prove our definitions equivalent to his in §4.

**Adya’s formalism** We summarize below some of the key definitions and results from Adya’s treatment of isolation [2]; a more complete description can be found in Appendix A.

Adya’s model is expressed in terms of histories, which consist of two parts: a partial order of events that reflect the operations of a set of transactions, and a version order that imposes a total order on committed object versions. Every history is associated with a directed serialization graph DSG(H) [17], whose nodes consist of committed transactions and whose edges mark the conflicts (read-write, write-write, or write-read) that occur between them. For specific isolation levels, Adya further augments the model with logical start and commit timestamps for transactions, leading
to start-ordered serialization graphs (SSG(H)) that add start-dependency edges to the nodes and edges of the corresponding DSG(H) (two transactions $T$, $T'$ are start-ordered if the commit timestamp of one precedes the start timestamp of the other).

An execution satisfies a given isolation level if it disallows aborted reads, intermediate reads, and circularity. The first two conditions prevent a transaction $T_1$ from reading, respectively, (i) values produced by an aborted transaction $T_2$ and (ii) a version of an object $x$ written by a transaction $T_2$ that $T_2$ subsequently overwrites. The third condition is more complex. Disallowing circularity prevents cycles in the serialization graph; the specific edges that compose the cycle, however, depend on the particular isolation level: read-uncommitted disallows cycles consisting only of write-read edges in the DSG(H) (referred to by Adya as phenomenon G0)\(^2\), while all remaining ANSI SQL isolation levels disallow cycles consisting of write-write/read edges (phenomenon G1). Serializability also disallows cycles that include read-write edges (G2). In contrast, snapshot isolation disallows write-write/read edges without corresponding start edges (G-SI(a)) as well as cycles containing a single read-write edge in the SSG(H) (G-SI(b)).

Towards a new formalism Adya’s formalism, like its other existing counterparts, specifies isolation guarantees as constraints on the ordering of the read and write operations that the storage system performs, and relies on low-level implementation details like timestamps or version order. Unfortunately, applications cannot directly observe this ordering: to them, the storage system is a black box. All they can observe are the values returned by the read operations they issue: they experience the storage system as if it were going through a sequence of atomic state transitions, of which they observe a subset. To make it easier for applications to reason about different levels of isolation, we adopt the viewpoint of the applications that must ultimately use their guarantees and introduce a new formalization of isolation based on application-observable states.

### 3 A STATE-BASED MODEL

To the best of our knowledge, our model is the first to specify isolation without relying on some notion of history. Instead, it associates with each transaction the set of candidate states (called read states) from which the transaction may have retrieved the values it read during its execution. Read states perform a role similar to Kripke structures [35]: they inform the application of the set of possible worlds (i.e., states) consistent with what a transaction observed during its execution.

Intuitively, a storage system guarantees a specific isolation level $I$ if it can produce an execution (a sequence of atomic state transitions) that satisfies two conditions. First, the execution must be consistent with the values observed by each transaction $T$; in our model, this requirement is expressed by associating every transaction $T$ with a set of read states, representing the states that the storage could have been in when the application executed $T$’s operations. Second, the execution must be valid, in that it must satisfy the constraints imposed by $I$; effectively narrows down which transactions’ read states can be used to build an acceptable execution. If no read state proves suitable for some transaction, then $I$ does not hold.

More formally, we define a storage system $S$ with respect to a set $K$ of keys and $V$ of values; a system state $s$ is a unique mapping from keys to values produced by writes from aborted or committed transactions. For simplicity, we assume that each value is uniquely identifiable, as is common practice both in existing formalisms [2, 17] and in practical systems (ETags in Azure [40] and S3 [7], timestamps in Cassandra [8]). There can thus be no ambiguity, when reading an object, as to which transaction wrote its content. In the initial system state, all keys have value $\bot$; later states similarly include every key, possibly mapped to $\bot$. As is common in database systems, we assume that applications modify the storage system’s state using transactions. A transaction $T$ is a tuple $(S_T, \rightarrow_T)$, where $S_T$ is the set of operations in $T$, and $\rightarrow_T$ is a total order on $S_T$. Operations can be either reads or writes. Read operation $r(k,v)$ retrieves value $v$ by reading key $k$; write operation $w(k,v)$ updates $k$ to its new value $v$. The read set of $T$ comprises the keys read by $T$: $\mathcal{R}_T = \{k | r(k,v) \in S_T\}$. Similarly, the write set of $T$ comprises the keys that $T$ updates: $\mathcal{W}_T = \{k | w(k,v) \in S_T\}$. For simplicity of exposition, we assume that a transaction only writes a key once. Finally, we assume the existence of a time oracle $\mathcal{T}$ that assigns distinct real-time start and commit timestamps $(T.start$ and $T.commit$) to every transaction $T \in T$. A transaction $T_1$ time-precedes $T_2$ (we write $T_1 <_T T_2$) if $T_1.start < T_2.start$. Applying a transaction $T$ to a state $s$ transitions the system to a state $s'$ that is identical to $s$ in every key except those written by $T$. Formally,

**Definition** 1 $s \xrightarrow{T} s' \iff \left\{ \begin{array}{ll} & (k,v) \in s' \land (k,v) \notin s \Rightarrow k \in \mathcal{W}_T \land w(k,v) \in S_T \Rightarrow (k,v) \in s' \end{array} \right\}$.

We refer to $s$ as the parent state of $T$ (denoted as $s_p(T)$)\(^3\); to the transaction that generated $s$ as $T_s$; and to the set of keys in which $s$ and $s'$ differ as $\Delta(s,s')$. An execution $e$ for a set of transactions $T$ is a totally ordered set defined by the pair $(S_e, \rightarrow_T)$, where $S_e$ is the set of states generated by applying, starting from the system’s initial state, a permutation of all the transactions in $T$. We write $s \xrightarrow{e} s'$ (respectively, $s \xrightarrow{e} s''$) to denote a sequence of zero (respectively, one) or more state transitions from $s$ to $s'$ in $e$. For example, in Figure 2, $T$ comprises five transactions, operating on a state that consists of the current version of keys $x$, $y$, and $z$.

Note that while $e$ identifies the state transitions produced by each transaction $T \in T$, it does not specify from which states in $S_e$ each operation in $T$ reads. In particular, reading a key in replicated distributed systems will not necessarily return the value produced by the latest write to that key, as writes may become visible in different orders at different replicas. In general, multiple states in

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\(^2\)We will use Adya’s shorthand for this and other phenomena in §4, when we prove that our new state-based definitions of isolation guarantees are equivalent to his.

\(^3\)Henceforth, we will drop the subscripted $T$ unless there is ambiguity.
SLs may be compatible with the values returned by any given operation. We call this subset the operation’s read states. To prevent operations from reading from the future, we restrict the valid read states for the operations in \( T \) to be no later than \( s_p \). Further, once an operation in \( T \) writes \( v \) to \( k \), we require all subsequent operations in \( T \) that read \( k \) to return \( v \) [2]: in this case, their set of read states by convention includes all states in \( S_L \) up to and including \( s_p \).

**Definition 2** Given an execution \( e \) for a set of transactions \( T \), let \( T \in T \) and let \( s_p \) denote \( T \)'s parent state. The read states for a read operation \( o \) in \( T \in T \) define the set of states

\[
\mathcal{RS}_e(o) = \{ s \in S_L | s \rightarrow s_p \land \exists (k,v) \in \mathcal{R}_T : (k,v) \in \mathcal{R}_e \}
\]

Figure 2 illustrates the notion of read states for the operations executed by transaction \( T_b \). Since \( r_2 \) returns \( y_1 \), its only possible read state is \( s_2 \), i.e., the only state containing \( y_1 \). When it comes to \( r_3 \), however, \( z_0 \) could have been read from any of \( s_0, s_1, \) or \( s_2 \): from the perspective of the client executing \( T_b \), these read states are indistinguishable. By convention, write operations have read states too: for a write operation in \( T \), they include all states in \( S_L \) up to and including \( T \)'s parent state. It is easy to prove that the read states of any operation \( o \) define a sequence of contiguous states in the total order that \( e \) defines on \( S_L \). We refer to the first state in that sequence as \( s_f \) and to the last state as \( s_l \). For instance, in Figure 2, \( s_f \) is \( z_0 \) (the first state that contains \( z_0 \)) and \( s_l \) is \( s_2 \) (\( z_0 \) is overwritten in \( s_3 \)). When the predicate \( \text{PREREAD}_e(T) \) holds, then such states exist for all transactions in \( T \):

**Definition 3** Let \( \text{PREREAD}_e(T) \equiv \forall o \in \mathcal{R}_T : \mathcal{RS}_e(o) \neq \emptyset \) Then \( \text{PREREAD}_e(T) \equiv \forall T \in T : \text{PREREAD}_T \).

We say that a state \( s \) is complete for \( T \) in \( e \) if every operation in \( T \) can read from \( s \). We write:

**Definition 4** \( \text{COMPLETE}_e(T)(s) \equiv s \in \bigcap_{o \in \mathcal{R}_T} \mathcal{RS}_e(o) \).

Looking again at Figure 2, \( s_2 \) is a complete state for transaction \( T_b \), as it is in the set of candidate read states of both \( r_2(y_1) \) (\( s_2 \)) and \( r_3(z_0) \) (\( \{s_0, s_1, s_2\} \)). A complete state is not guaranteed to exist: no such state exists for \( T_r \), as the sole candidate read states of \( r_4 \) and \( r_5 \) (respectively, \( s_0 \) and \( s_3 \)) are distinct. As we will see in §4, complete states are key to determining whether read states from a consistent snapshot.

4 ISOLATION

Isolation guarantees specify the valid set of executions for a given set of transactions \( T \). We show that it is possible to formalize these guarantees solely in terms of each transaction’s read and commit states, without relying on histories of low-level operations or on implementation details such as timestamps. The underlying reason is simple: ultimately, it is through the visible states produced during an execution that the storage system can prove to its users that a given isolation guarantee holds. Histories are just the mechanism that generates those probationary states; indeed, multiple histories can map to the same execution.

In a state-based model, isolation guarantees constrain each transaction \( T \in T \) in two ways. First, they limit which states, among those in the candidate read sets of the operations in \( T \), are admissible. Second, they restrict which states can serve as parent states for \( T \). We express these constraints by means of a commit test: for an execution \( e \) of a set \( T \) of transactions to be valid under a given isolation level \( I \), each transaction \( T \) in \( e \) must satisfy the commit test \( CT_I(T, e) \) for \( I \).

**Definition 5** Given a set of transactions \( T \) and their read states, a storage system satisfies an isolation level \( I \) iff \( \exists e: \forall T \in T : CT_I(T, e) \).

Table 1 summarizes the commit tests that define the isolation guarantees most commonly used in research and industry: the ANSI SQL isolation levels (serializability, read committed, read uncommitted, and snapshot isolation) as well as parallel snapshot isolation [20, 53], strict serializability [47], and the recently proposed read atomic [13] isolation level. Though our state-based definitions make no reference to histories, we prove that they are equivalent to those in Adya’s classic treatment. As the proofs follow a similar structure, we provide an informal proof sketch only for serializability and snapshot isolation, deferring a more complete and formal treatment to Appendices A, B and E.

**Serializability** Serializability requires the values observed by the operations in each transaction \( T \) to be consistent with those that would have been observed in a sequential execution. The commit test enforces this requirement through two complementary conditions on observable states. First, all of \( T \)'s operations must read from the same state \( s \) (i.e., \( s \) must be a complete state for \( T \)). Second, \( s \) must be the parent state of \( T \), i.e., the state that \( T \) transitions from. These two conditions suffice to guarantee that \( T \) will observe the effects of all transactions that committed before it. This definition is equivalent to Adya’s cycle-based definition. Specifically, we prove that (a more formal proof can be found in Appendix A.2):

**Theorem 1** \( \exists e: \forall T \in T : CT_{SER}(T, e) \equiv \neg G_1 \land \neg G_2 \).

**Proof sketch.** \( \exists e: \forall T \in T : CT_{SER}(T, e) \Rightarrow \neg G_1 \land \neg G_2 \). By definition, \( e \) is a totally-ordered execution where the parent state of every transaction \( T \) is a complete state for \( T \). Considering the order of transactions in \( e \), we make three observations. First, all write-write edges in the DSG point in the same direction, as they map to state transitions in the totally-ordered execution \( e \). Second, all write-read edges point in the same direction as write-write edges: given any transaction \( T \), since all operations in \( T \) read from \( T \)'s parent state, all write-read edges that end in \( T \) must originate from a transaction that precedes \( T \) in \( e \)'s total order. Finally, all read-write dependency edges point in the same direction as write-write and write-read edges: as all read operations in \( T \) read from \( T \)'s parent state, the value they return cannot be later overwritten by a transaction \( T' \) ordered before \( T \) in \( e \). Since all edges point in the same direction, no cycle can form in the DSG.

\( \exists e: \forall T \in T : CT_{SER}(T, e) :\neg (G_1 \land \neg G_2) \). If no cycle exists in the DSG, we can construct an execution \( e' \) such that the parent state \( s_p \) of each transaction \( T \) is a complete state for \( T \). We construct \( e' \) by topologically sorting the DSG (it is acyclic) and by applying every transaction in the resulting order. Thus, if a transaction \( T' \) writes a value that \( T \) subsequently reads (write-read edge), the state associated with \( T' \) is guaranteed to precede \( T \)'s state in the execution \( e' \). Moreover, as there are no backpointing read-write edges, no other
transaction in \( e' \) will update an object read by \( T \) between the state produced by \( T' \) and \( s_p \). \( s_p \) is therefore a valid read state for every operation in \( T \) and, consequently, a complete state for \( T \). \( \square \)

**Snapshot isolation (SI)** Like serializability, SI prevents transaction \( T \) from seeing the effects of concurrently running transactions. The commit test enforces this requirement by having all operations in \( T \) read from the same state, produced by a transaction that precedes \( T \) in the execution \( e \). However, SI no longer insists on that state \( s \) being \( T \)'s parent state \( s_p \); other transactions, whose operations \( T \) will not observe, may commit in between \( s \) and \( s_p \). The commit test only forbids \( T \) from modifying any of the keys that changed value as the system's state progressed from \( s \) to \( s_p \). Denoting the set of keys in which \( s \) and \( s' \) differ as \( \Delta(s, s') \), we express this as \( NO-CONF(s) \equiv \Delta(s, s') \land \forall w_T \neq 0 \). We prove that this definition is equivalent to Adya’s (a more formal proof can be found Appendix A.3):

**Theorem 2** \( \forall e' : e' \in T : CT_{SI}(T, e) \equiv \neg G1 \land \neg G-SI \).

**Proof sketch.** (\( \forall e' : e' \in T : CT_{SI}(T, e) \equiv \neg G1 \land \neg G-SI \)). We construct a valid execution for any history satisfying \( \neg G1 \land \neg G-SI \) using the time-precedes partial order introduced by Adya. First, we topologically sort transactions according to their commit point, then, and apply them in the resulting order to generate an execution \( e \). Next, we prove that every transaction \( T \) satisfies the commit test: we first show that the state created by the last transaction \( T_{rs} \) on which \( T \) start-depends is a complete state. As \( T \) start-depends on \( T_{rs} \), it must also start-depend on all transactions that precede \( T_{rs} \), since, by construction, these transactions have a commit timestamp smaller than \( T_{rs} \). Moreover, as \( T_{rs} \) is the last transaction that \( T \) starts-depend on, all subsequent transactions will either be concurrent with \( T \), or start-depend on \( T \). Adya’s (\( \neg G-SI \)) requirement (formally, that there can be neither a write-read /write-write edge without also a start dependency edge nor a cycle including a single read-write edge) implies that \( T \) can only read or overwrite a value written by a transaction \( T' \) if \( T \) start-depends on \( T' \). Any such \( T' \) must either be \( T_{rs} \) or precede \( T_{rs} \) in \( e \). Similarly, if another transaction \( T'' \) overwrites a value that \( T \) reads, \( T \) cannot start-depend on \( T'' \) as it would otherwise create a cycle with a start-edge and a single read-write edge. \( T'' \) is therefore ordered after \( T_{rs} \) in \( e \). We conclude that \( s_{rs} \) necessarily contains all the values that \( T \) reads: it is a complete state. Next, we show that \( \Delta(s_{rs}, s_T) = 0 \). By construction, \( T \) cannot start-depend on any transaction \( T' \) that follows \( T_{rs} \) in the execution but precedes \( T \). By G-SI, there cannot be a write-write dependency edge from \( T' \) to \( T \), and their write-sets must therefore be distinct. Consequently: \( \Delta(s_{rs}, s_T) = 0 \).

\( (\exists e' : e' \in T : CT_{SI}(T, e) \equiv \neg G1 \land \neg G-SI) \). We show that the serial-ization graph \( SSG(H) \) corresponding to \( e \) does not exhibit phenomena \( G1 \) or \( G-SI \). Every transaction in \( e \) reads from some previous state and commits in the total order defined by \( e \). It follows that all write-write and write-read edges follow the total order introduced by \( e \): there can be no cycle consisting of write-write/ write-read dependencies. \( \neg G1 \) is thus satisfied. To show that \( SSG(H) \) does not exhibit G-SI, we first select the start and commit point of each transaction. We assign commit points to transactions according to their order in \( e \). We assign the start point of each transaction \( T \) to be directly after the commit point of the first transaction \( T_{rs} \) in \( e \) whose generated state satisfies \( COMPLETE_T(s_p) \land (\forall w_T \neq 0) \). It follows that \( T_{rs} \) (and all the transactions that precede it in \( e \)) start-precede \( T \). Proving \( \neg G-SI \) is then straightforward: any transaction \( T' \) that \( T \) write-read/write-write depends on precedes \( T_{rs} \) in the execution and, consequently start-precedes \( T \). Proving \( \neg G-SI \) for \( G-SI \) requires a little more care. By \( \neg G-SI \), there necessarily exists a corresponding start-depend edge for any write-read or write-write edge between two transactions \( T \) and \( T' \); if there exists a cycle with exactly one read-write edge in the state \( SSG(H) \), there must exist a cycle with exactly one read-write edge and only start-depend edges in \( SSG(H) \). Assuming by contradiction that \( G-SI \) holds and that there exists a cycle with one read-write edge and multiple start-depend edges (we reduce this cycle to a single start-depend edge as start-edges are transitive). Let \( T \) read-write depend on \( T' \); \( s_T \) is ordered after \( s_{rs} \) in \( e \) (otherwise \( s_{rs} \) cannot be a valid read state for \( T \)). However, as previously mentioned, \( T \) only has start-depend edges with transactions that precede \( T_{rs} \) (included) in \( e \). \( T' \) thus does not start-depend on \( T \), a contradiction. \( \square \)

Unlike Adya’s, however, the correctness of our state-based definition does not rely on using start and commit timestamps. This is a crucial difference. Including these low-level attributes in the definition has encouraged the development of variations of SI that differ in their use of timestamps, whose fundamental guarantees are, as a result, difficult to compare. In §5.2 we show that, when expressed in terms of application-observable states, several of these variations, thought to be distinct, are actually equivalent!

**Read committed** Read committed allows \( T \) to see the effects of concurrent transactions, as long as they are committed. The commit test therefore no longer constrains all operations in \( T \) to read from the same state; instead, it only requires each of them to read from a state that precedes \( T \) in the execution \( e \). We prove in Appendix A.4 that:

**Theorem 3** \( \exists e' : e' \in T : CT_{GC}(T, e) \equiv G1 \).

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4 As proofs follow a similar structure, we defer all subsequent proofs to Appendices.

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</tr>
</tbody>
</table>
**Read uncommitted** Read uncommitted allows \( T \) to see the effects of concurrent transactions, whether they have committed or not. The commit test reflects this permissiveness, to the point of allowing transactions to read arbitrary values. Still, we prove in Appendix A.5 that:

**Theorem 4** \( \exists e : \forall T \in \mathcal{T}: C_{RU}(T,e) \equiv \neg G_{0} \).

The reason behind the laxity of the state-based definition is that isolation models in databases consider only committed transactions and are therefore unable to distinguish values produced by aborted transactions from those produced by future transactions. This distinction, however, is not lost in environments, such as transactional memory, where correctness depends on providing guarantees such as opacity [30] for all live transactions. We discuss this further in Section 7.

**Strict Serializability** Strict serializability guarantees that the real-time order of transactions will be reflected in the final history or execution. It can be expressed by adding the following condition to the serializability commit test: \( \forall T' \in \mathcal{T} : T' \prec s \Rightarrow sy \rightarrow sy' \).

**Parallel Snapshot Isolation** Parallel snapshot isolation (PSI) was recently proposed by Sovran et al. [53] to address SI’s scalability issues in geo-replicated settings. Snapshot isolation requires transactions to read from a snapshot (a complete state in our parlance) that reflects a single commit ordering of transactions. The coordination implied by this requirement is expensive to carry out in a geo-replicated system and must be enforced even when transactions do not conflict. PSI aims to offer a scalable alternative by allowing distinct geo-replicated sites to commit transactions in different orders. The specification of PSI is given as an abstract specification code that an implementation must emulate. Specifically, a PSI execution must enforce three properties. First, *site snapshot read*: all operations read the most recent committed version at the transaction’s origin site as of the time when the transaction began (P1). Second, *no write-write conflicts*: the write sets of each pair of somewhere-concurrent committed transactions must be disjoint (two transactions are somewhere-concurrent if they are concurrent on site\( (T_1) \) or site\( (T_2) \)) (P2). And finally, *commit causality across sites*: if transaction \( T_1 \) commits at a site A before transaction \( T_2 \) starts at site A, then \( T_1 \) cannot commit after \( T_2 \) at any site.

Our first step towards a state-based definition of PSI is to populate, using solely client-observable states, the predecessor set of each transaction \( T \), i.e., the set of transactions after which \( T \) must be ordered. A transaction \( T' \) is in \( T \)'s predecessor set if (i) \( T \) reads a value that \( T' \) wrote; or (ii) \( T \) writes an object modified by \( T' \) and the execution orders \( T' \) before \( T \); or (iii) \( T' \) precedes \( T'' \) and \( T'' \) precedes \( T \). Formally:

\[
D_{\text{prec},e}(T) = \{ T \circ e \in \Sigma_T : T = T_f r_{o} \} \cup \{ T \circ \Sigma_T \rightarrow s_{e} \wedge W_{T_f} \cap W_{T_f} \neq \emptyset \}.
\]

We write \( T_i \rightarrow T_j \) if \( T_i \in D_{\text{prec},e}(T_j) \) and \( T_i \rightarrow T_j \) if \( T_i \) transitively precedes \( T_j \). PSI guarantees that the state observed by a transaction \( T \)'s operation includes the effects of all transactions that precede it. We can express this requirement in PSI’s commit test as follows:

**Definition 6** \( C_{PSI}(T,e) \equiv \text{PRE_READ}(T) \wedge \forall T' : \forall o \in \Sigma_T : o,k \in \text{W}_T \rightarrow sy \rightarrow s_{f_T} \).

---

**Figure 3: Simple Banking Application. Alice and Bob share checking and savings accounts. Withdrawals are allowed as long as the sum of both account is greater than zero.**

This client-centric definition of PSI makes immediately clear that the state which operations observe is not necessarily a complete state, and hence may not correspond to a snapshot of the database at a specific time. We prove the following theorem in Appendix E.3:

**Theorem 5** \( \exists e : \forall T \in \mathcal{T} : C_{PSI}(T,e) \equiv \text{PSI} \).

**Read Atomic** Read atomic [13], like PSI, aims to be a scalable alternative to snapshot isolation. It preserves atomic visibility (transactions observe either all or none of a committed transaction’s effects) but does not preclude write-write conflicts nor guarantees that transactions will read from a causally consistent prefix of the execution. These weaker guarantees allow for efficient implementations and nonetheless ensure *synchronization independence*: one client’s transactions cannot cause another client’s transactions to fail or stall. Read atomic can be expressed in our state-based model as follows:

**Definition 7** \( C_{RA}(T,e) \equiv \forall T_1(k_1, c_1), r_2(k_2, c_2) \in \Sigma_T \land k_2 \in W_{T_f r_{1}} \Rightarrow s_{f_r_{1}} \rightarrow s_{f_r_{2}} \).

Intuitively, if an operation \( o_1 \) observes the writes of a transaction \( T_i \)'s, all subsequent operations that read a key included in \( T_i \)'s write set must read from a state that includes \( T_i \)'s effects. We prove the following theorem in Appendix B:

**Theorem 6** \( \exists e : \forall T \in \mathcal{T} : C_{RA}(T,e) \equiv \text{RA} \).

---

**5 BENEFITS OF A STATE-BASED APPROACH**

Specifying isolation using client-observable states rather than histories is not only equally expressive, but brings forth several benefits: it gives application developers a clearer intuition for the implications of choosing a given isolation level (§ 5.1), brings additional clarity to how different isolation levels relate (§ 5.2), and opens up opportunities for performance improvements in existing implementations (§ 5.3).

**5.1 Minimizing the intuition gap**

A state-based model makes it easier for application programmers to understand the anomalies allowed by weak isolation levels, as it precisely captures the *root cause* of these anomalies. Consider,
for example, snapshot isolation: it allows for a non-serializable behavior called write-skew (see §1), illustrated in the simple banking example of Figure 3, Alice and Bob share checking (C) and savings (S) accounts, each holding $30, the sum of which should never be negative. Before performing a withdrawal, they check that the total funds in their accounts allow for it. They then withdraw the amount from the specified account, using the other account to cover any overdraft. Suppose Alice and Bob try concurrently to withdraw $40 from, respectively, their checking and savings account, and issue transactions T_1 and T_2. Figure 3(a) shows an execution under serializability. Because transactions read from their parent state (see Table 1), T_2 observes T_1’s withdrawal and, since the balance of Bob’s accounts is below $40, aborts. In contrast, consider the execution under snapshot isolation in Figure 3(b). As it is legal for both T_1 and T_2 to read from a complete but stale state s_1, Alice and Bob can both find that the combined funds in the two accounts exceed $40, and, unaware of each other, proceed to generate an execution whose final state s_3 is illegal. The state-based definitions of snapshot isolation and serializability make both the causes and the danger of write-skew immediately clear: to satisfy snapshot isolation, it suffices that both transactions read from the same complete state s_1, even though this behavior is clearly not serializable, as s_1 is not the parent state of T_2. The link is, arguably, less obvious with the history-based definition of snapshot isolation, which requires “disallowing all cycles consisting of write-write and write-read dependencies and a single anti-dependency”.

5.2 Removing implementation artefacts

By cleanly separating high-level properties from low-level implementation details, a state-based model makes the plethora of isolation guarantees introduced in recent years easier to compare. We leverage this newfound clarity below to systematize snapshot-based guarantees, including ANSI SI [15], Adya SI [2], Weak SI [24], Strong SI [24], parallel snapshot isolation (PSI) [53], Strong Session SI [24], PC-SI [24], PL-2+ (Lazy Consistency) [3], Prefix-Consistent SI (PC-SI) [24]. We find that several of these isolation guarantees, previously thought to be distinct, are in fact equivalent from a client’s perspective, and establish a clean hierarchy that encompasses them.

Snapshot-based isolation guarantees, broadly speaking, are defined as follows. A transaction is assigned both a start and a commit timestamp; the first determines the database snapshot from which the transaction can read (it includes all transactions with a smaller commit timestamp), while the second maintains the “first-committer-wins” rule: no conflicting transactions should write to the same objects. The details of these protocols, however, differ. Each strikes a different performance trade-off in how it assigns timestamps and computes snapshots that influences its high-level guarantee in ways that can only be understood by applications with in-depth knowledge of the internals of the underlying systems. As a result, it is hard for application developers and researchers alike to compare and contrast them.

In contrast, formulating isolation in terms of client-observable states forces definitions that specify guarantees according to how they are perceived by clients. It then becomes straightforward to understand what guarantees are provided, and to observe their differences and similarities. Specifically, it clearly exposes the three dimensions along which snapshot-based guarantees differ: (i) time (whether timestamps are logical [2, 37, 53] or based on real-time [15, 24, 48]); (ii) snapshot recency (whether the computed snapshot contains all transactions that committed before the transaction start time [2, 24] or can be stale [9, 24, 48, 53]); and state completeness (in our parlance, whether snapshots must correspond to a complete state [2, 15, 24, 48] or whether a causally consistent [4] snapshot suffices [3, 9, 37, 53]).

Grouping isolation guarantees in this way highlights a clean hierarchy between these definitions, and suggests that many of the newly proposed isolation levels proposed are in fact equivalent to prior guarantees. We summarize the different commit tests in Table 2 and the resulting hierarchy in Figure 4. As the existence of the hierarchy follows straightforwardly from the commit tests, we defer the proof of its soundness to Appendix F, along with proofs of the corresponding equivalences.

At the top of the hierarchy is Strong SI [24]. It requires that a transaction T observe the effects of all transactions that have committed (in real-time) before T (in other words, read from the most recent database snapshot) and obtain a commit timestamp greater than any previously committed transaction. We express this (Table 2, first row) by requiring that the last state in the execution generated by a transaction that happens before T in real time must be complete (\( \forall T' <_c T : s_{T'} \rightarrow s \)), and that the total order defined by the execution respects commit order (\( \text{commit}(T) \rightarrow \text{commit}(T') \)). We prove that this formulation is equivalent to its traditional implementation specification in Appendix C.4:

**Theorem 7** \( \exists e : \forall T : CT_{\text{Strong SI}}(T,e) \equiv \text{Strong SI} \).

Skipping for a moment one level in the hierarchy, we consider next ANSI SI [15]. ANSI SI weakens Strong SI’s requirement that the snapshot from which T reads include all transactions that precede T in real-time (including those that access objects that T does not access). This weakening, which improves scalability by avoiding the coordination needed to generate Strong SI’s snapshot, can be expressed in our state-based approach by relaxing the requirement that the complete state be the most recent in real-time (Table 2, third line). An attractive consequence of this new formulation is that it clarifies the relationship between ANSI SI and generalized snapshot isolation [48], a refinement of ANSI SI for lazily replicated databases. We prove that these two decade-old guarantees are actually equivalent in Appendices C.2 and D.2:

**Theorem 8** \( \exists e : \forall T : CT_{\text{ANSI SI}}(T,e) \equiv \text{GSI} \equiv \text{ANSI SI} \).
ANSI SI and Strong SI: they introduce the notion of C-ORD on transactions (than Strong SI, their definitions still include several red flags for PSI requires that transactions read from a committed snapshot moment in time (and hence read from a database snapshot that corresponds to a single transaction reading a row), allowing for maximum flexibility for how snapshot isolation can be implemented without affecting client-side guarantees. Our model can capture this distinction by simply removing the timestamps consistent with the transactions’ observations. Logical conclusions by removing the dependence on real-time, instead allocating timestamps consistent with the transactions’ observations. In its original specification, the definition of parallel snapshot isolation [53] requires datacenters to enforce snapshot isolation, even as it globally only offers (as we prove in Theorem 10) the guarantees of lazy consistency/PL-2+. This baked-in implementation decision makes the very definition of PSI unsuitable for large-scale partitioned datacenters as it makes the definition (and therefore any system that implements it) susceptible to slowdown cascades. Sloowdown cascades (common in large-scale systems) arise when a slow or failed node/partition delays operations that do not access that node, and have been identified by industry [5] as the primary barrier to adoption of stronger consistency guarantees. By enforcing SI on every site, the history-based definition of PSI creates a total commit order across all transactions within a datacenter, even as they may access different keys. Transactions thus become dependent on all previously committed transactions on that datacenter, and cannot be replicated to other sites until all these transactions have been applied. If a single partition is slow, all transactions that artificially depend on transactions on that node will be unnecessarily delayed, creating a cascading slowdown.

| Strong SI (CT_{Strong SI}(T,e)) | C-ORD(T_{p}, T) \land \exists s \in S_{p} \cdot COMPLETE_{e}(T) \land \NO-CONF_{T}(T) \land (T_{s} < T) \land (VT \sqsubseteq T : sT_{p} \rightarrow sT) |
|---------------------------------------------------------------|
| Strong Session SI/PC-SI (CT_{Session SI}(T,e)) | C-ORD(T_{p}, T) \land \exists s \in S_{p} \cdot COMPLETE_{e}(T) \land \NO-CONF_{T}(T) \land (T_{s} < T) \land (VT \sqsubseteq T : sT_{p} \rightarrow sT) |
| ANSI SI (GSI (CT_{ANSI SI}(T,e))) | C-ORD(T_{p}, T) \land \exists s \in S_{p} \cdot COMPLETE_{e}(T) \land \NO-CONF_{T}(T) \land (T_{s} < T) |
| Adya SI (CT_{SI}(T,e)) | \exists s \in S_{e} \cdot COMPLETE_{e}(T) \land \NO-CONF_{T}(T) |
| PSI/PL-2+ (CT_{PSI}(T,e)) | \text{PREREAD}_{e}(T \land \forall T' \rightarrow T ; \text{CAUS-VIS}(e,T)) |

Table 2: Commit tests for snapshot-based protocols

5.3 Identifying performance opportunities

Beyond improving clients’ understanding, defining isolation guarantees in terms of client-observable states helps prevent them from subjecting transactions to stronger requirements than what these guarantees require end-to-end. Indeed, by removing all implementation-specific details (timestamps, replicas) present in system-centric formulations, our model gives full flexibility to how these guarantees can be implemented. We illustrate this danger, and highlight the benefits of our approach, using the specific example of PSI/PL-2+.
An approach based on client-observable states, in contrast, makes no such assumptions: the depend-set of a transaction is computed using client observations and read states only, and thus consists exclusively of transactions that the application itself can perceive as ordered with respect to one another. Every dependency created stems from an actual observation: the number of dependencies that a client-centric definition creates is consequently minimal (and the fewer dependencies a system creates, the less likely it will be subject to slowdown cascades). To illustrate this potential benefit, we simulated the number of transactional dependencies created at each datacenter by the traditional definition of PSI as compared to the “true” dependencies generated by the proposed client-centric definition, using an asynchronously replicated transactional key-value store, TARDiS [23]. On a workload consisting of read-write transactions (three reads, three writes) accessing data uniformly over 10,000 objects (Figure 5), we found that a client-centric approach decreased dependencies, per transaction, by two orders of magnitude (175x), a reduction that can yield significant dividends in terms of scalability and robustness.

State-based specifications of isolation guarantees can also benefit performance, as they abstract away the details of specific mechanisms used to enforce isolation, and instead focus on how different flavors of isolation constrain permissible read states. A case-in-point is Ardekani et al.’s non-monotonic snapshot isolation (NMSI) [9]: NMSI logically moves snapshots forward in time to minimize the risk of seeing stale data (and consequent aborts due to write-write conflicts), without violating any consistency guarantees. This technique is premised on the observation that, given the values read by the client, the states at the earlier and later snapshot are indistinguishable.

6 RELATED WORK

Most past definitions of isolation and consistency [2, 9, 15–19, 27, 31, 47, 53, 55] refer to specific orderings of low-level operations and to system properties that cannot be easily observed or understood by applications. To better align these definitions with what clients perceive, recent work [10, 20, 36, 57] distinguishes between concrete executions (the nuts-and-bolts implementations details) and abstract executions (the system behaviour as perceived by the client). Attiya et al., for instance, introduce the notion of observable causal consistency [10], a refinement of causal consistency where causality can be inferred by client observations. Likewise, Cerone et al. [20, 21] introduce the dual notions of visibility and arbitration to define, axiomatically, a large number of existing isolation levels. The simplicity of their formulation, however, relies on restricting their model to consider only isolation levels that guarantee atomic visibility [13], which prevents them from expressing guarantees like read-committed, the default isolation level of most common database systems [39, 42, 45, 46, 50, 51, 58], and the only supported level for some [45]. Shapiro and Ardekani [52] adopt a similar approach to identify three orthogonal dimensions (total order, visibility, and transaction composition) that they use to classify consistency and isolation guarantees. All continue, however, to characterize correctness by constraining the ordering of read and write operations and often let system specific details (e.g., system replicas) leak through definitions. Our model takes their approach a step further: it directly defines consistency and isolation in terms of the observable states that are routinely used by developers to express application invariants [6, 12, 23]. Finally, several practical systems have recognized the benefits of taking a client-centric approach to system specification and development. These systems target very different concerns, from file I/O [44] to cloud storage [38], and from Byzantine fault-tolerance [33] to efficient Paxos implementations [49]. In the specific context of databases and key-value stores, in addition to Ardekani et al.’s work [9], Mehdi et al. [37] recently proposed a client-centric implementation of causal consistency that is both scalable and resilient to slowdown cascades (§3.3).

7 CONCLUSION

We present a new way to reason about isolation based on application-observable states and prove it to be as expressive as prior approaches based on histories. We present evidence suggesting that this approach (i) maps more naturally to what applications can observe and illuminates the anomalies allowed by distinct isolation/consistency levels; (ii) makes it easy to compare isolation guarantees, leading us to prove that distinct, decade-old guarantees are in fact equivalent; and (iii) facilitates reasoning end-to-end about isolation guarantees, enabling new opportunities for performance optimization.

Limitations Nonetheless, our model currently has two main limitations, which we plan to address in future work. First, it does not constrain the behavior of ongoing transactions. It thus cannot express consistency models, like opacity [30] or virtual world consistency [32], designed to prevent STM transactions from accessing an invalid memory location. This limitation is consistent with the assumption, made in most isolation and consistency research, that applications never make externally visible decisions based on uncommitted data, so that their actions can be rolled back if the transaction aborts. Second, our model focuses on the traditional transactional/read/write model, predominant in database theory and modern distributed storage systems. To support semantically rich operations, abstract data types, and commutativity, we will start from Weikum et al.’s theory of multi-level serializability [59], which maps higher-level operations to reads and writes.

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