# Principled Programming

Introduction to Coding in Any Imperative Language

Tim Teitelbaum

*Emeritus Professor*

*Department of Computer Science*

*Cornell University*

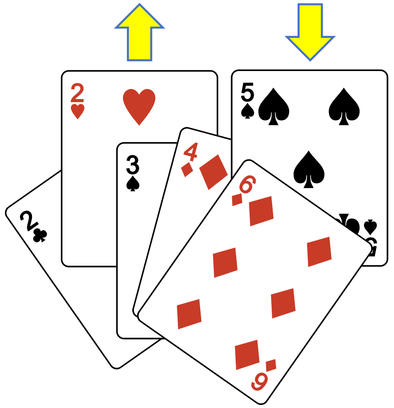
# One-Dimensional Array Rearrangements

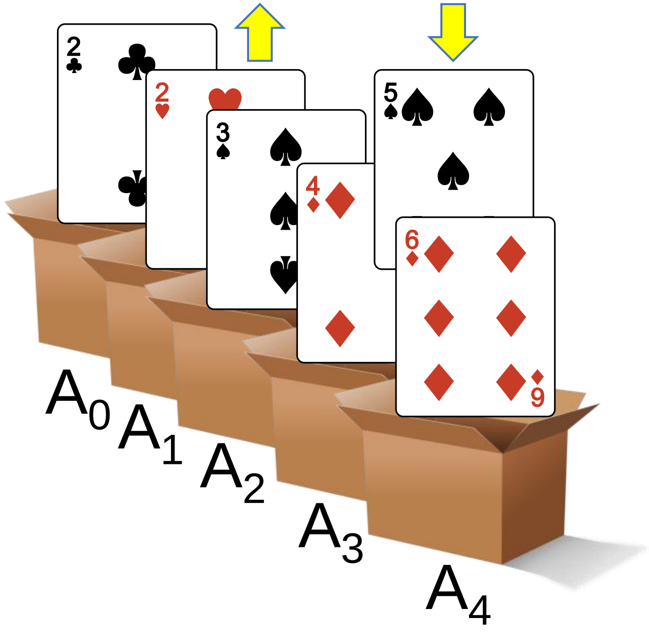
The need to rearrange values in an array is commonplace, and facility in doing so is important.

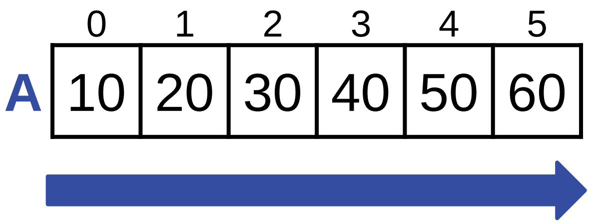
Everyday experience is helpful, e.g., manipulating a hand of playing cards. However, beware that when cards are deleted or inserted, others move over automagically. A better analogy is cards in boxes, but even this is flawed because values are *copied* from variables, not *pulled*, like cards.

We consider:

- Reverse
- LeftShift
- LeftRotate
- Partitioning
- Collation








**Application:** Reverse the order of an array.

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☞ **There is no shame in reasoning with concrete examples.**

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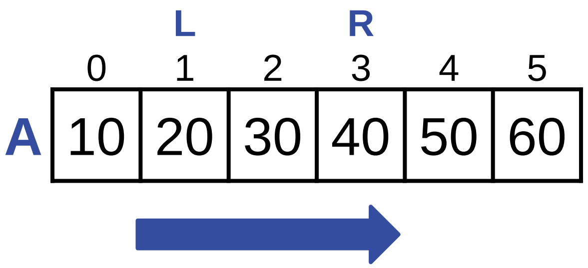


**Application:** Reverse the order of (a subsequence of) an array.

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/* Given int array A[0..n-1], reverse the order of the subsequence A[L..R]
   in situ without affecting the rest of A. */
static void Reverse( int A[], int L, int R ) {
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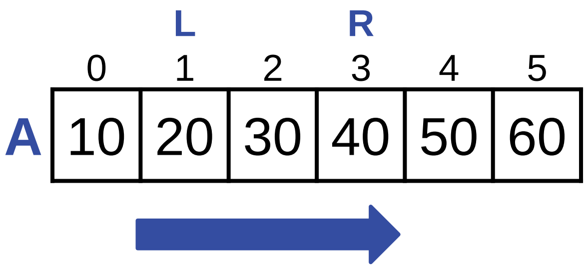


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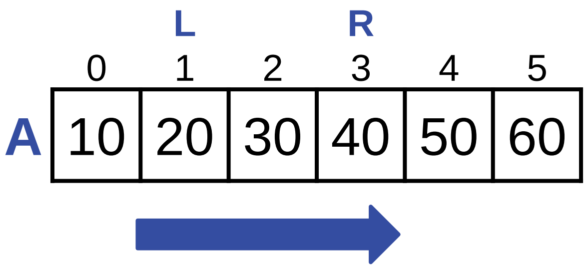


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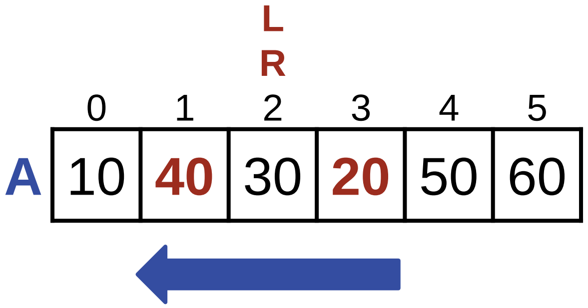


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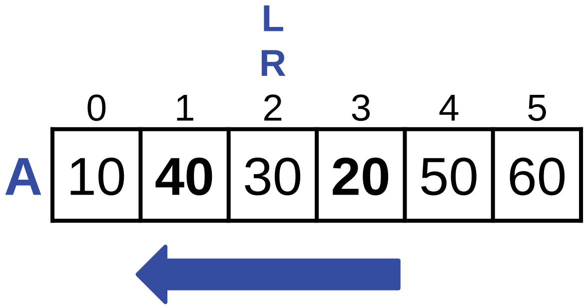


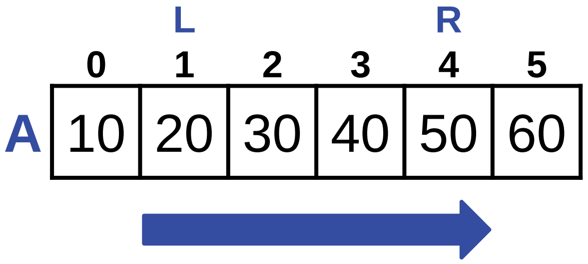


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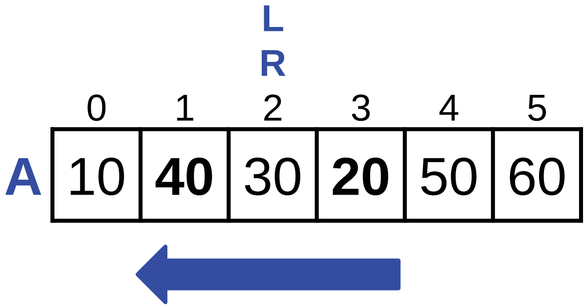
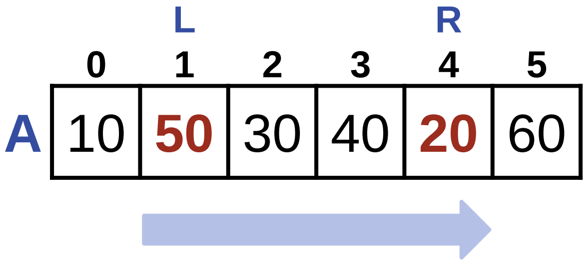
☞ **Validate output thoroughly.**

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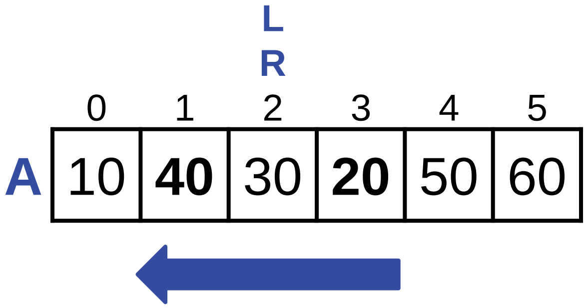


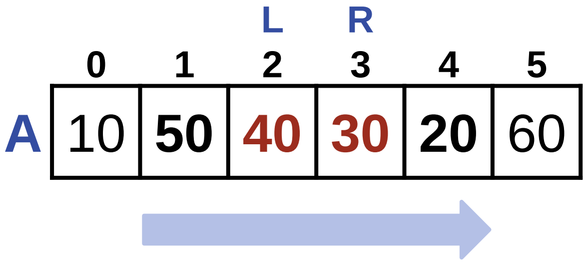


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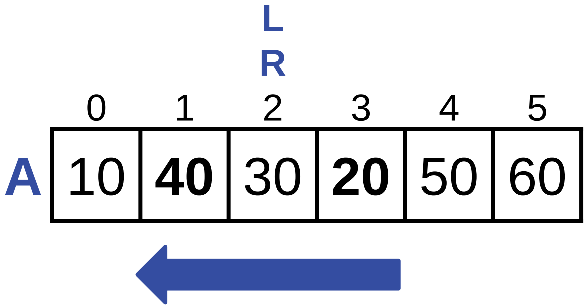


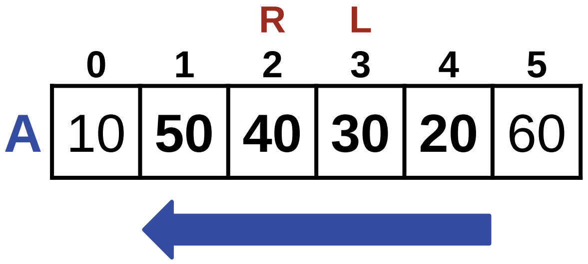


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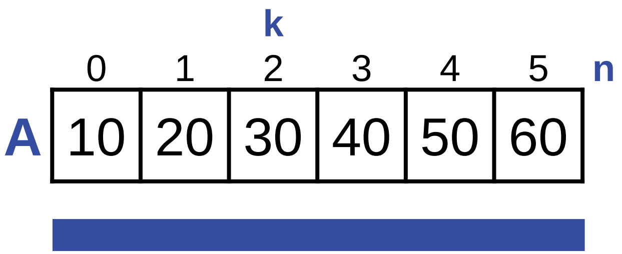
```

It would have been an error for the *condition* to have been `L!=R`.

☞ **Be alert to high-risk coding steps associated with binary choices.**

# Left-Shift-k




**New Application:** Shift an array left k places.

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static void LeftShiftK( int[] A, int n, int k ) {
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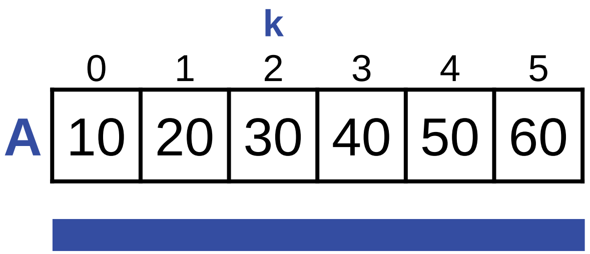


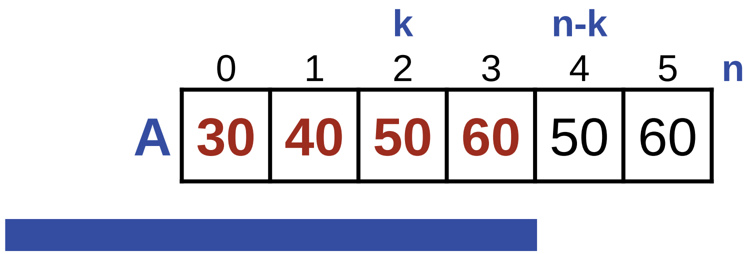


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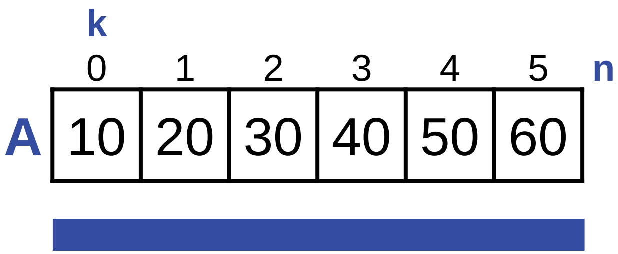


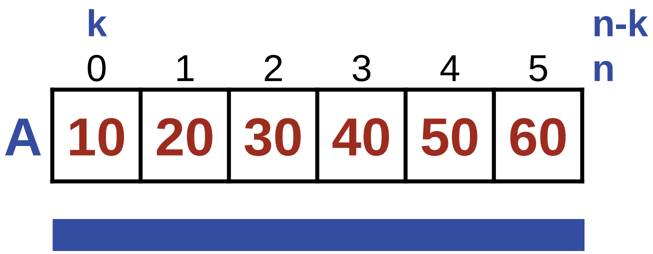


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☞ **Boundary conditions. Dead last, but don't forget them.**

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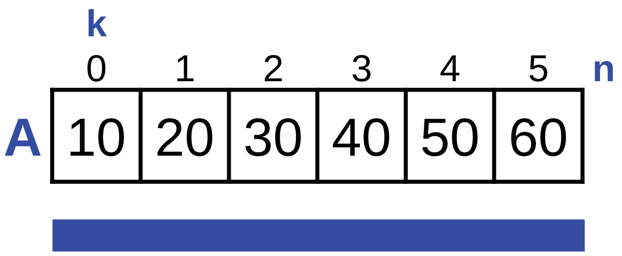


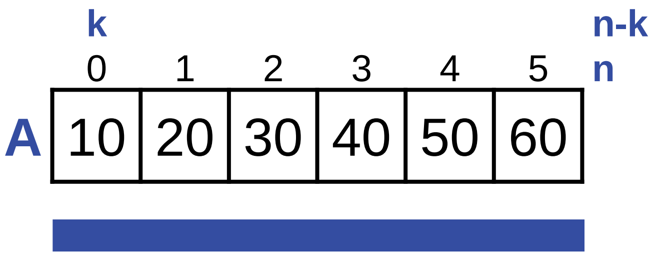


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static void LeftShiftK( int[] A, int n, int k ) {
   if ( k>0 )
      for (int j=0; j<n-k; j++) A[j] = A[j+k];
   } /* LeftShiftK */
```

It would have been correct without this test, but offensive that for k==0 we would do the most work.

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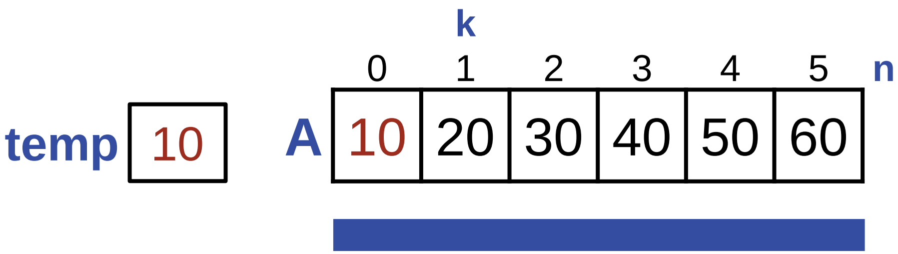
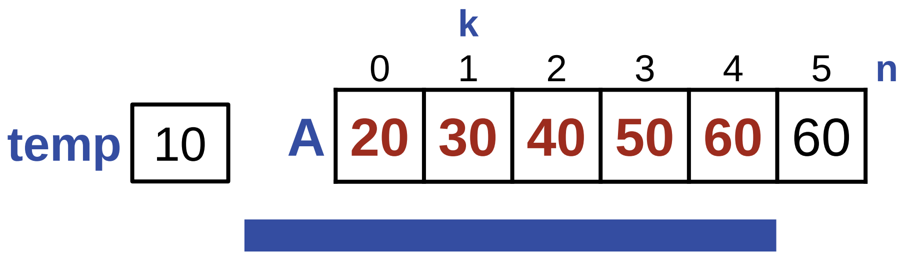
# Left-Rotate-1




**New application:** Rotate an array left 1 place.

```
/* Given int array A[0..n-1], shift A[1..n-1] 1 place left, with the value
   originally in A[0] reentering at right in A[n-1]. */
static void LeftRotateOne(int A[], int n) {
   int temp = A[0];
   LeftShiftK(A, n, 1);
   A[n-1] = temp;
   } /* LeftRotateOne */
```

# Left-Rotate-1

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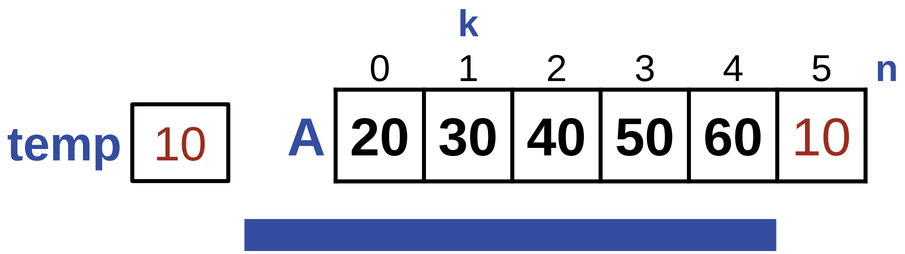


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**New Application:** Rotate an array left k places.

```
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
```


**Application:** Rotate an array left k places.

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/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
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## Left-Rotate-k

# Left-Rotate-k


**Application:** Rotate an array left k places.

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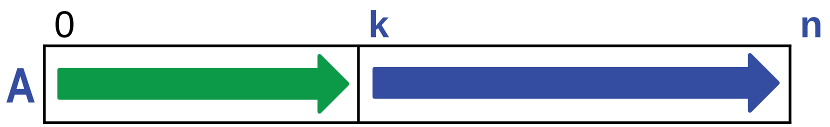
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
```

We shall consider four distinct approaches:

- Repeated Left-Rotate-1
- Swap Generalization
- Three Flips
- Juggle in Cycles

**Approach 1:** Repeated left rotation 1 place.

```
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
   for (int j=0; j<k; j++) LeftRotateOne(A, n);
```




**Approach 2:** k-wide generalization of swap.

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/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
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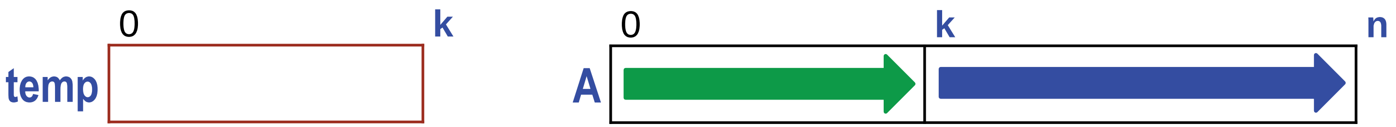


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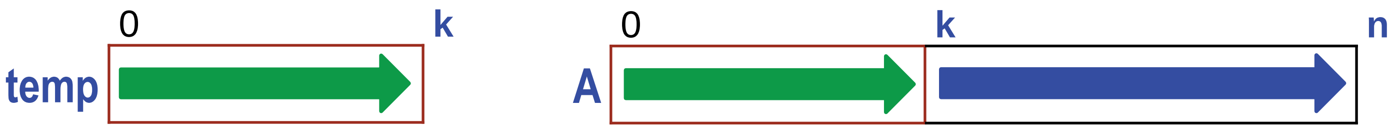
   int temp[] = new int[k];
```

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   /* temp[0..k-1] = A[0..k-1]; */
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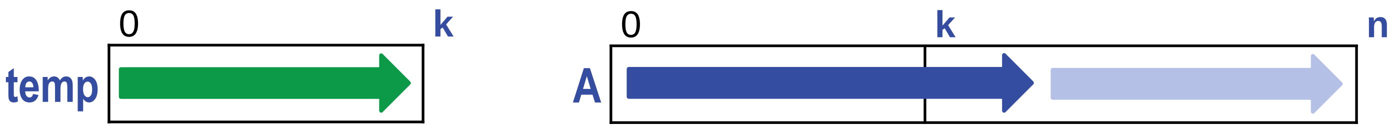


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   LeftShiftK(A, n, k);
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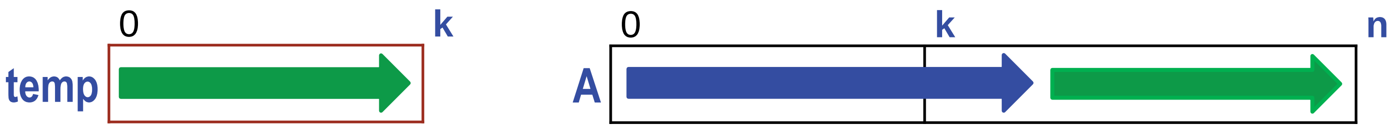


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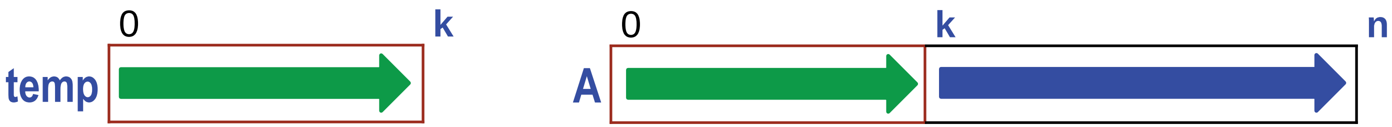
```

☞ **Defer challenging code for later; do the easy parts first.**

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   LeftShiftK(A, n, k);
   /* A[___..n-1] = temp[0..k-1]; */
```

# Left-Rotate-k
## Swap Generalization


**Approach 2:** k-wide generalization of swap.

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/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
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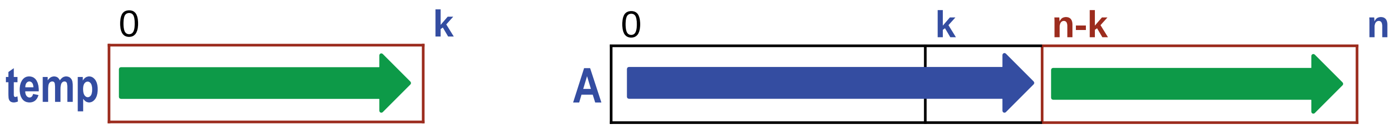
      for (int j=0; j<k; j++) temp[j] = A[j];
   LeftShiftK(A, n, k);
   /* A[___..n-1] = temp[0..k-1]; */
      for (int j=0; j<k; j++) A[_____] = temp[j];
```

☞ **Avoid gratuitous differences in code. Reuse code patterns, if possible.**

# Left-Rotate-k

## Swap Generalization

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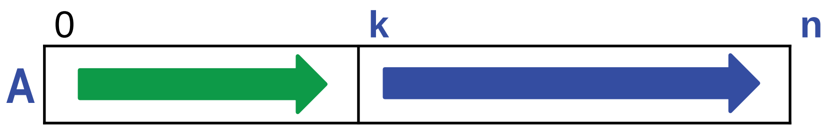
   int temp[] = new int[k];
   /* temp[0..k-1] = A[0..k-1]; */
      for (int j=0; j<k; j++) temp[j] = A[j];
   LeftShiftK(A, n, k);
   /* A[n-k..n-1] = temp[0..k-1]; */
      for (int j=0; j<k; j++) A[n-k+j] = temp[j];
```

Left-Rotate-k

Three Flips




**Approach 3:** Three Flips. Consider the two parts of the array.

```
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
```


**Approach 3:** Represent the values in those parts as **green** and **blue** arrows.

```
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
```

## Left-Rotate-k

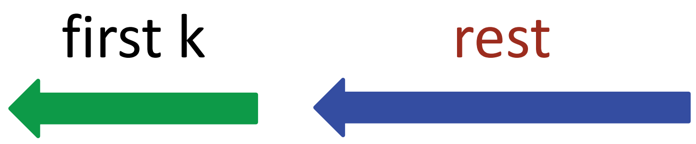
### Three Flips


**Approach 3:** Reverse first k

```
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
   Reverse(A, 0, k-1);
```

# Left-Rotate-k

## Three Flips




**Approach 3:** Reverse first k, then rest of elements

```
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
   Reverse(A, 0, k-1);
   Reverse(A, k, n-1);
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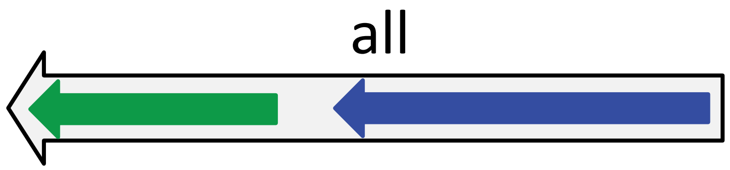


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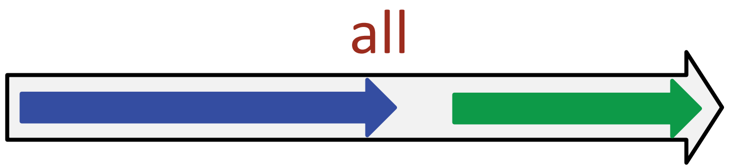


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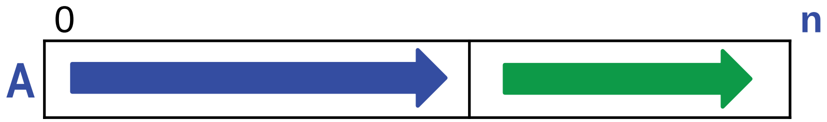


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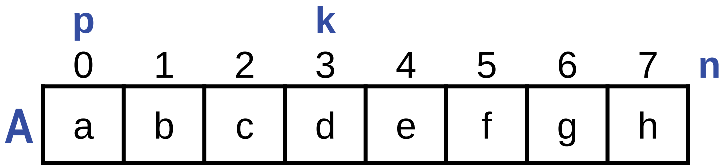
# Left-Rotate-k

## Juggle in Cycles




**Approach 4:** Juggle elements in a stride of k.

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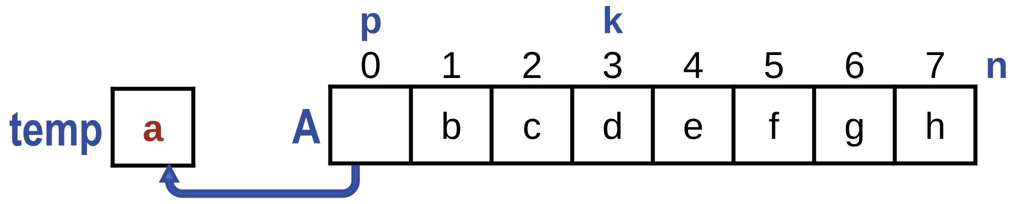


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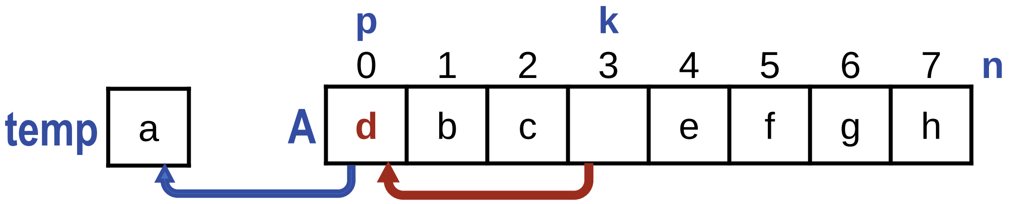


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# Left-Rotate-k

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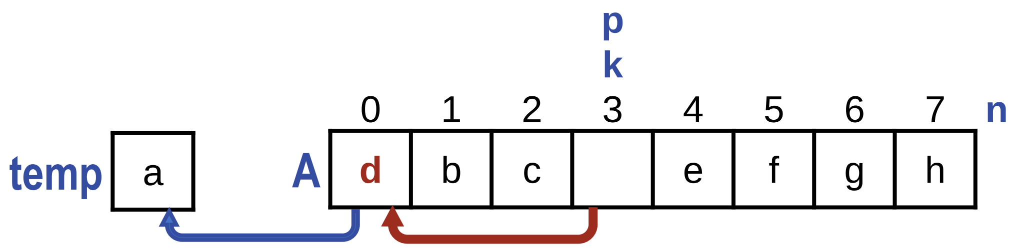


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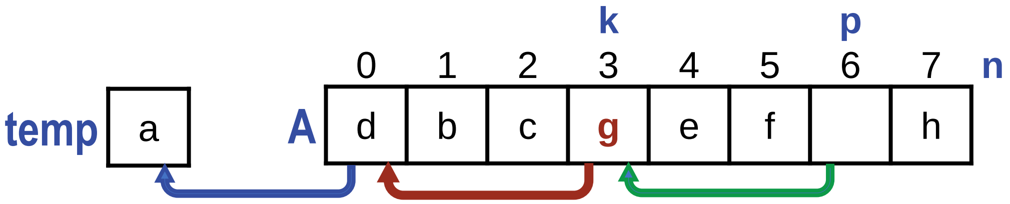


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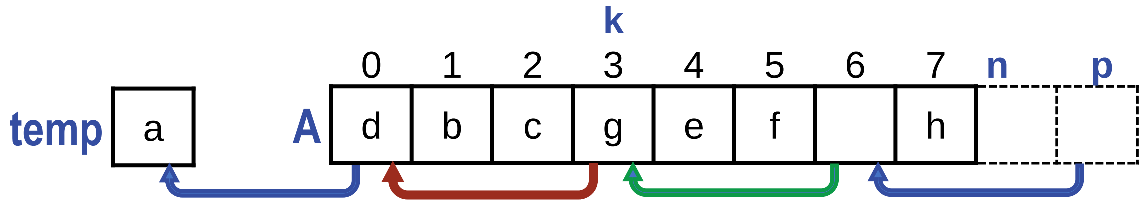


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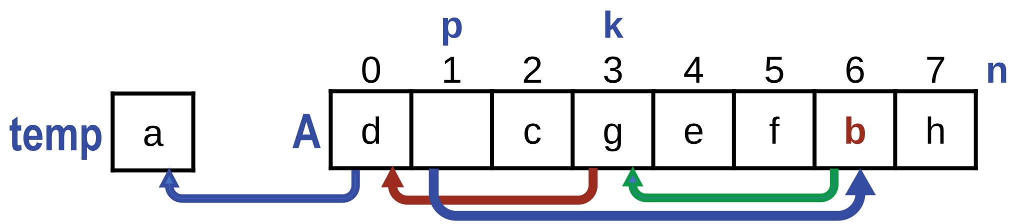


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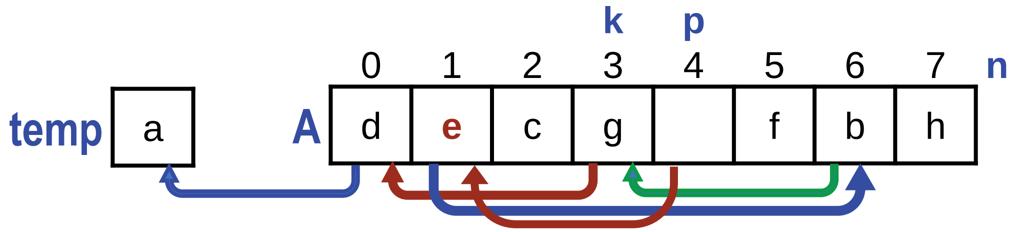


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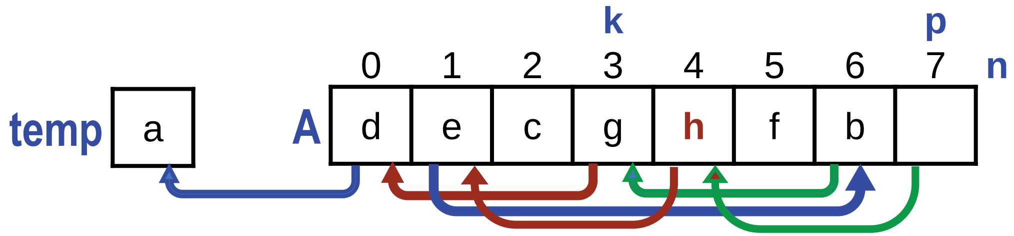


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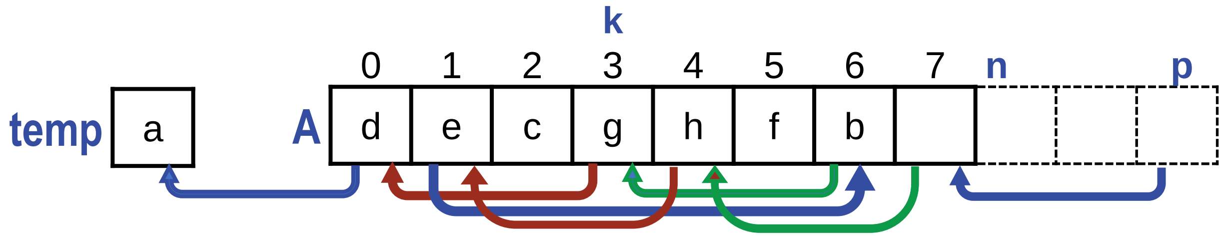


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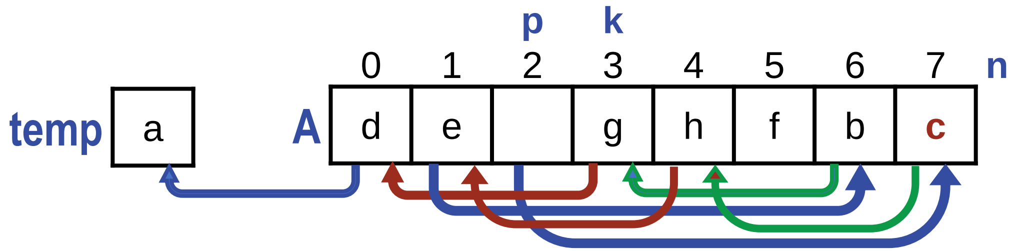
# Left-Rotate-k

## Juggle in Cycles

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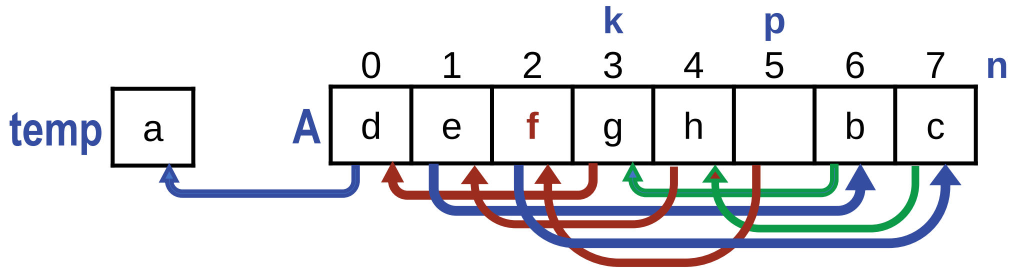


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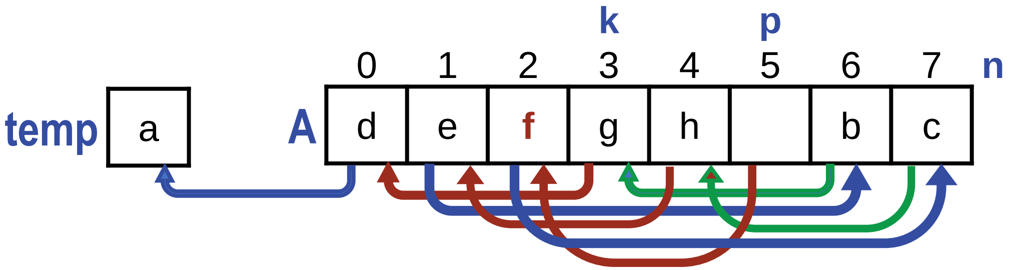


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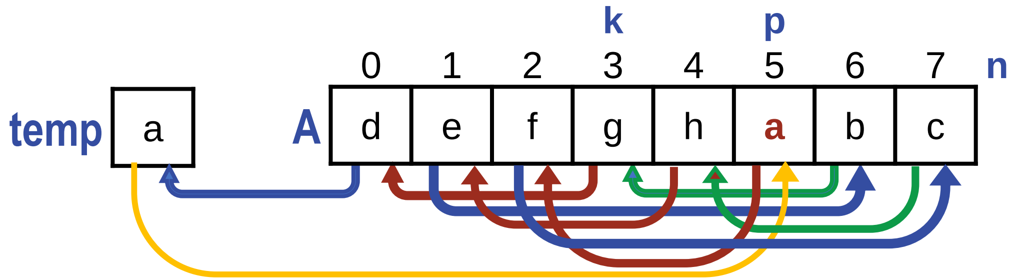


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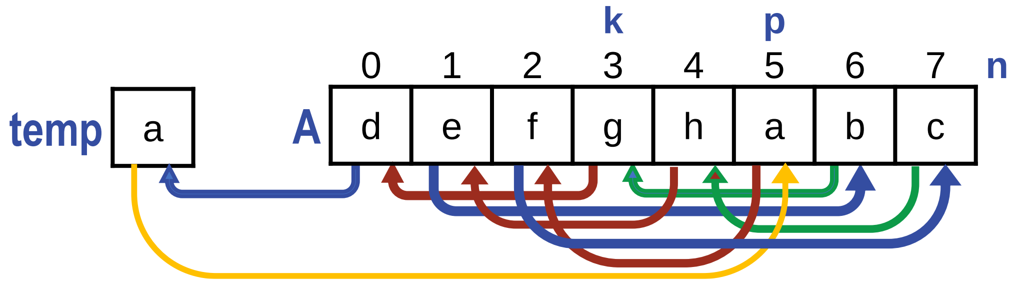


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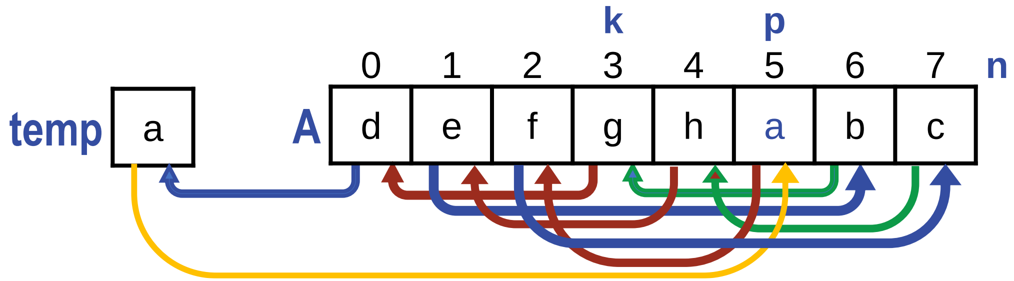
Are we done?

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☞ **Beware of premature self-satisfaction.**

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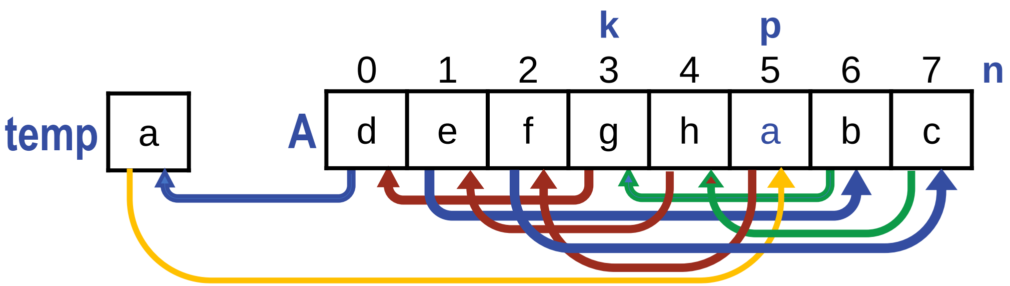


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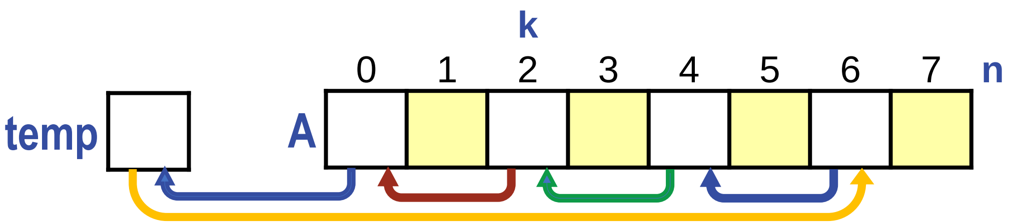
☞ **Validate output thoroughly.**

Are we done?

# Left-Rotate-k

## Juggle in Cycles

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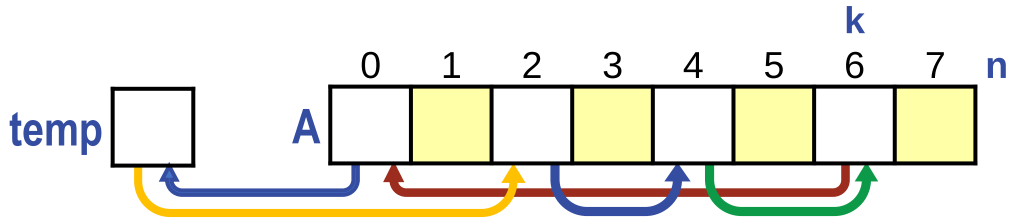
Are we done? Hardly.

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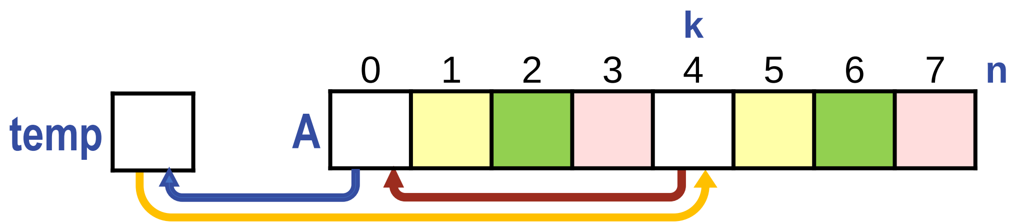


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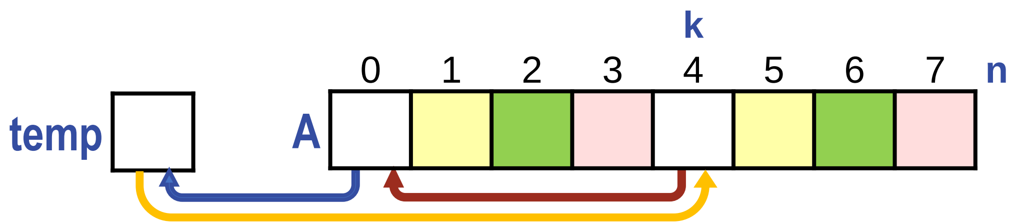
Are we done? Hardly. It only works if k and n are relatively prime!

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Are we done? Hardly. It only works if k and n are relatively prime! Now what?

Abandon, or learn that each of `A[0..gcd(k,n)-1]` begins a disjoint cycle.

**Approach 4:** Juggle elements in a stride of k.

```
/* Given int array A[0..n-1], and integer k, 0≤k<n, left shift A[k..n-1]
   k places, with values originally in A[0..k-1] reentering at right. */
   int g = gcd(n,k);
   for (int j=0; j<g; j++) {
      int p = j;              // Start at A[j]
      int temp = A[p];        // and make a hole there.
      while ( (p+k)%n!=j ) { // Stop if p is about to be j again.
         A[p] = A[(p+k)%n];  // Fill hole at p, making a new hole.
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## Assessment:

| Version of Left-Rotate-k | #moves | Explanation |
|---|---|---|
| Repeated Left-Rotate-1 | $k \cdot n$ | Each Left-Rotate-1 moves all $n$ elements. Done $k$ times. |
| Swap Generalization | $n+k$ | The copies into and out from `temp` do $2 \cdot k$ moves, and the shift does $n$-$k$ moves. |
| Three Flips | $2 \cdot n$ | Each element moves once during the 1st two reverses, and then again for the 3rd reverse. |
| Juggle in Cycles | $n+\gcd(n,k)$ | Each element moves once, plus the first element of each of the gcd(n,k) cycles must first be saved in temp. |

Worst #moves, by far. Easiest to understand.

## Assessment:

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Reasonable #moves but not *in situ*, i.e., requires extra space for `temp`.

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| Swap Generalization | $n+k$ | The copies into and out from `temp` do $2 \cdot k$ moves, and the shift does $n$-$k$ moves. |
| ☞ Three Flips | $2 \cdot n$ | Each element moves once during the 1st two reverses, and then again for the 3rd reverse. |
| Juggle in Cycles | $n+\text{gcd}(n,k)$ | Each element moves once, plus the first element of each of the gcd(n,k) cycles must first be saved in temp. |

Reasonable #moves. Good locality.

## Assessment:

| Version of Left-Rotate-k | #moves | Explanation |
|---|---|---|
| Repeated Left-Rotate-1 | $k \cdot n$ | Each Left-Rotate-1 moves all *n* elements. Done *k* times. |
| Swap Generalization | $n+k$ | The copies into and out from `temp` do $2 \cdot k$ moves, and the shift does $n-k$ moves. |
| Three Flips | $2 \cdot n$ | Each element moves once during the 1st two reverses, and then again for the 3rd reverse. |
| ☞ Juggle in Cycles | $n+\text{gcd}(n,k)$ | Each element moves once, plus the first element of each of the gcd(n,k) cycles must first be saved in temp. |

Hardest to understand. Poor locality.

**Assessment:**

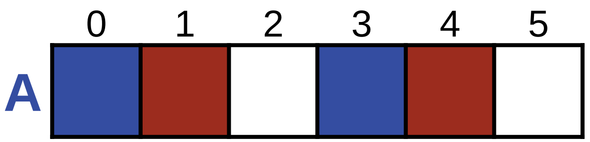
| Version of Left-Rotate-k | #moves | Explanation |
|---|---|---|
| Repeated Left-Rotate-1 | $k \cdot n$ | Each Left-Rotate-1 moves all *n* elements. Done *k* times. |
| Swap Generalization | n+k | The copies into and out from `temp` do $2 \cdot k$ moves, and the shift does *n*-*k* moves. |
| ✓ Three Flips | $2 \cdot n$ | Each element moves once during the 1st two reverses, and then again for the 3rd reverse. |
| Juggle in Cycles | *n*+gcd(*n*,*k*) | Each element moves once, plus the first element of each of the gcd(n,k) cycles must first be saved in temp. |

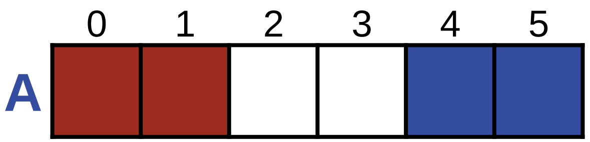
Personal favorite, and really elegant!

# Dutch National Flag

**New Application:** The Dutch National Flag problem.

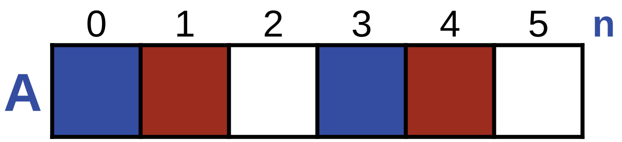
# Dutch National Flag

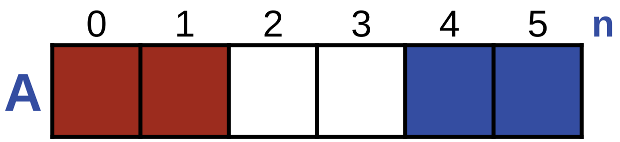



**Application:** Rearrange an array into all red, then all white, then all blue.

# Dutch National Flag




**Application:** Rearrange an array into all red, then all white, then all blue.

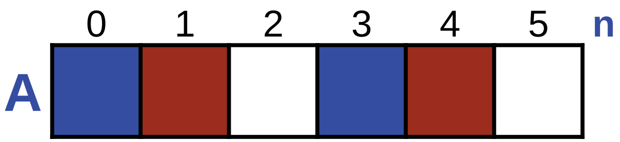
```
/* Given array A[0..n-1] consisting of only three values (red, white,
   and blue), rearrange A into all red, then white, then blue. */
```

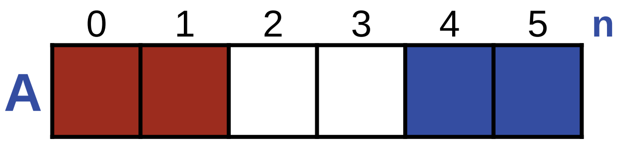
---

☞ **A statement-comment says exactly what code must accomplish, not how it does so.**

---

# Dutch National Flag

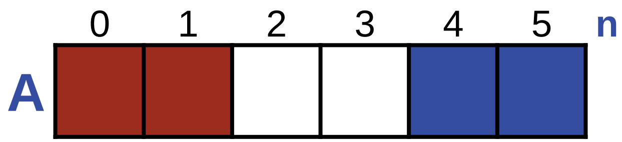



**Application:** Rearrange an array into all red, then all white, then all blue.

```
/* Given array A[0..n-1] consisting of only three values (red, white,
   and blue), rearrange A into all red, then white, then blue. */
   while ( ______ ) __________
```

☞ **If you “smell a loop”, write it down.**



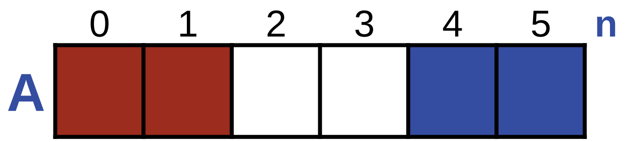

POST

**Application:** Rearrange an array into all red, then all white, then all blue.

```
/* Given array A[0..n-1] consisting of only three values (red, white,
   and blue), rearrange A into all red, then white, then blue. */
   while ( ______ ) __________
```

☞ **To get to POST iteratively, choose a weakened POST as INVARIANT.**

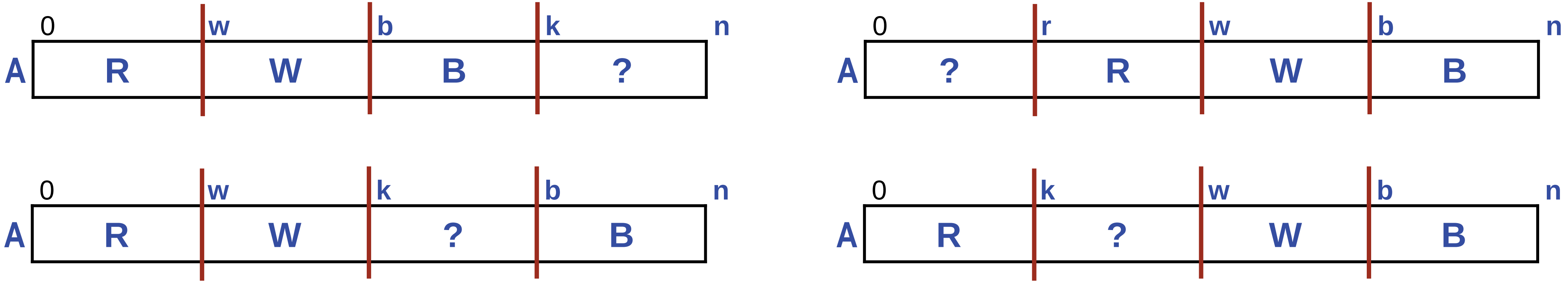
# Dutch National Flag




POST

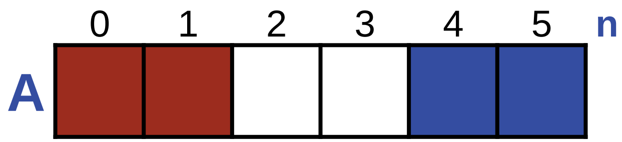
**Application:** Rearrange an array into all red, then all white, then all blue.

Here are four choices:




☞ **To get to POST iteratively, choose a weakened POST as INVARIANT.**

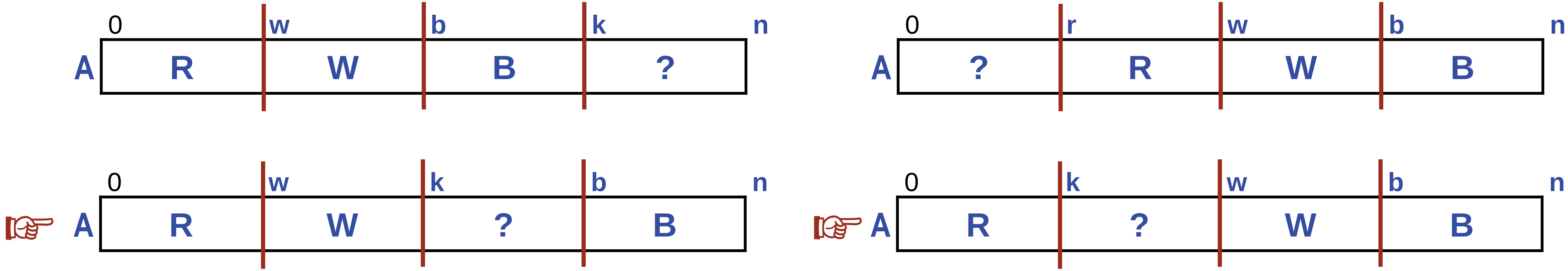
# Dutch National Flag



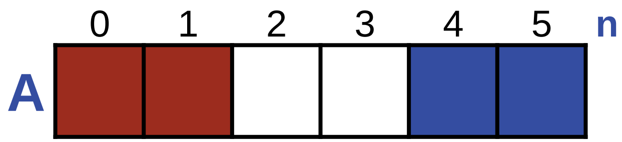

**POST**

**Application:** Rearrange an array into all red, then all white, then all blue.

Here are four choices: How shall we choose?



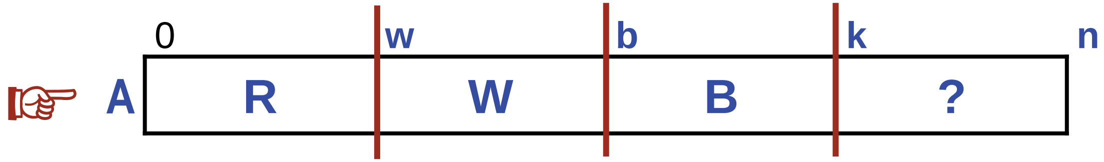

Symmetric, so discard one arbitrarily.

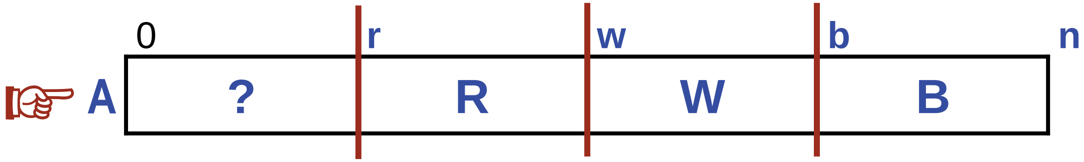


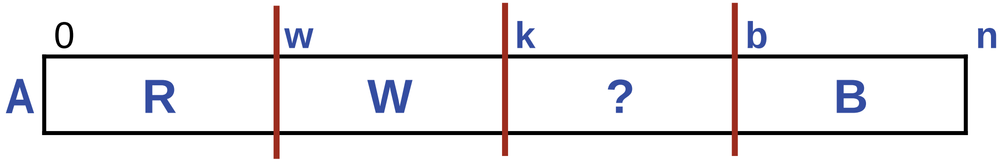

POST

**Application:** Rearrange an array into all red, then all white, then all blue.

Here are four choices: How shall we choose?

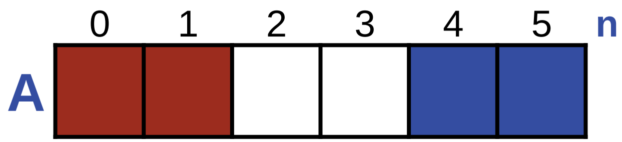



Left more intuitive, because the ? region seems more familiar, so discard right.

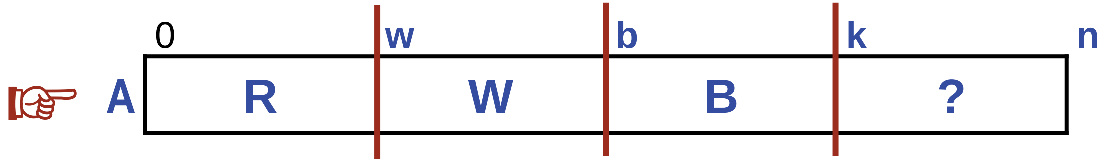
# Dutch National Flag

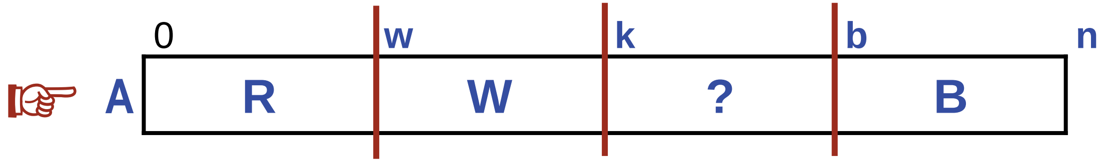



POST

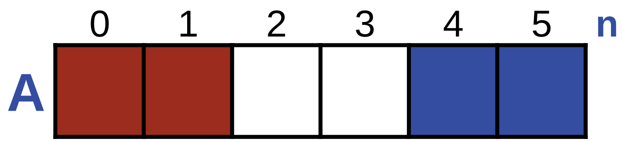
**Application:** Rearrange an array into all red, then all white, then all blue.

Here are four choices: How shall we choose?



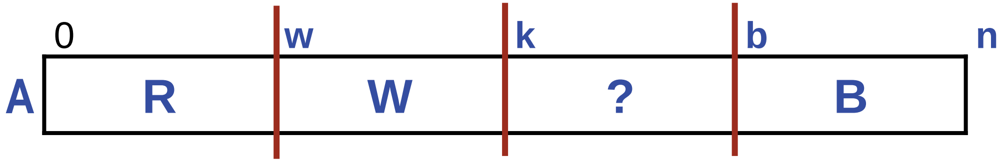

? region of top has only one degree of freedom, but bottom has two. Discard top.



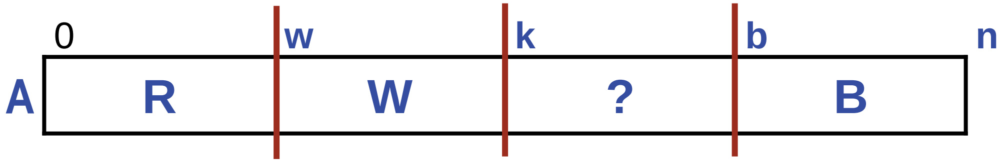

POST

**Application:** Rearrange an array into all red, then all white, then all blue.

Here are four choices: How shall we choose?




This will be our **INVARIANT**.

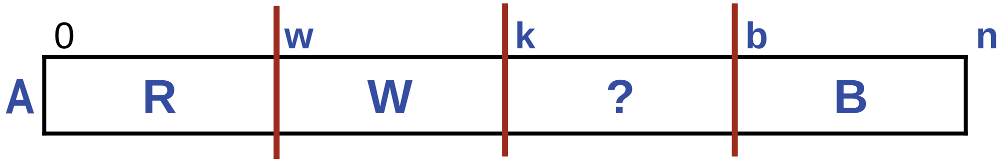



**VARIANT: b-k**

**Application:** Rearrange an array into all red, then all white, then all blue.

```
/* Given array A[0..n-1] consisting of only three values (red, white,
   and blue), rearrange A into all red, then white, then blue. */
   /* A[0..w-1] red, A[w..k-1] white, A[b..n-1] blue, for 0≤w≤k≤b≤n. */
      int k = ____; int w = ____; int b = ____;
   while ( ______ ) __________
```

# Dutch National Flag

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      int k = ____; int w = ____; int b = ____;
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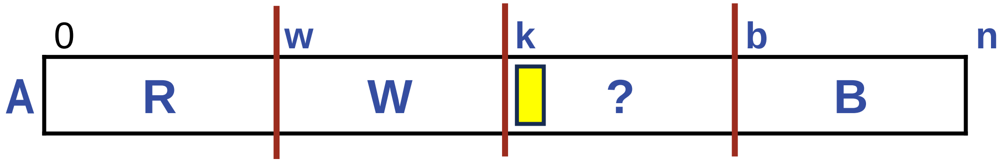
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

# Dutch National Flag

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      int k = ____; int w = ____; int b = ____;
   while ( ______ )
      if ( A[k] == ______ ) __________
      else if ( A[k] == ______ ) __________
      else __________
```

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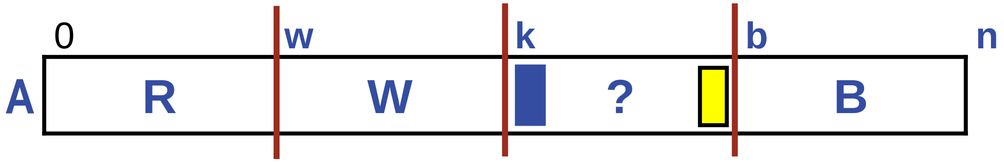
# Dutch National Flag

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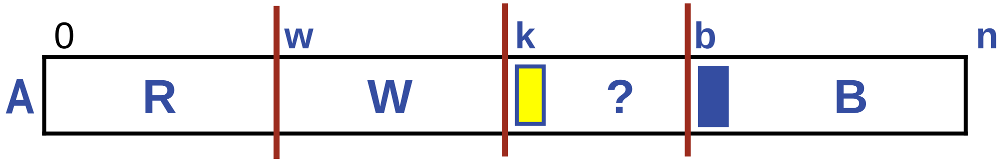
      int k = ____; int w = ____; int b = ____;
   while ( ______ )
      if ( A[k]==B ) { /* Swap A[b-1] and A[k]. */  b--; }
      else if ( A[k] == ______  ) __________
      else __________
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

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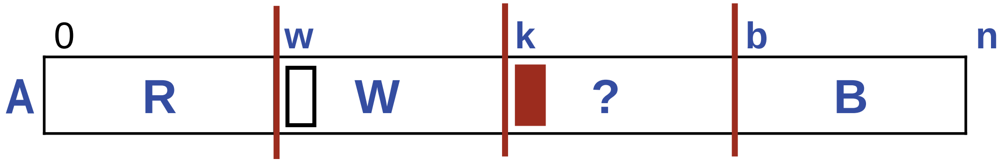
# Dutch National Flag

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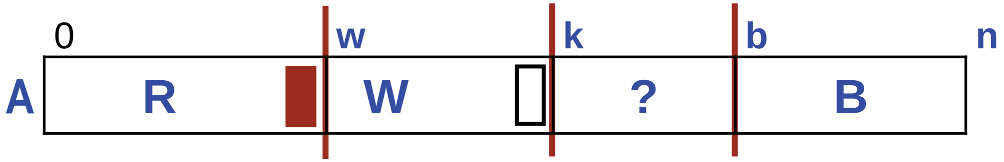
      int k = ____; int w = ____; int b = ____;
   while ( ______ )
      if ( A[k]==B ) { /* Swap A[b-1] and A[k]. */  b--; }
      else if ( A[k]==R ) { /* Swap A[w] and A[k]. */  w++; k++; }
      else __________
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

# Dutch National Flag

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**Application:** Rearrange an array into all red, then all white, then all blue.

```
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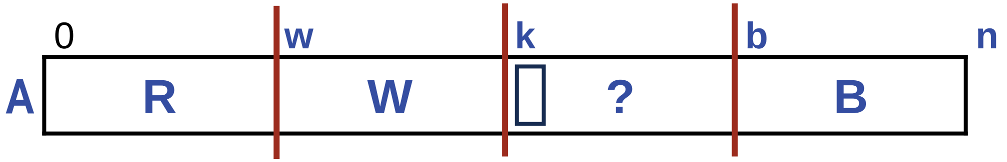
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

# Dutch National Flag




**VARIANT: b-k**

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/* Given array A[0..n-1] consisting of only three values (red, white,
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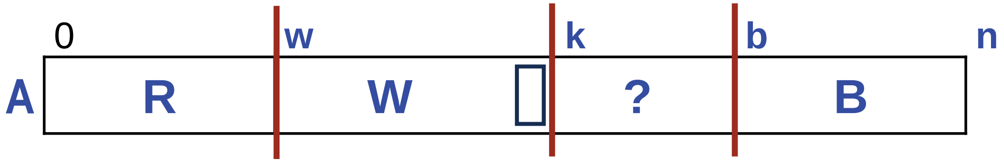
      int k = ____; int w = ____; int b = ____;
   while ( ______ )
      if ( A[k]==B ) { /* Swap A[b-1] and A[k]. */  b--; }
      else if ( A[k]==R ) { /* Swap A[w] and A[k]. */  w++; k++; }
      else /* A[k]==W */ k++;
```

---

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

---

# Dutch National Flag




**VARIANT: b-k**

**Application:** Rearrange an array into all red, then all white, then all blue.

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/* Given array A[0..n-1] consisting of only three values (red, white,
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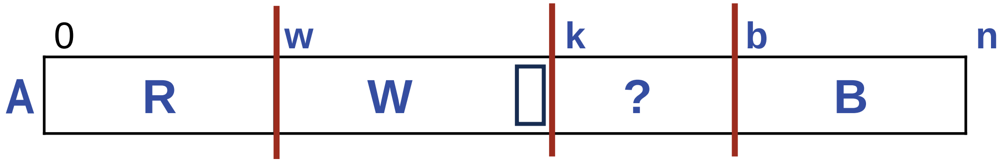
      int k = ____; int w = ____; int b = ____;
   while ( ______ )
      if ( A[k]==B ) { /* Swap A[b-1] and A[k]. */  b--; }
      else if ( A[k]==R ) { /* Swap A[w] and A[k]. */  w++; k++; }
      else /* A[k]==W */ k++;
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**




**VARIANT: b-k**

# Dutch National Flag

**Application:** Rearrange an array into all red, then all white, then all blue.

```
/* Given array A[0..n-1] consisting of only three values (red, white,
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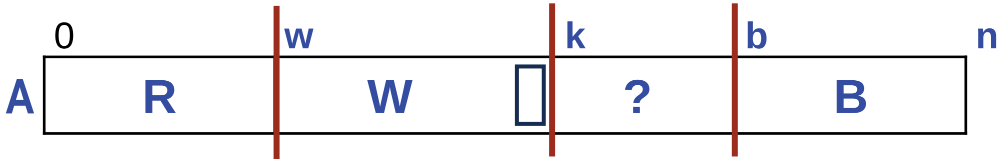
      int k = ____; int w = ____; int b = ____;
   while ( k!=b )
      if ( A[k]==B ) { /* Swap A[b-1] and A[k]. */  b--; }
      else if ( A[k]==R ) { /* Swap A[w] and A[k]. */  w++; k++; }
      else /* A[k]==W */ k++;
```

# Dutch National Flag




**VARIANT: b-k**

**Application:** Rearrange an array into all red, then all white, then all blue.

```
/* Given array A[0..n-1] consisting of only three values (red, white,
   and blue), rearrange A into all red, then white, then blue. */
   /* A[0..w-1] red, A[w..k-1] white, A[b..n-1] blue, for 0≤w≤k≤b≤n. */
      int k = 0; int w = 0; int b = n;
   while ( k!=b )
      if ( A[k]==B ) { /* Swap A[b-1] and A[k]. */  b--; }
      else if ( A[k]==R ) { /* Swap A[w] and A[k]. */  w++; k++; }
      else /* A[k]==W */ k++;
```

**Application:** Rearrange an array into all red, then all white, then all blue.

```
/* Given array A[0..n-1] consisting of only three values (red, white,
   and blue), rearrange A into all red, then white, then blue. */
   /* A[0..w-1] red, A[w..k-1] white, A[b..n-1] blue, for 0≤w≤k≤b≤n. */
      int k = 0; int w = 0; int b = n;
   while ( k!=b )
      if ( A[k]==B ) {
         /* Swap A[b-1] and A[k]. */
         b--;
         }
      else if ( A[k]==R ) {
         /* Swap A[w] and A[k]. */
         w++; k++;
         }
      else /* A[k]==W */ k++;
```

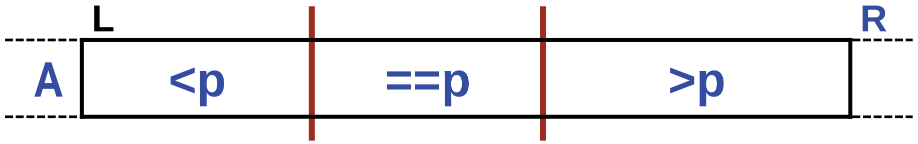
## Dutch National Flag

**Application:** Rearrange an array into all red, then all white, then all blue.

```
/* Given array A[0..n-1] consisting of only three values (red, white,
   and blue), rearrange A into all red, then white, then blue. */
   /* A[0..w-1] red, A[w..k-1] white, A[b..n-1] blue, for 0≤w≤k≤b≤n. */
      int k = 0; int w = 0; int b = n;
   while ( k!=b )
      if ( A[k]==B ) {
         /* Swap A[b-1] and A[k]. */
            int temp = A[b-1]; A[b-1] = A[k]; A[k] = temp;
         b--;
         }
      else if ( A[k]==R ) {
         /* Swap A[w] and A[k]. */
            int temp = A[k]; A[k] = A[w]; A[w] = temp;
         w++; k++;
         }
      else /* A[k]==W */ k++;
```

**Performance:**

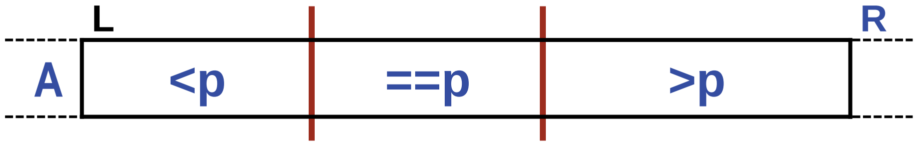
- Constant work per iteration.
- Variant reduced by 1 on each iteration.
- Thus, running time linear in n.




**New Application:** Rearrange (a segment of) an array into <p, ==p, and >p.

```
/* Rearrange A[L..R-1] into all <p, then all ==p, then all >p. */
static void Partition( int A[], int L, int R, int p ) {
   } /* Partition */
```

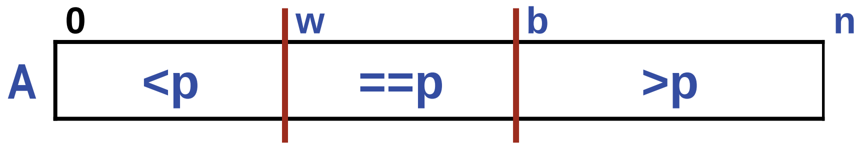
☞ **A header-comment says exactly what a method must accomplish, not how it does so.**




**Application:** Rearrange (a segment of) an array into <p, ==p, and >p.

```
/* Rearrange A[L..R-1] into all <p, then all ==p, then all >p. */
static void Partition( int A[], int L, int R, int p ) {
   ⟨body of Dutch National Flag problem⟩
   } /* Partition */
```

☞ **Don't type if you can avoid it; clone. Cut and paste, then adapt.**



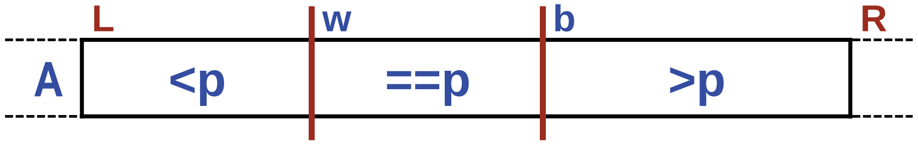

**Application:** Rearrange (a segment of) an array into <p, ==p, and >p.

Dutch National Flag problem

```
/* Rearrange A[L..R-1] into all <p, then all ==p, then all >p. */
static void Partition( int A[], int L, int R, int p ) {
   /* A[0..w-1] red, A[w..k-1] white, A[b..n-1] blue, for 0≤w≤k≤b≤n. */
      int k = 0; int w = 0; int b = n;
   while ( k!=b )
      if ( A[k]==B ) {
         /* Swap A[b-1] and A[k]. */
            int temp = A[b-1]; A[b-1] = A[k]; A[k] = temp;
         b--;
         }
      else if ( A[k]==R ) {
         /* Swap A[w] and A[k]. */
            int temp = A[k]; A[k] = A[w]; A[w] = temp;
         w++; k++;
         }
      else /* A[k]==W */ k++;
   } /* Partition */
```

**Application:** Rearrange (a segment of) an array into <p, ==p, and >p.

```
/* Rearrange A[L..R-1] into all <p, then all ==p, then all >p. */
static void Partition( int A[], int L, int R, int p ) {
   /* A[L..w-1]<p, A[w..k-1]==p, A[b..R-1]>p, for L≤w≤k≤b≤R. */
      int k = L; int w = L; int b = R;
   while ( k!=b )
      if ( A[k]>p ) {
         /* Swap A[b-1] and A[k]. */
            int temp = A[b-1]; A[b-1] = A[k]; A[k] = temp;
         b--;
         }
      else if ( A[k]<p ) {
         /* Swap A[w] and A[k]. */
            int temp = A[k]; A[k] = A[w]; A[w] = temp;
         w++; k++;
         }
      else /* A[k]==p */ k++;
   } /* Partition */
```




**Application:** Rearrange (a segment of) an array into <p, ==p, and >p.

What value of p would tend to create “<p” and “>p” regions of near equal size?

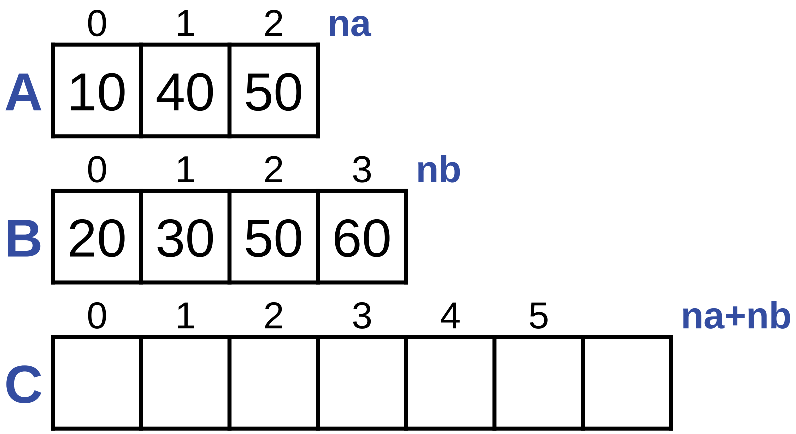
If `A[L..R-1]` are in random order, any of those values is equally good for `p`.

Choosing a p of `(A[L]+A[R-1])/2` guards against poor performance when `A[L..R-1]` is already ordered.

This is central to a Divide and Conquer approach to sorting called QuickSort.

**New Application:** Collate ordered arrays A and B into array C.

# Collation




**Application:** Collate ordered arrays A and B into array C.

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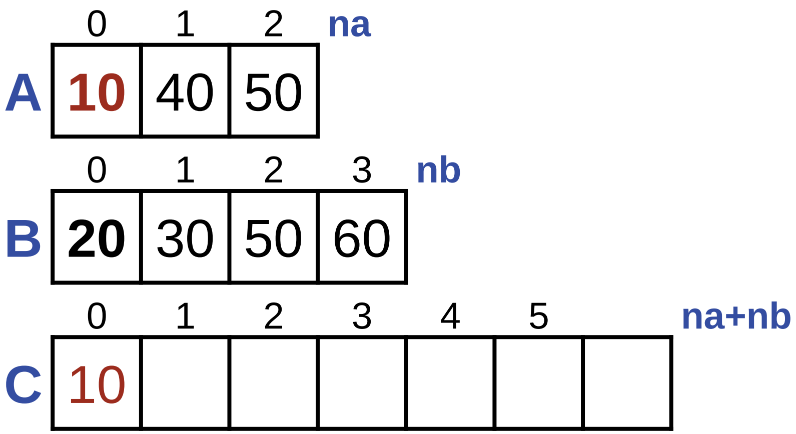
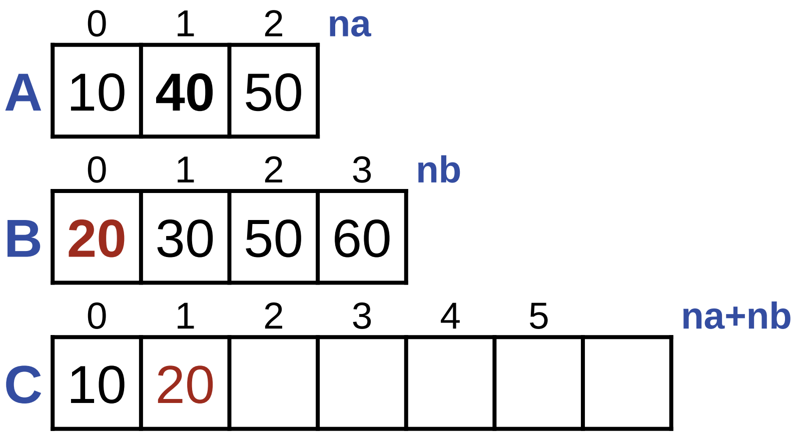



**Application:** Collate ordered arrays A and B into array C.

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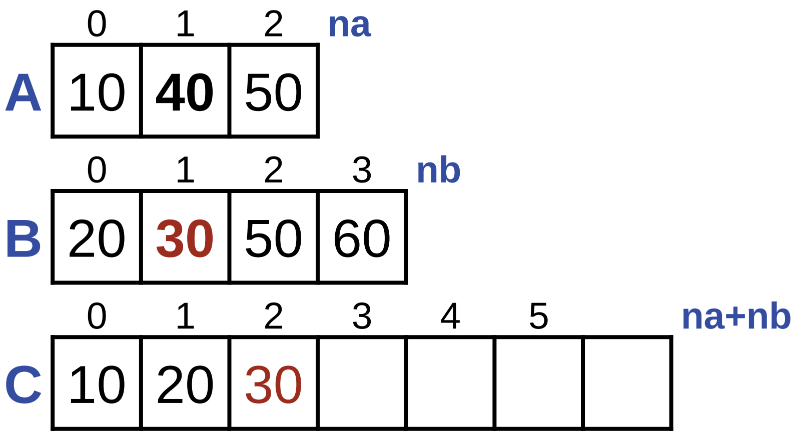


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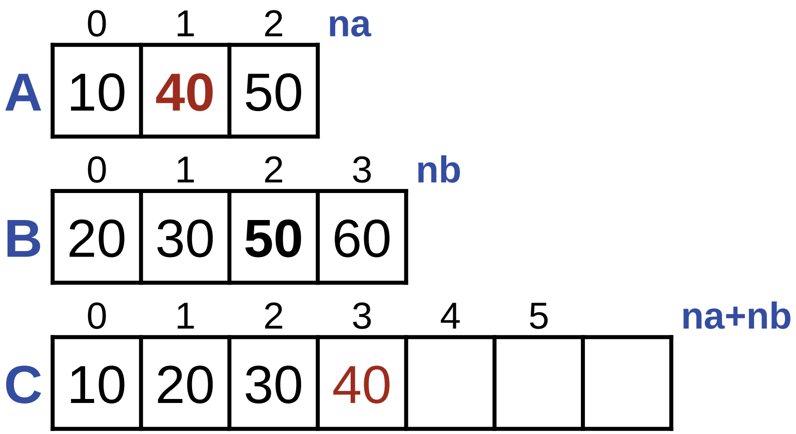


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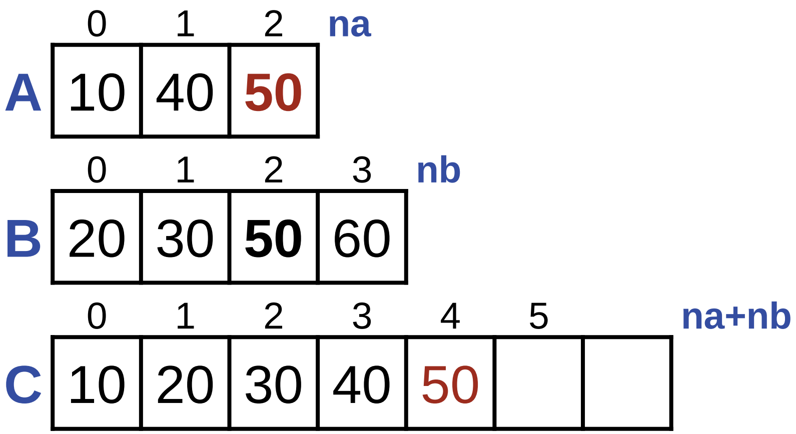


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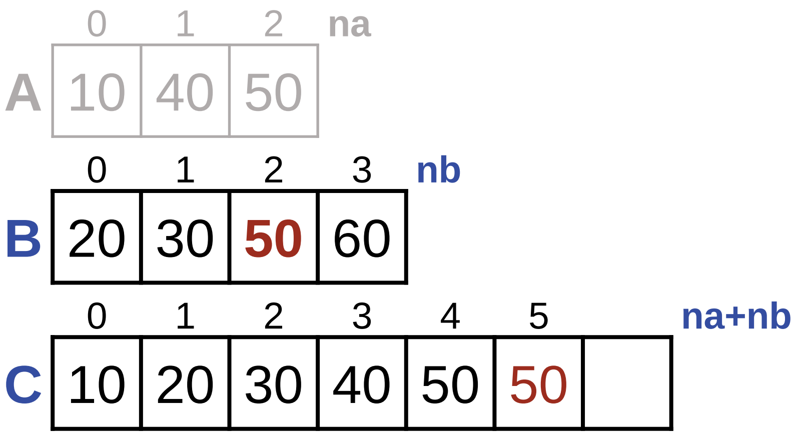



**Application:** Collate ordered arrays A and B into array C.

# Collation

A (indices 0, 1, 2; na)

| 0 | 1 | 2 |
|---|---|---|
| 10 | 40 | 50 |

B (indices 0, 1, 2, 3; nb)

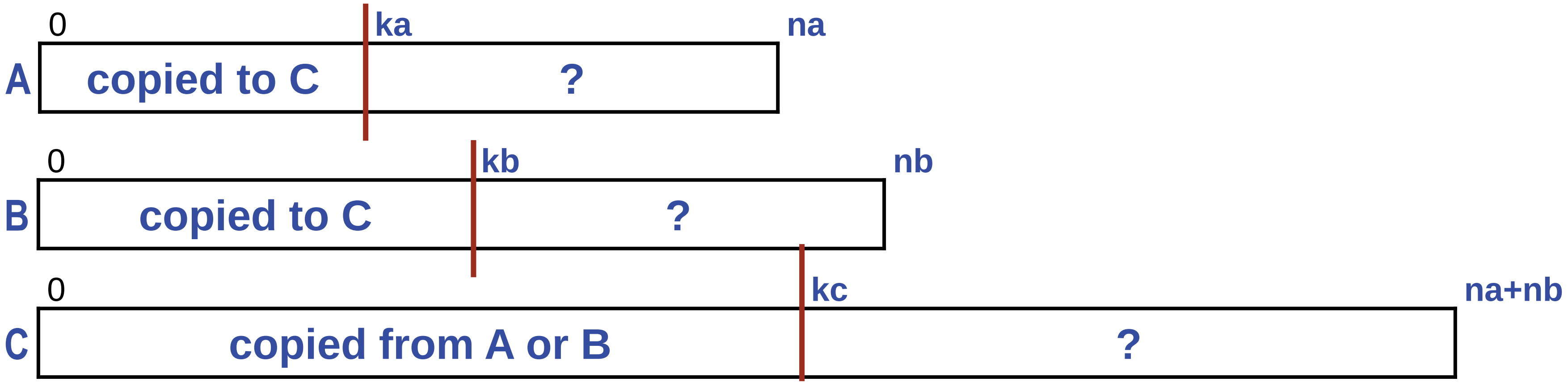
| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 20 | 30 | 50 | **60** |

C (indices 0, 1, 2, 3, 4, 5; na+nb)

| 0 | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|
| 10 | 20 | 30 | 40 | 50 | 50 | 60 |

**Application:** Collate ordered arrays A and B into array C.

Collation is central to a Divide and Conquer approach to sorting called MergeSort.




**Application:** Collate ordered arrays A and B into array C.

```
/* Given ordered arrays A and B of lengths na and nb, create ordered
   array C of length na+nb consisting of those values. */
   int C[] = new int[na+nb];                     // C[0..kc-1] is collation of
                                                 // A[0..ka-1] and B[0..kb-1].
int ka = ___; int kb = ___; int kc = ___; // Indices in A, B, and C.
/* Copy values from A or B into C until one array is exhausted. */
/* Copy remaining values into C from the unexhausted array, A or B. */
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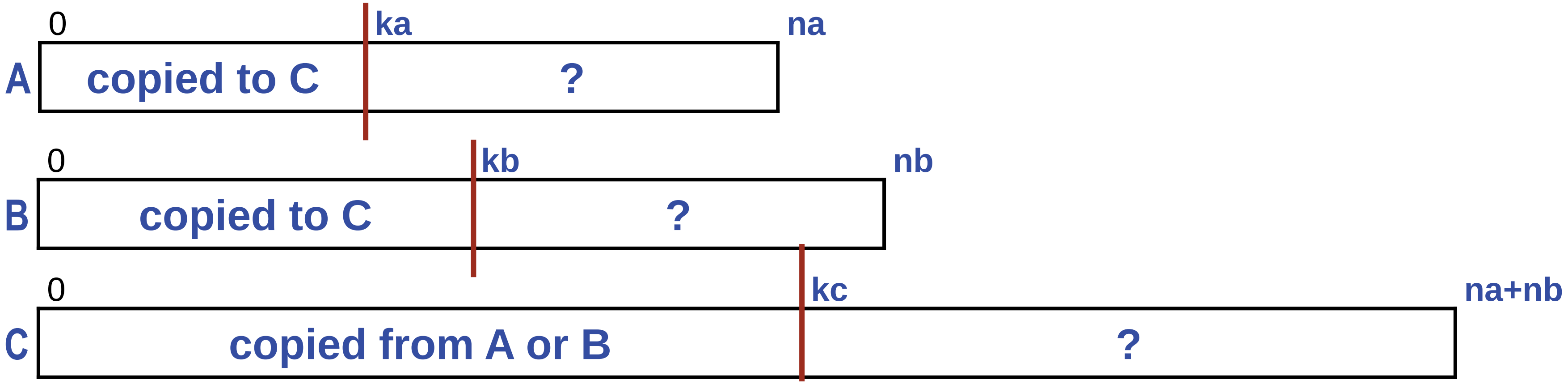


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      if ( ___________ ) { __________________________ }
      else               {                            }
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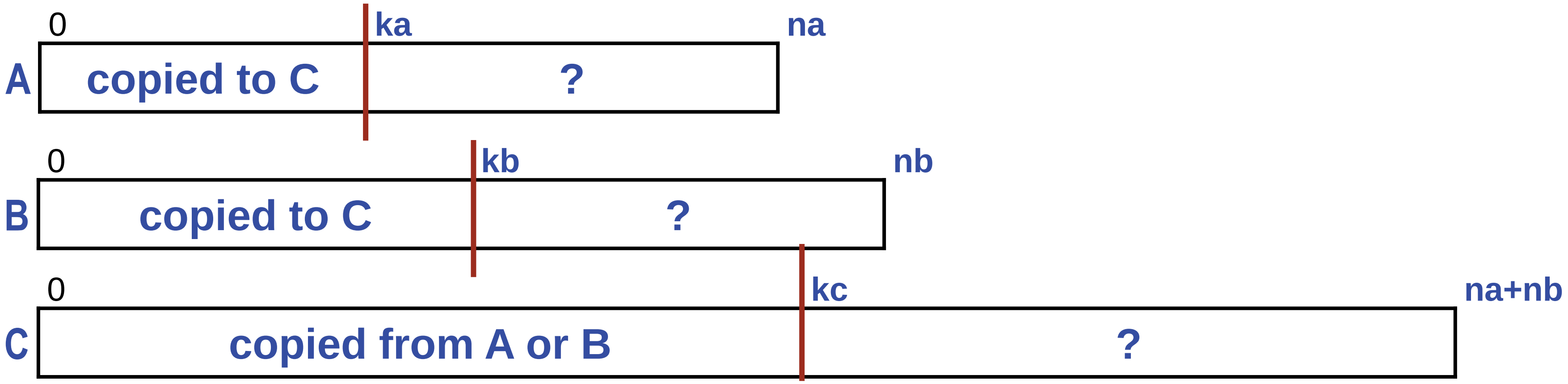


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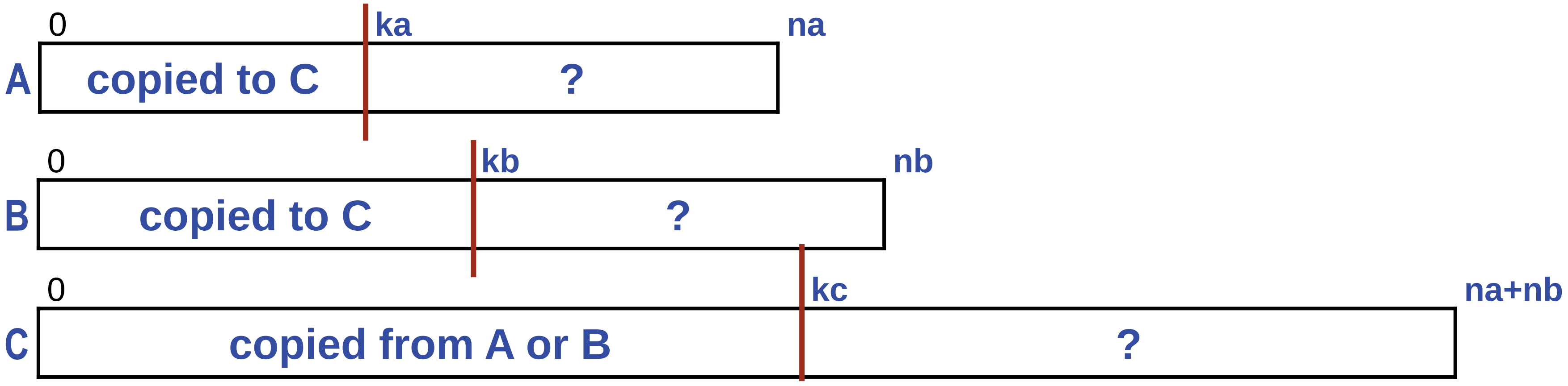


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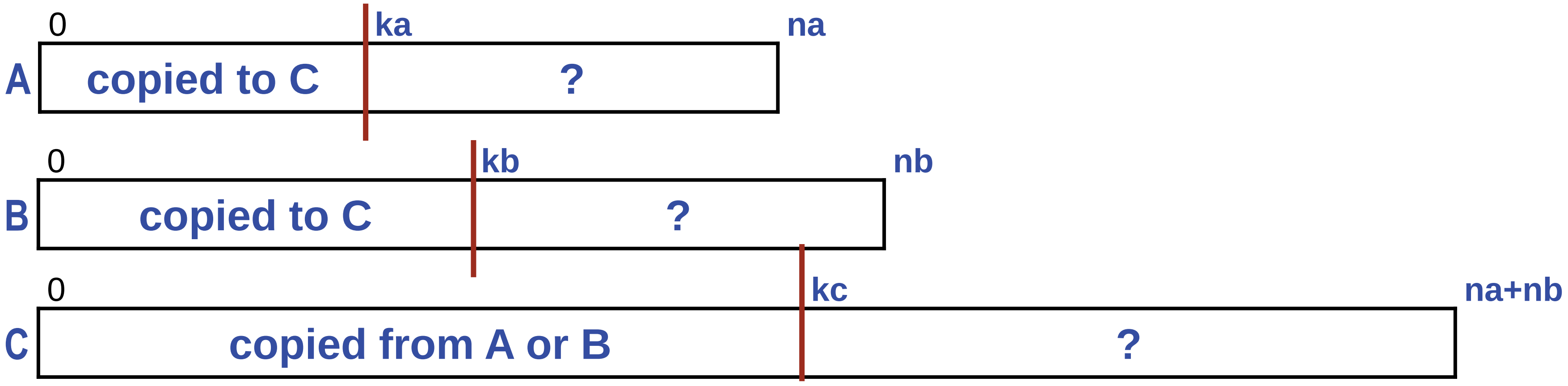


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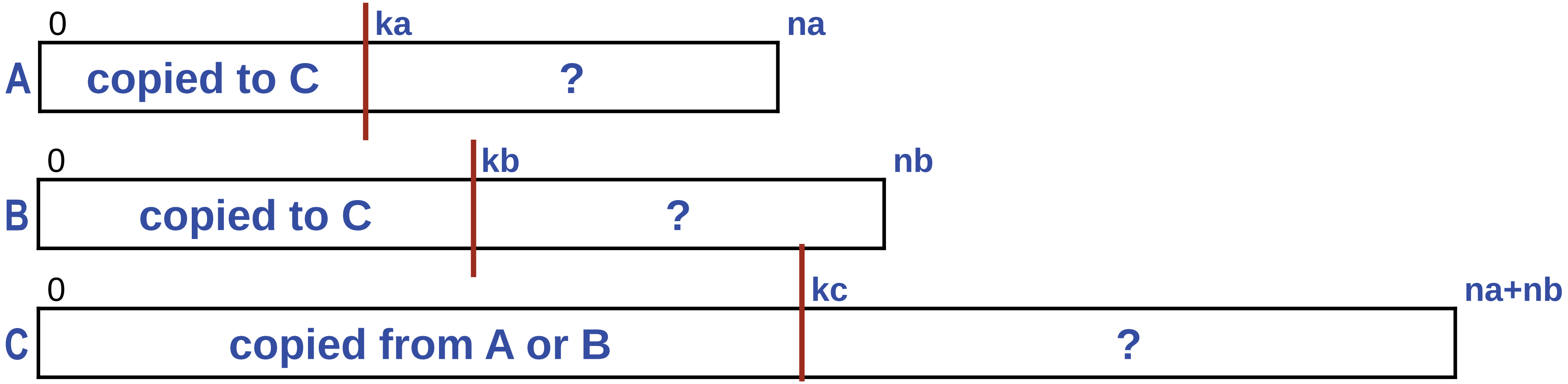


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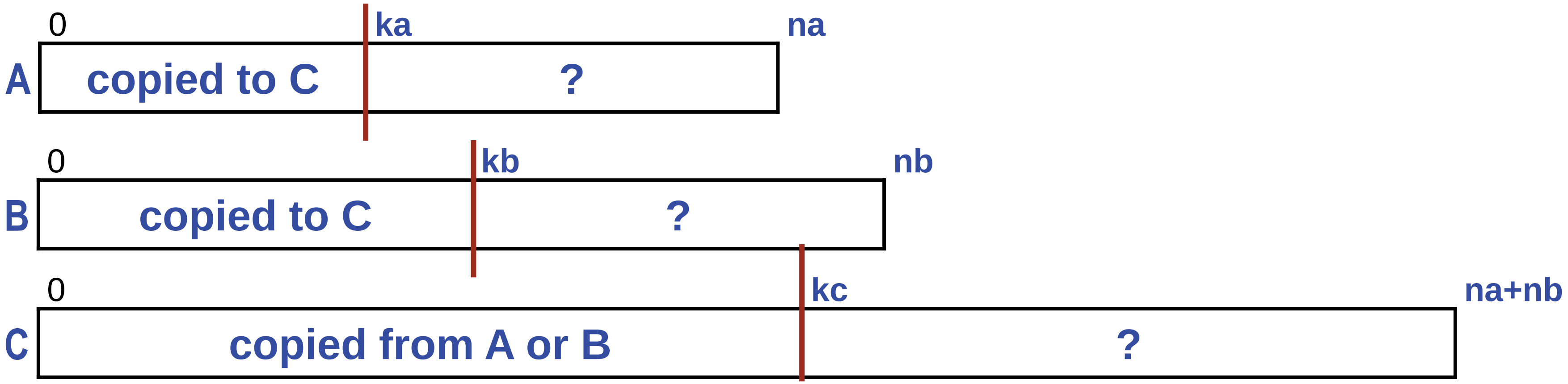


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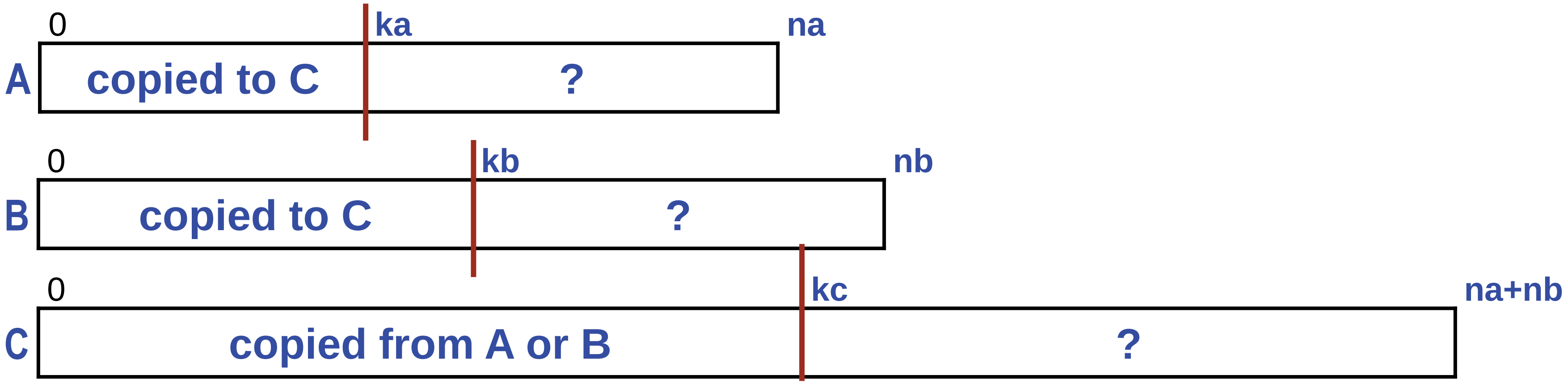


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